

Comp 311 - Review 1

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This review sheet provides a few exercises. You do not need to hand them in and will not get points for your work. You are encouraged to do the exercises to make sure you understand the material. This material will be tested on the first exam.

1 LC

Inductive definition for LC:

Exp	$::=$	Var		variable
		Num		number (constant)
		$(\text{lambda } (Var) Exp)$		lambda (function)
		$(Exp Exp)$		application
Var	$::=$	$Sym \setminus \{\text{lambda}\}$		all symbols except for lambda

Examples of strings that are in the language defined above:

- 1
- a
- $(\text{lambda } (x) 15)$
- $(\text{lambda } (x) y)$
- $((\text{lambda } (x) x) 3)$
- $(5 7)$

Note that some of these programs will generate run-time errors when executed. 2. has free occurrences (a and y , respectively), and 6. is an application of a number to a number.

2 Bound, Binding, and Free Occurrences

A variable can occur in an expression either as "bound", "binding", or "free". Consider the following LC expression:

$$\begin{array}{cccc} ((\text{lambda } (& x &) & x &) & y) \\ & \uparrow & & \uparrow & \uparrow \\ & 1. & & 2. & 3. \end{array}$$

1. is a binding occurrence of x .
2. is a bound occurrence of x . It is bound by occurrence 1.
3. is a free occurrence of y .

2.1 Exercise: Draw an arrow from each bound occurrence to its binding occurrence. Circle all free occurrences.

1. x
2. $(\text{lambda } (x) x)$
3. $((\text{lambda } (a) 2) a)$
4. $((\text{lambda } (n) n) 3)$
5. $(\text{lambda } (z) ((\text{lambda } (y) ((\text{lambda } (x) x) y)) z))$
6. $(\text{lambda } (t) ((\text{lambda } (t) ((\text{lambda } (t) t) t)) t)) t$

Solutions in ¹

3 Nameless Representation

The last of the LC expressions above is an interesting example. There are several binding occurrences, but all of them use the variable t . This suggests that names for a variable are not necessary as long as we know what binding occurrence a bound occurrence refers to. This idea gives rise to static distance coordinates and de Bruijn notation.

In the LC expression

$$\begin{array}{cccccc}
 (\text{lambda } (& x &) (\text{lambda } (& t &) ((\text{lambda } (& t &) & t &) & x &))) \\
 & \uparrow & & \uparrow & & \uparrow & \uparrow & \uparrow & & \uparrow \\
 & 1. & & 2. & & 3. & 4. & 5. & &
 \end{array}$$

1. is a binding occurrence of x .
2. is a binding occurrence of t .
3. is a binding occurrence of a different t .
4. is a bound occurrence of t . It is bound by occurrence 3.
5. is a bound occurrence of x . It is bound by occurrence 1.

Consider the notion of scope from class: The scope of a variable is the textual region where the variable may occur free. In the expression above, 4. is bound by occurrence 3., the binding occurrence that creates the innermost scope. 5., however, is bound by occurrence 1., the binding occurrence that creates the outermost scope. Nothing is bound by the middle scope that is created by the binding occurrence 2.

To reach the binding occurrence for 4., we consider the 1st scope. For the binding occurrence for 5., we do not consider the 1st scope, but the 2nd scope. We can therefore drop variable names and use numeric representations for "1st scope outwards" and "2nd scope outwards":

$$(\text{lambda } (\text{lambda } ((\text{lambda } \#1) \#2)))$$

We use the form $\#Number$ to avoid confusing static distance coordinates (e.g. $\#1, \#2, \dots$) with number constants (e.g. $0, 1, \dots$).

3.1 Exercise: Convert to static distance form.

1. $(\lambda x) x$
2. $((\lambda n) n) 3$
3. $(\lambda z) ((\lambda y) ((\lambda x) x) y) z$
4. $(\lambda t) ((\lambda t) ((\lambda t) t) t) t$
5. $(\lambda r) (\lambda s) ((\lambda t) (r (r (r t)))) r$

Solutions in ²

Notes

¹Solutions to Exercise 2.1:

1. x free.
2. First x binding occurrence, second x bound occurrence bound by first x .
3. First a binding occurrence, second a free occurrence.
4. First n binding occurrence, second n bound occurrence bound by first n .
5. First z binding occurrence, first y binding occurrence, first x binding occurrence. Second x bound occurrence bound by the first x , second y bound occurrence bound by the first y , second z bound occurrence bound by the first z .
6. First t binding occurrence, second t binding occurrence, third t binding occurrence. Fourth t bound occurrence bound by the third t , fifth t bound occurrence bound by the second t , sixth t bound occurrence bound by the first t . Seventh t free.

²Solutions to Exercise 3.1:

1. $(\lambda \#1)$
2. $((\lambda \#1) 3)$
3. $(\lambda (\lambda (\lambda (\lambda \#1) \#1)) \#1)$
4. $(\lambda (\lambda (\lambda (\lambda \#1) \#1)) \#1)$
5. $(\lambda (\lambda (\lambda (\lambda (\lambda \#3) (\#3 (\#3 \#1)))) \#2))$