Parallel Depth-first (PDF) Scheduling

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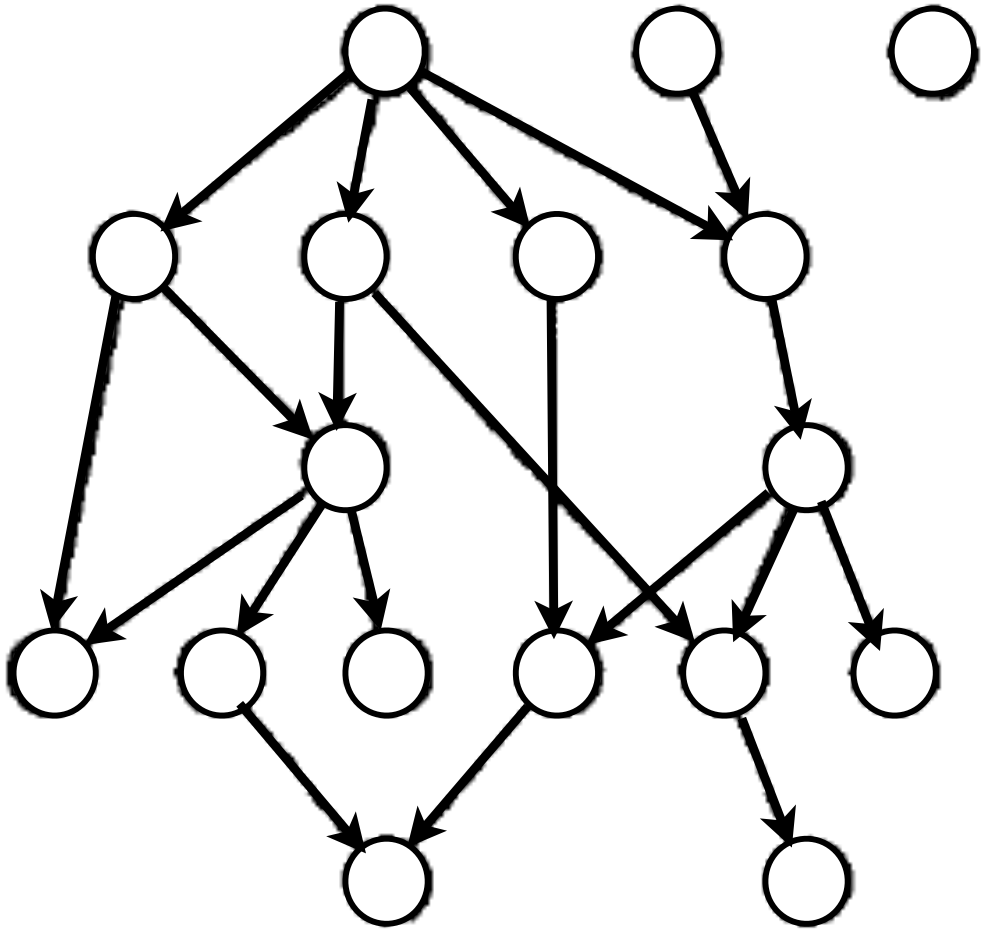
Computation DAGs and Scheduling

DAG Model for Computations



DAG Scheduling

- Map a DAG computation onto P threads
- Determine what executes on each thread at each time step
- A schedule must obey the following constraints
  - —each node appears only once in a schedule
  - —all predecessors of a node must execute before a node executes
    - node is ready at time step t if its ancestors executed at times ≤ t - 1
  - —any ready node may be scheduled
- Valid schedules must exist since computation graph is a DAG

Dynamic DAGs and Online Scheduling

A DAG is incrementally revealed as execution proceeds

- **When a node is scheduled, its outgoing edges are revealed**
- **When all its incoming edges are revealed, a node is revealed and available for scheduling**
- **Online scheduling: based only on revealed nodes**

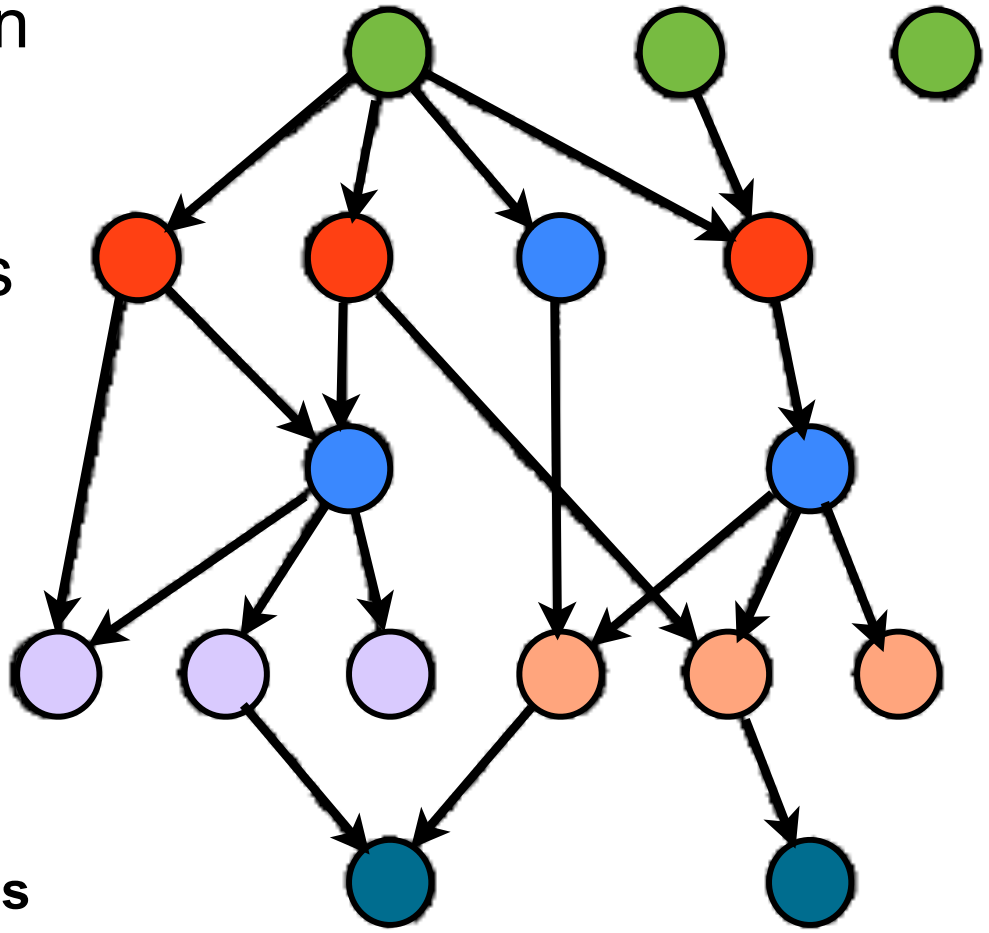
Greedy Scheduling

- **Definition: At each step in an execution ...**
  - **if P tasks are ready, then P execute**
  - **if fewer than P tasks are ready, all execute**
- **Theorem**
  - **for any multithreaded computation with work T_1 and DAG depth T_∞, for any number P of processors, any greedy schedule achieves $T_P \leq T_1/P + T_\infty$**
- **Commentary**
  - **generally, want linear speedup, i.e., $T_P = O(T_1/P)$**
  - **greedy schedules achieve linear speedup when avg. parallelism, defined as $T_1/T_\infty = \Omega(P)$; namely $T_1/T_\infty \geq cP$, or $T_1/P \geq cT_\infty$**
    - **if # processors is ≤ average parallelism, there will be enough work to keep the processors adequately busy for good speedup**
    - **too many processors leads to idleness, which degrades speedup**

A Greedy Schedule on 3 Processors

Schedule constraints

- Each node appears once in the schedule
- Each node is scheduled after all of its predecessors
- Each step consists of at most P nodes



$T_1 = 17$, $T_\infty = 5$

Parallel execution time $T_P = 6$ steps

$T_1/P + T_\infty = 17/3 + 5 = 10.66 \geq T_P = 6$

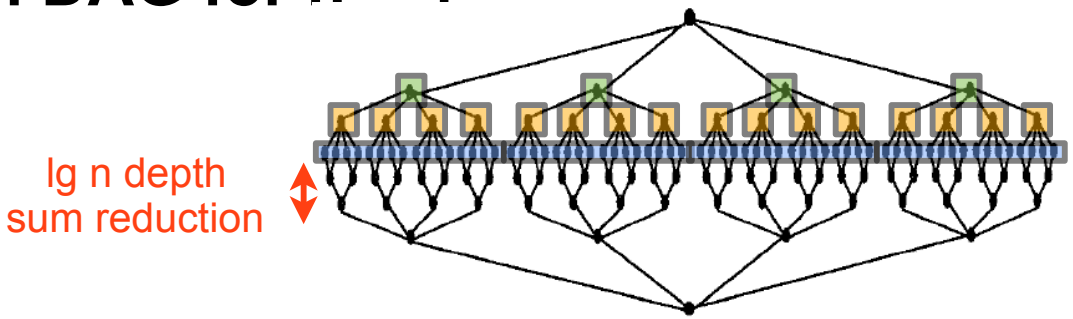
The Importance of Schedule Choice

Scheduling and space

- **Consider matrix multiplication with fine-grain parallelism**

```
In parallel for i from 1 to n
    In parallel for j from 1 to n
        r[i,j] = TreeSum(In parallel for k from 1 to n
                             a[i,k]*b[k,j])
```

- **Computation DAG for n = 4**



- **Sequential schedule uses only $O(n^2)$ space**
- **Level-by-level parallel schedule would use $O(n^3)$ space**

What Schedules are Best?

- Greedy parallel schedules deliver good speedup
  - —good bound on the number of steps
- Which greedy schedules (if any) have good bounds on space?
  - —memory and cache

Provably Efficient Scheduling for Languages with Fine-grain Parallelism

Effectively Sharing a Cache Among Threads

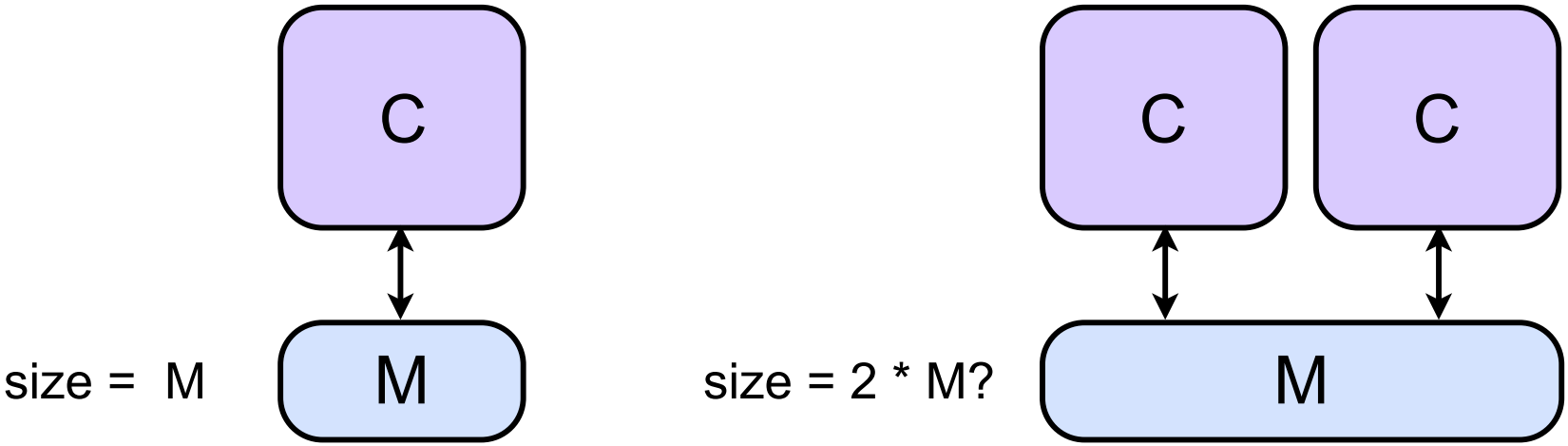
Space-efficient Scheduling of Nested Parallelism

NESL DAG Schedules

- Task structure is a dynamically unfolding series-parallel DAG
- Certain nodes can have arbitrary fan out or fan in
  - —fork or join nodes respectively
  - —more general than that of Blumofe and Leiserson
    - fanout 2

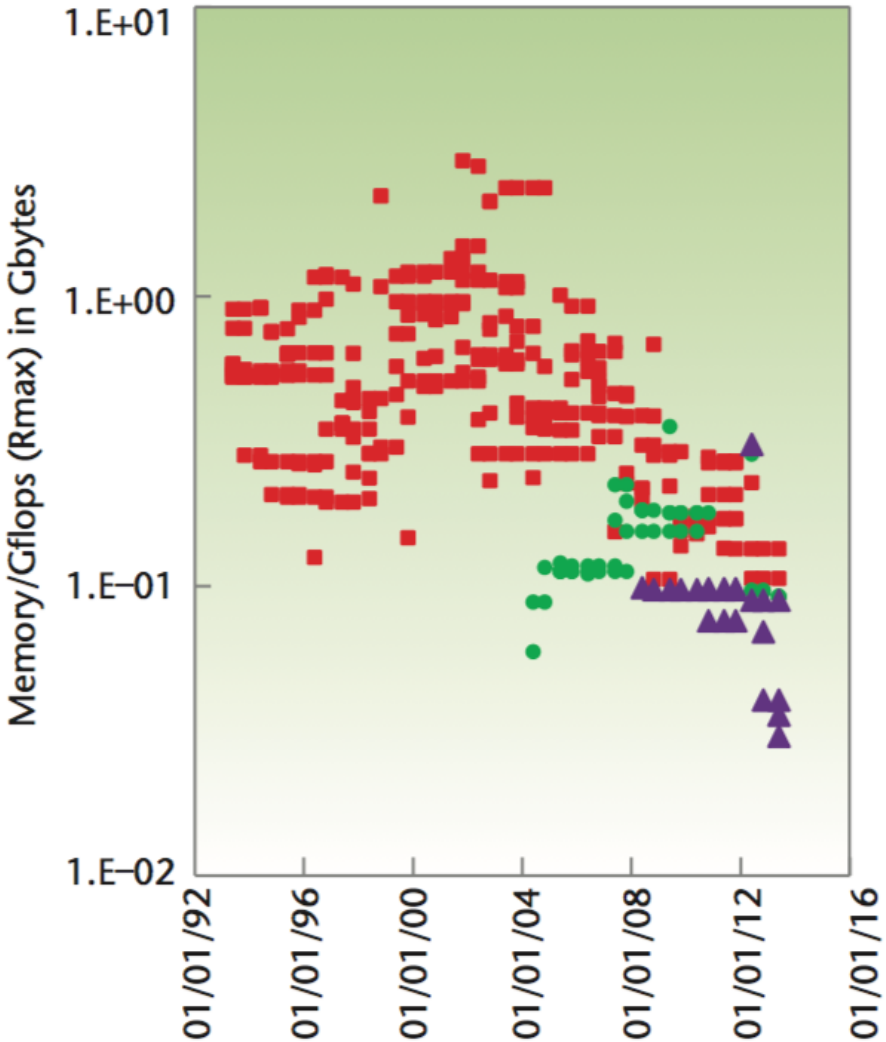
Motivation: Memory Footprint

- **Single core system uses memory of size M**
- **For a multicore with P cores, do we need memory of size PM?**



Why Does Memory Footprint Matter?

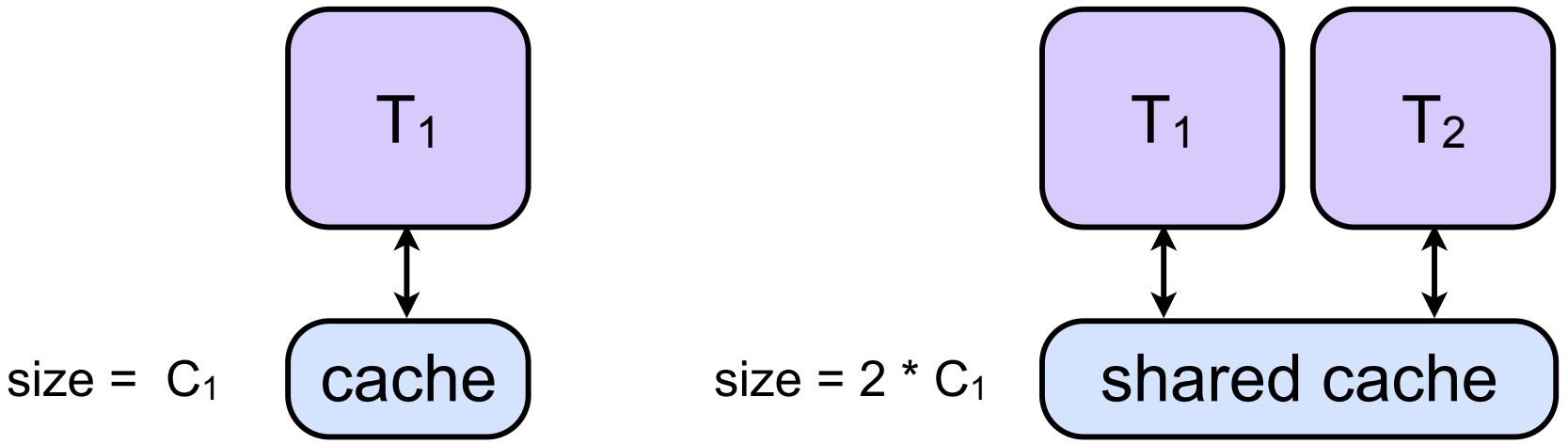
Decrease in relative memory capacity per FLOP/second



Peter Kogge and John Shalf. 2013. Exascale Computing Trends: Adjusting to the "New Normal"' for Computer Architecture. *Computing in Science and Engineering.* 15, 6 (November 2013), 16-26.

What about Cache Footprint?

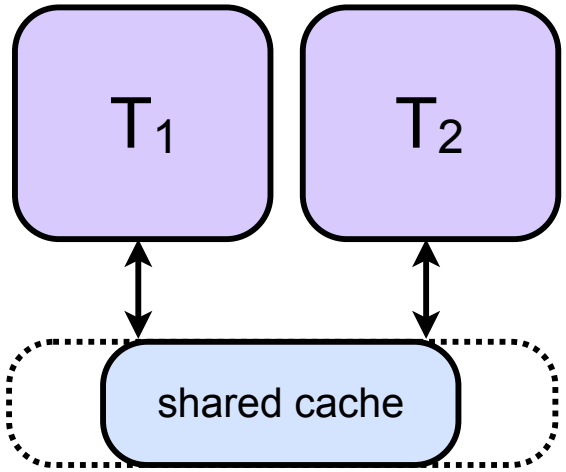
- **Single core system uses cache of size C_1**
- **For a core with T HW threads, do we need cache of size TC_1 ?**



Problem Formulation for Cache

- **Consider a parallel execution of a computation DAG**
  - **—on a multithreaded core with T threads and a shared ideal cache**
- **How much extra cache is needed for same hit rate as 1 thread?**

size = ?



Outline

- Efficient parallel DAG schedules
- Cache miss rate bounds
- Practical issues when implementing PDF schedules
- Value of PDF schedules
- Space-efficient scheduling of nested parallelism
- Summary

Toward Space-efficient Parallel Schedules

- Greedy parallel schedules evaluate nodes out of order with respect to a sequential execution
- Poor choice of schedule may require excessive space
  - —see the parallel matrix multiply example
- Three key ideas for space-efficient scheduling
  - —define a class of parallel schedules based on sequential ones
  - —focus on nodes scheduled "prematurely" ahead of their position in normal sequential order
  - —bound number of premature nodes to establish bounds on space

DAG Scheduling Strategies

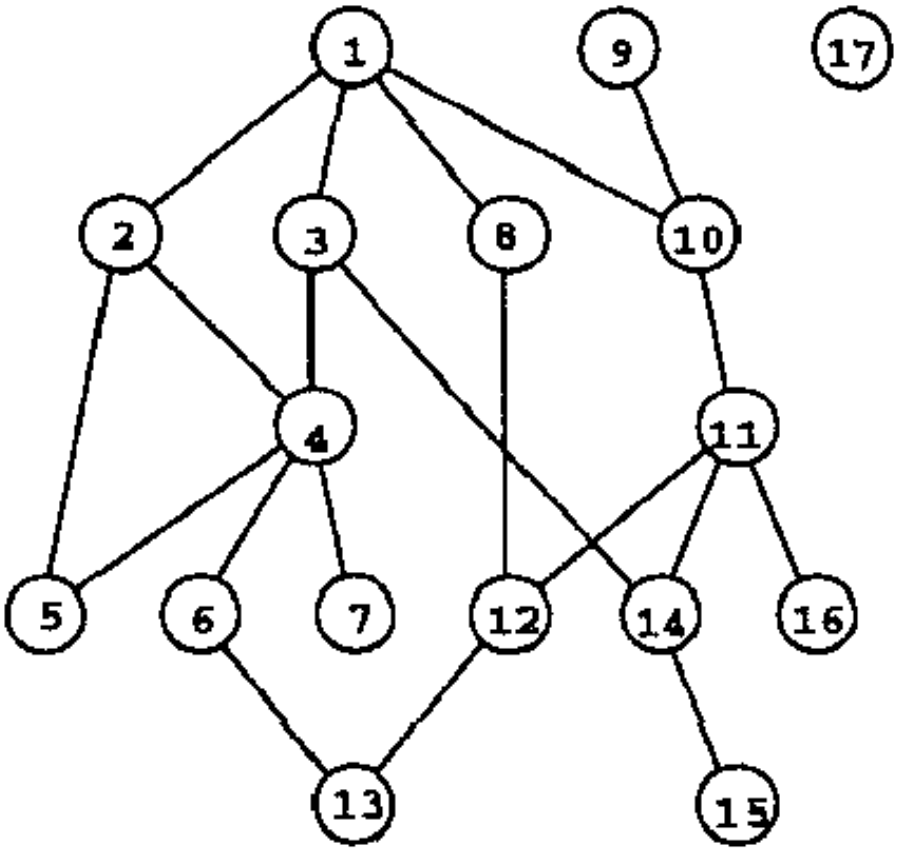
- 1DF schedule
  - —schedule one ready node per time step
  - —schedule in depth first order
    - simple using stack of ready nodes
- PDF schedule
  - —schedule up to P ready nodes per time step
  - —bias towards the 1DF ordering
    - if u precedes v in 1DF schedule, both are scheduled, neither are scheduled, or only u is scheduled

1DF Schedule

```
for each v in G (in any order)
    if v is ready (i.e., indegree == 0)
        push(v, stack)
while (v = pop(stack) != null)
    if v has not already been scheduled
        schedule(v)
        for each ready child u of v
            push(u, stack)
```

DAG Scheduling

1 DF Schedule



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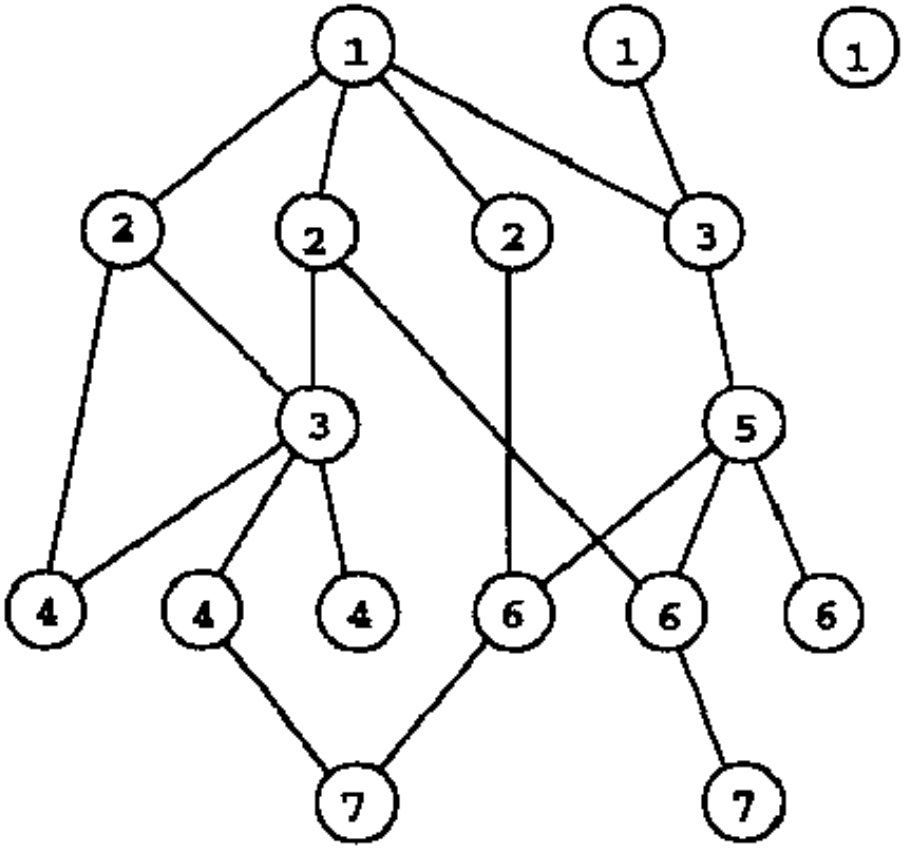

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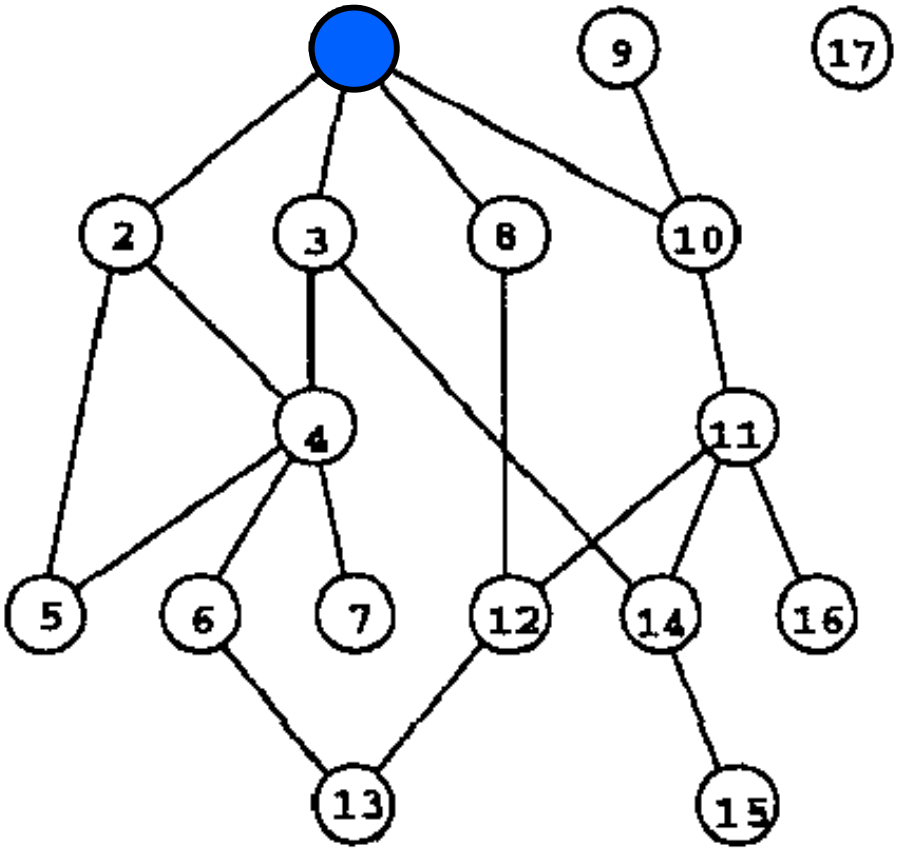

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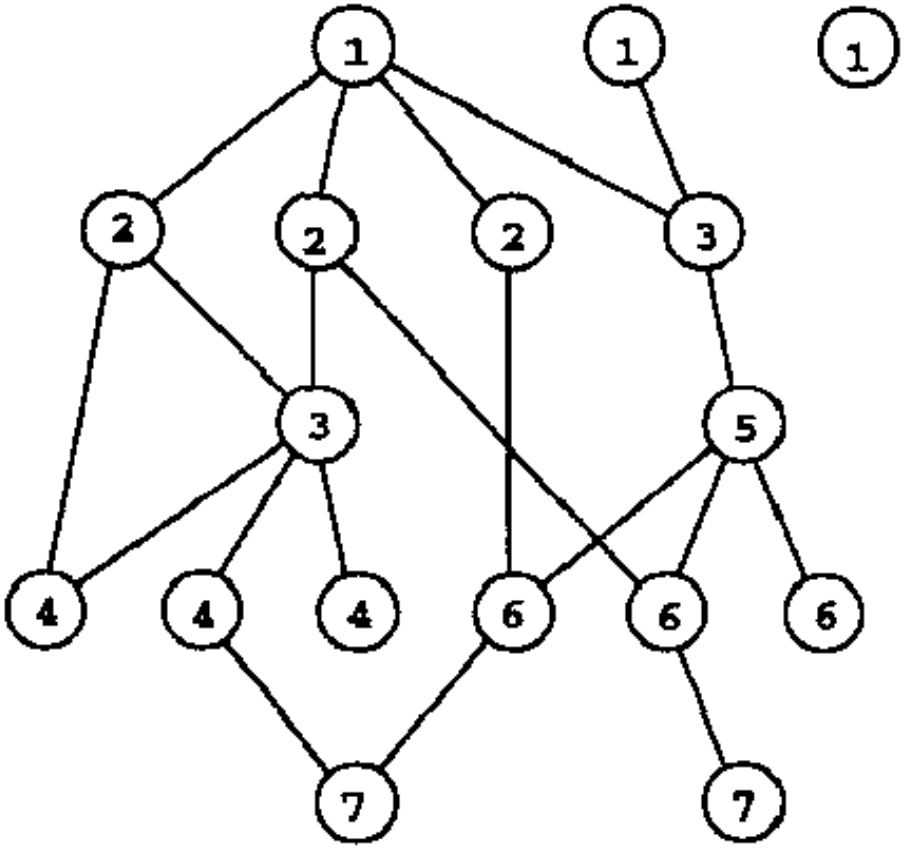

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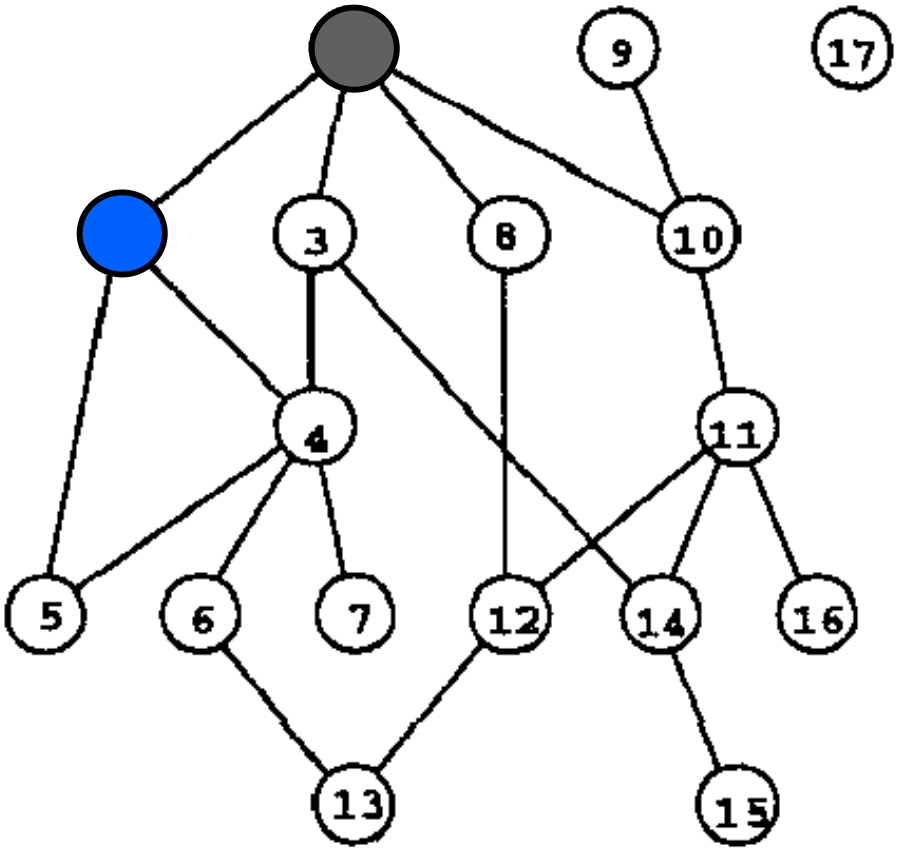

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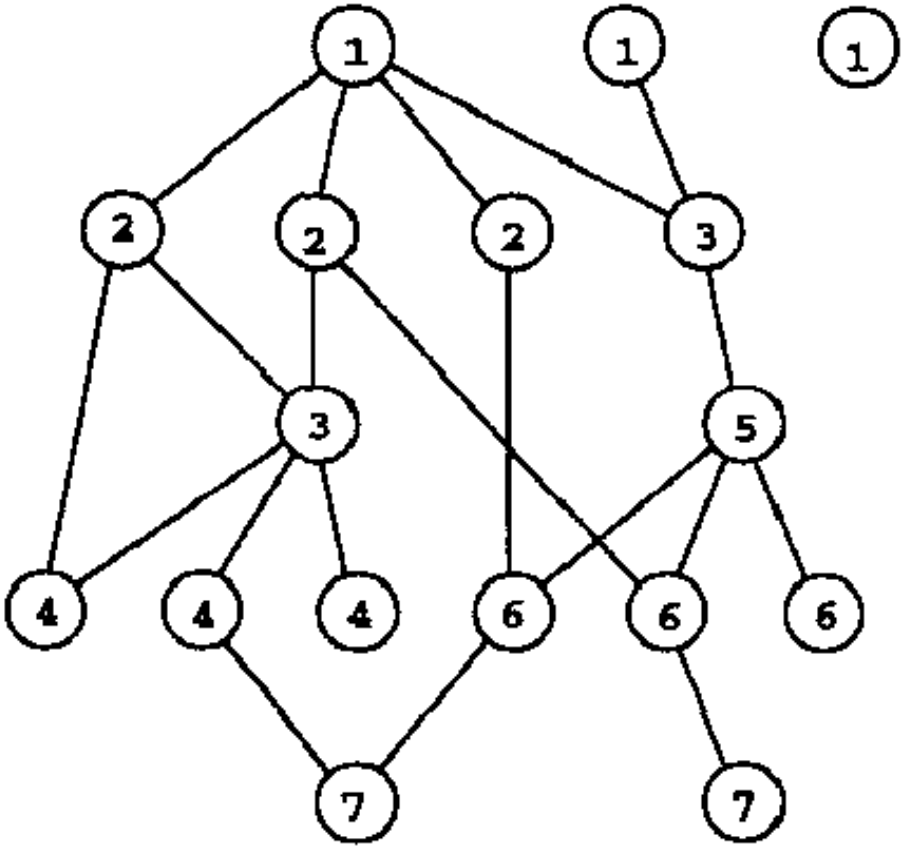

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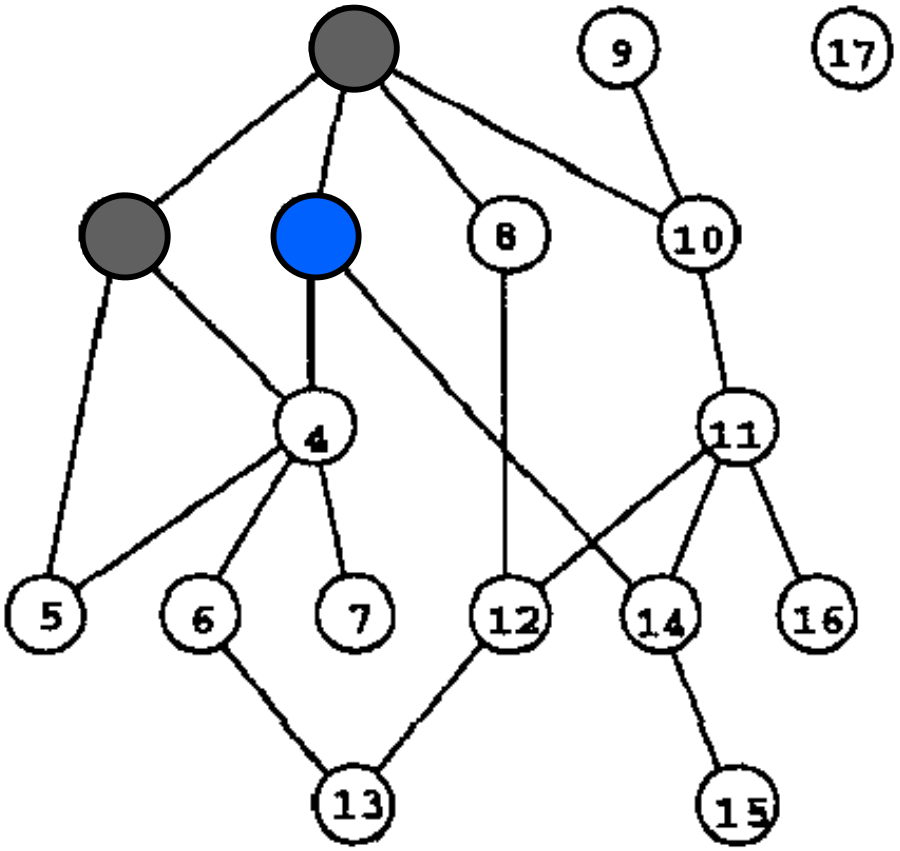

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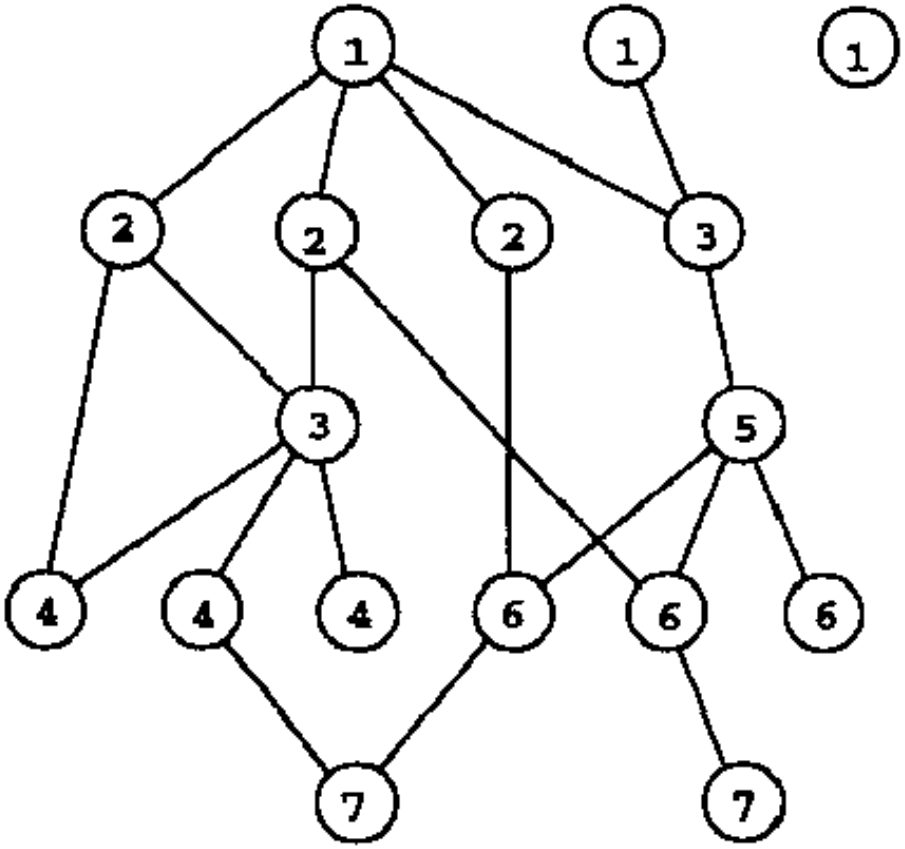

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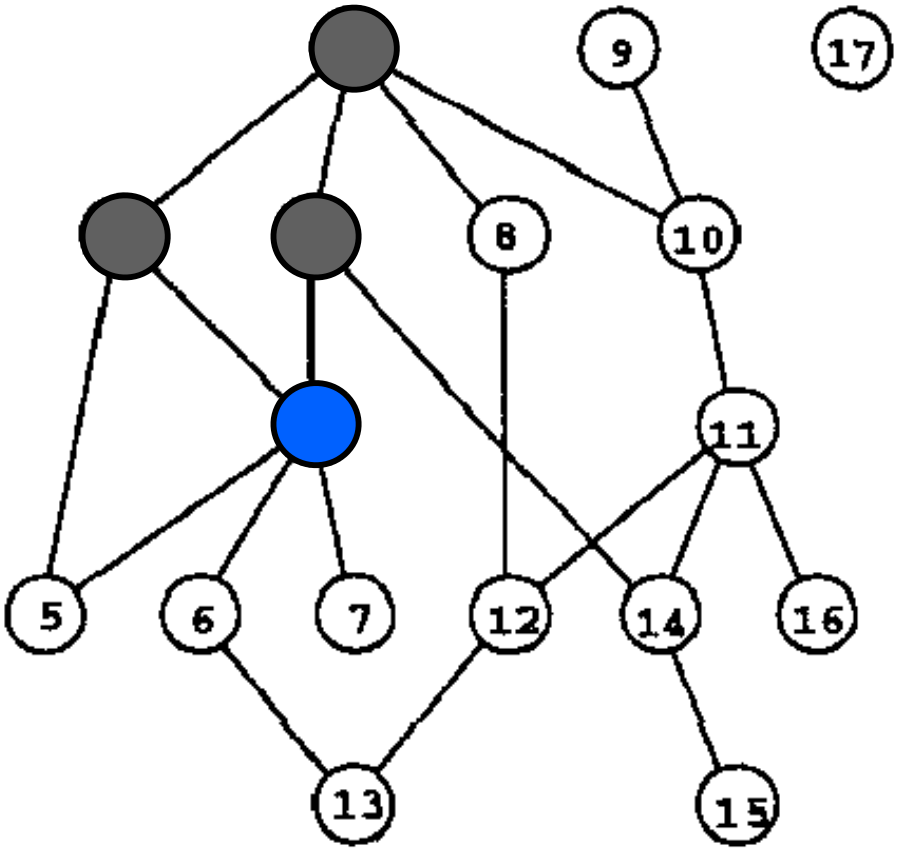

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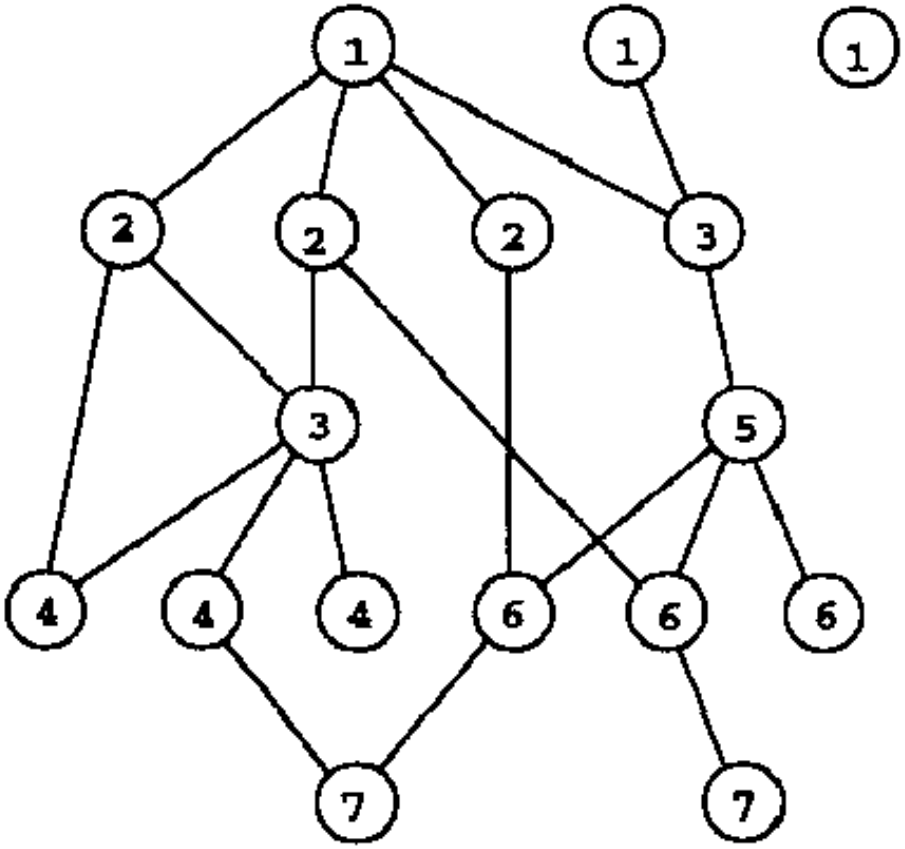

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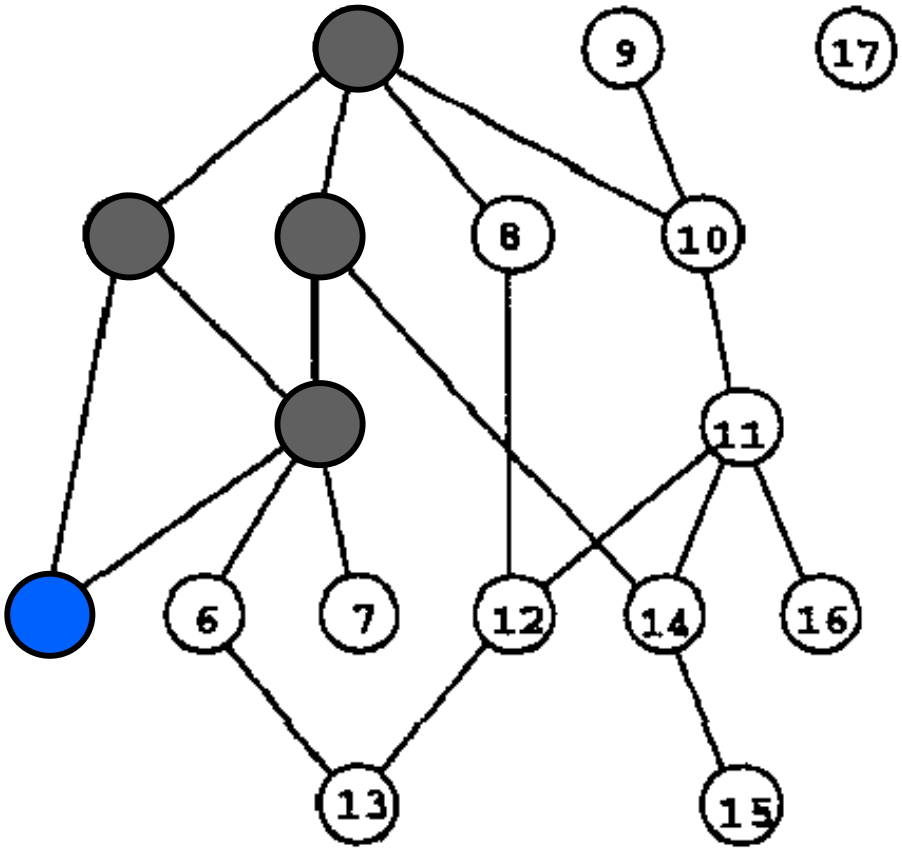

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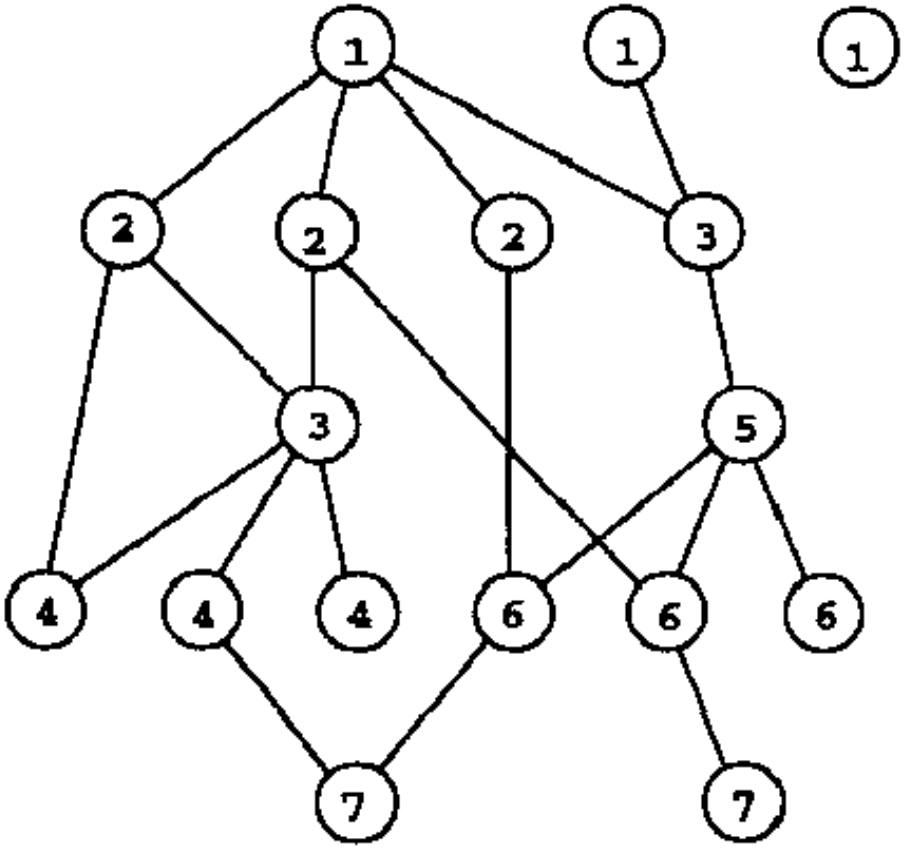

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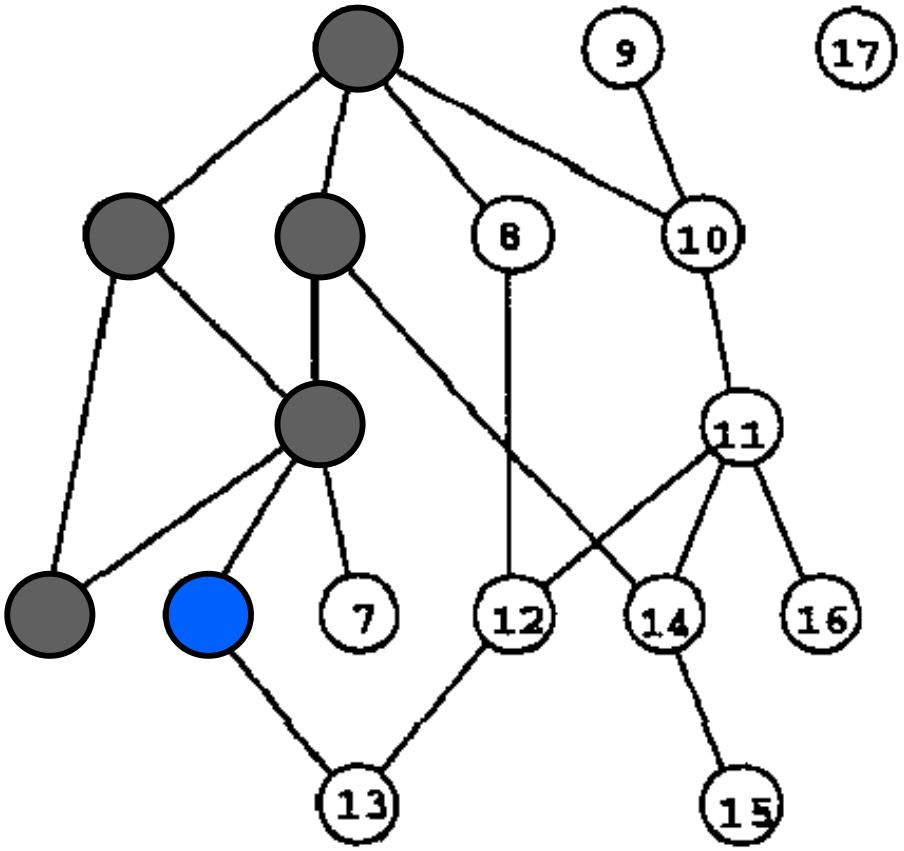

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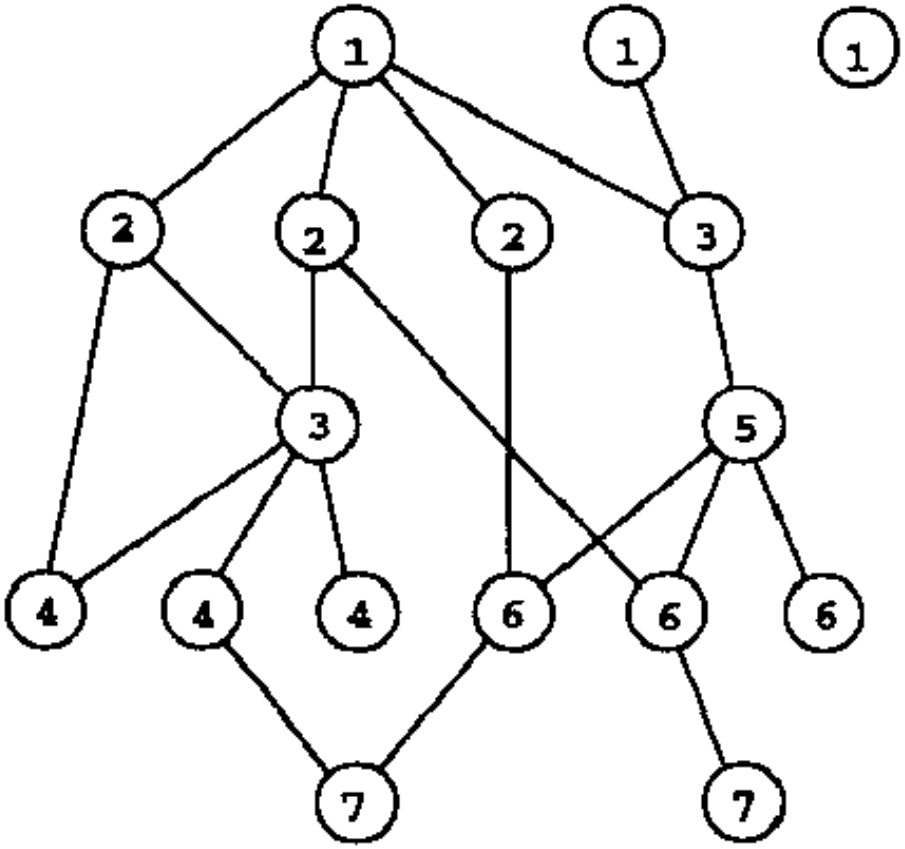

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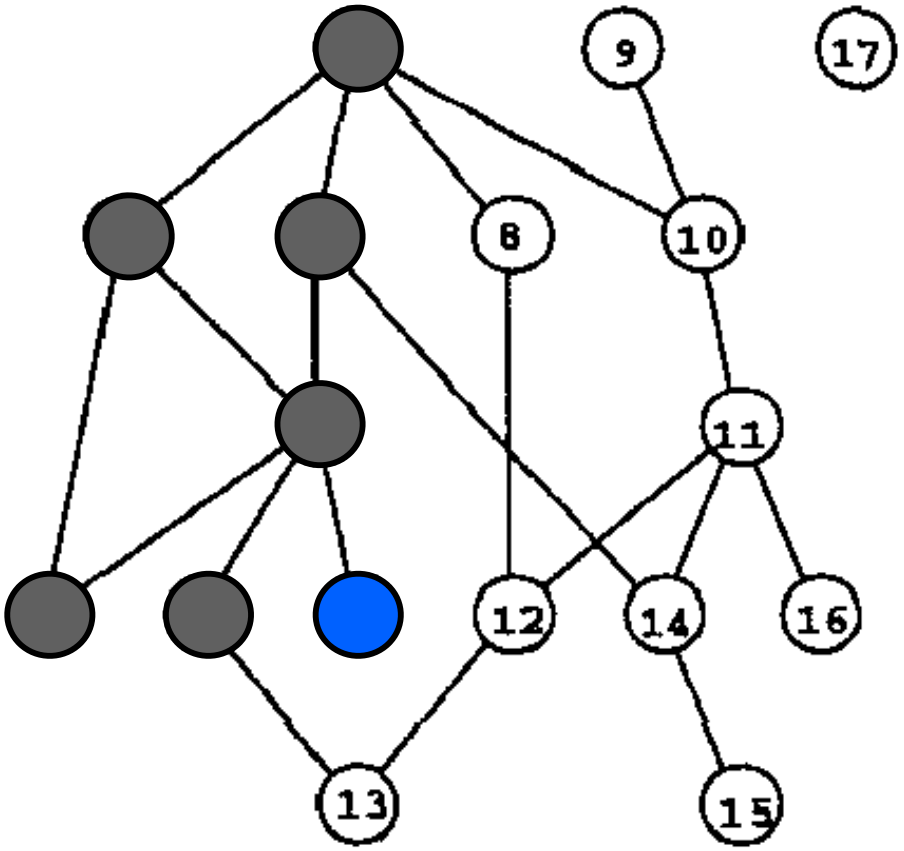

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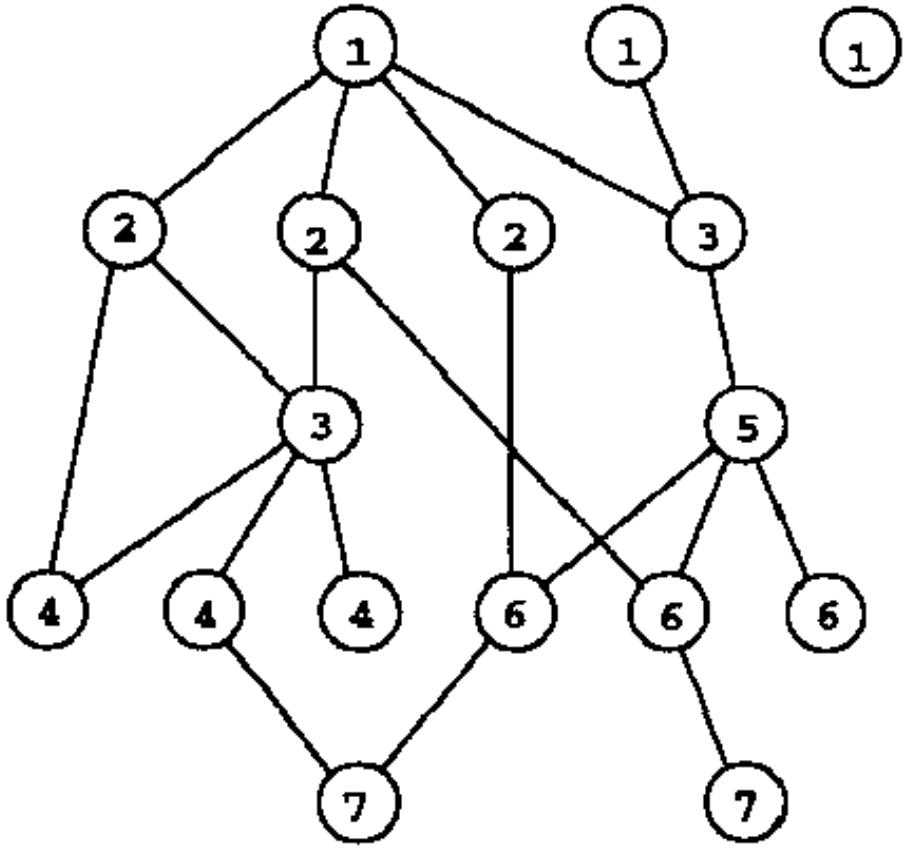

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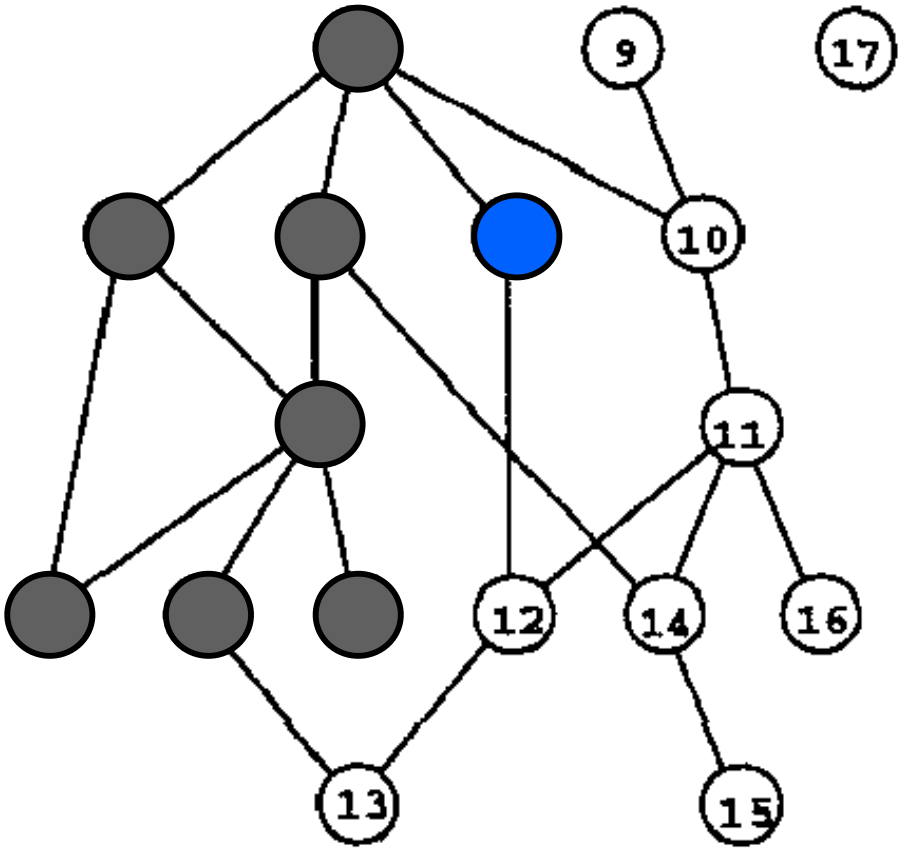

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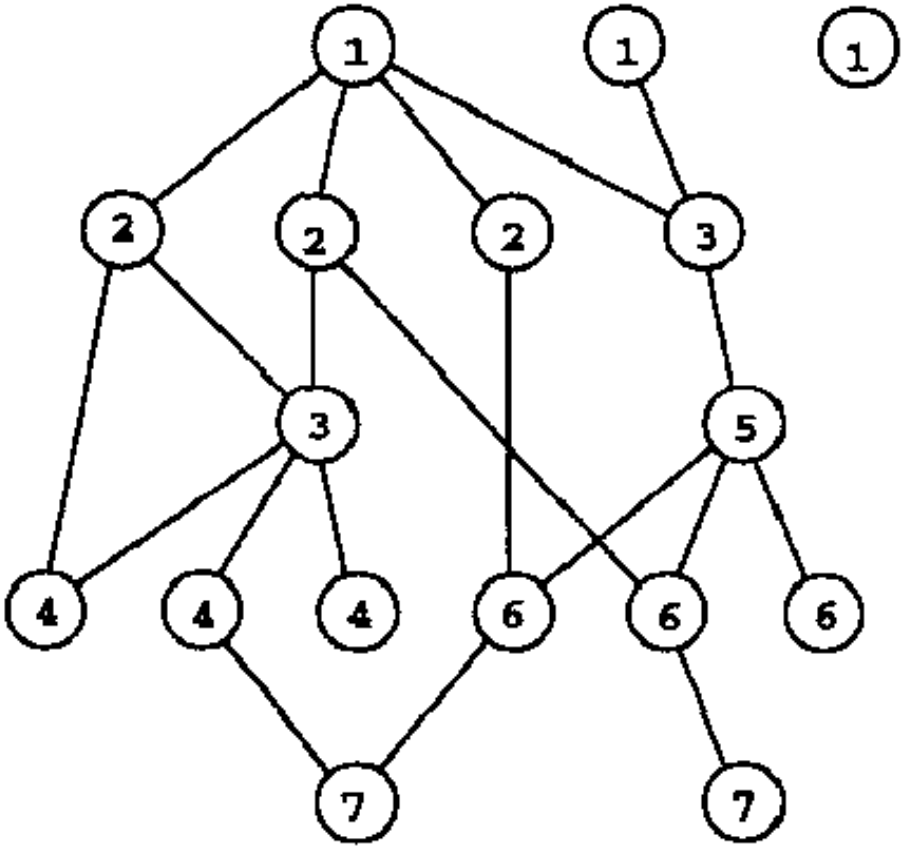

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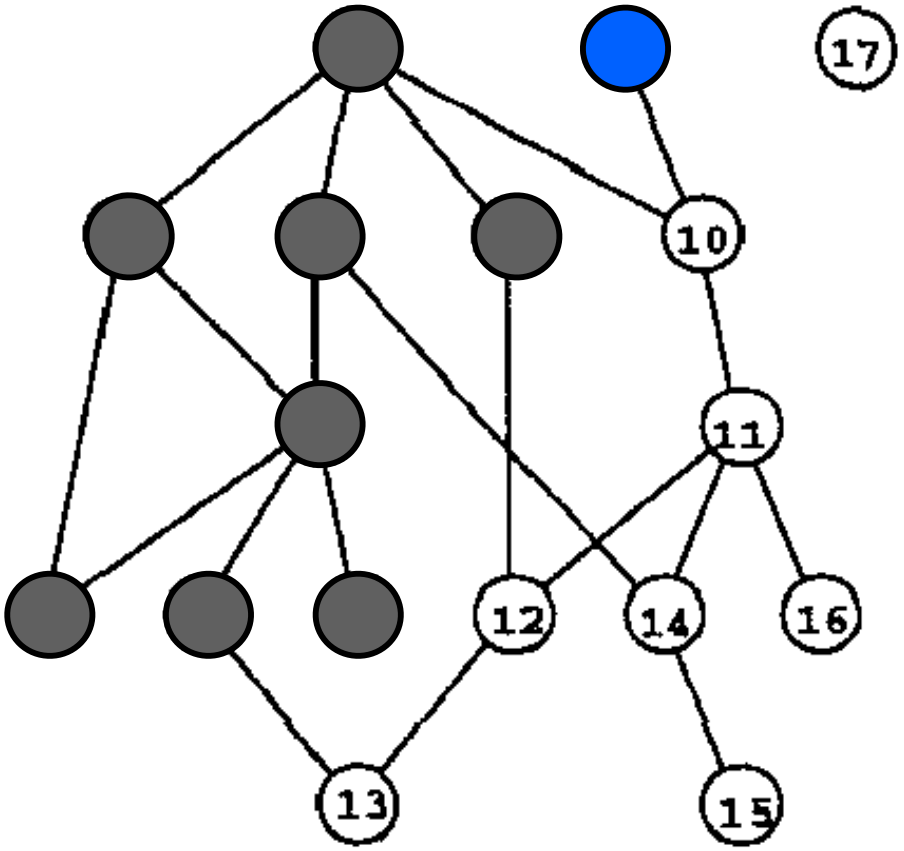

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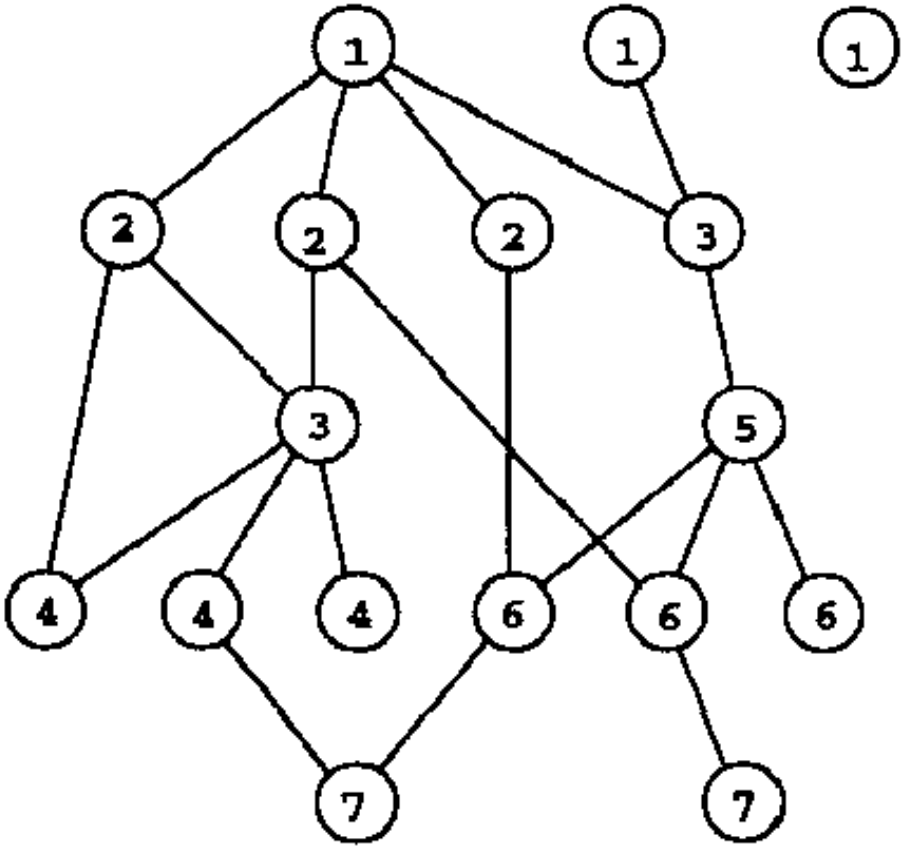

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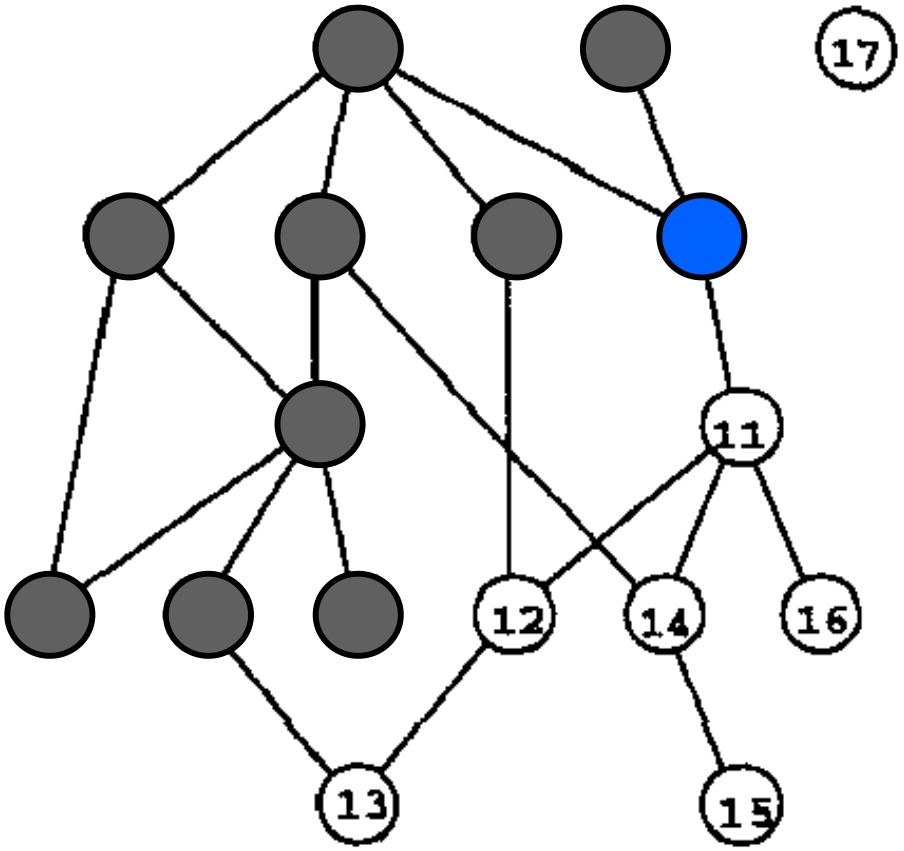

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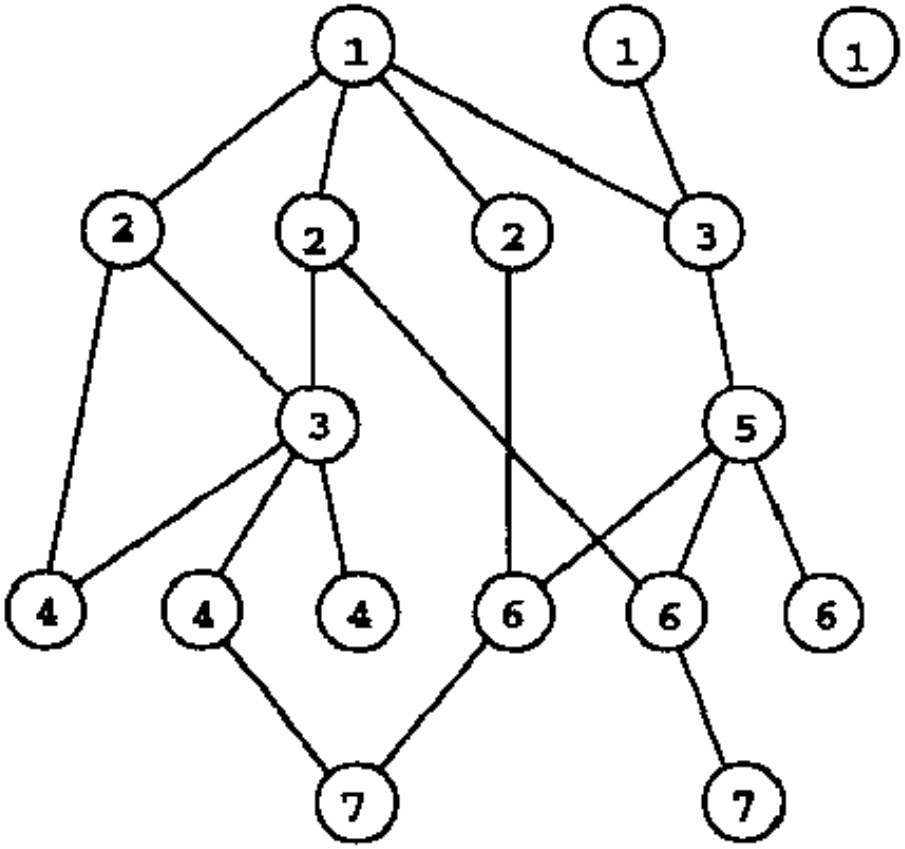

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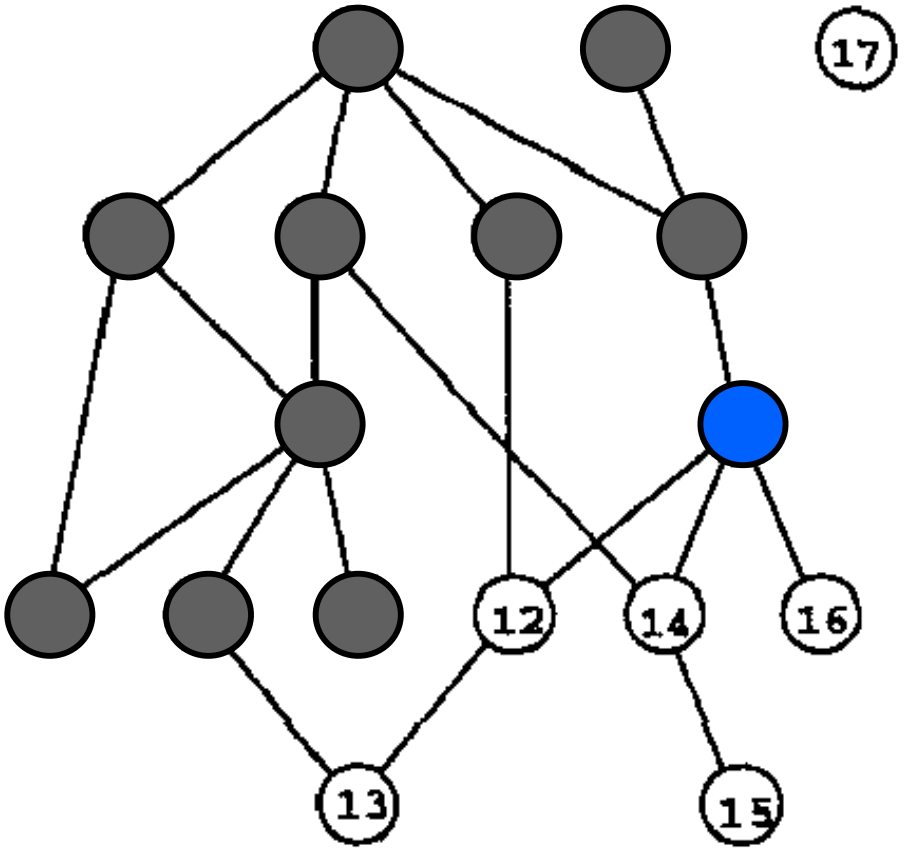

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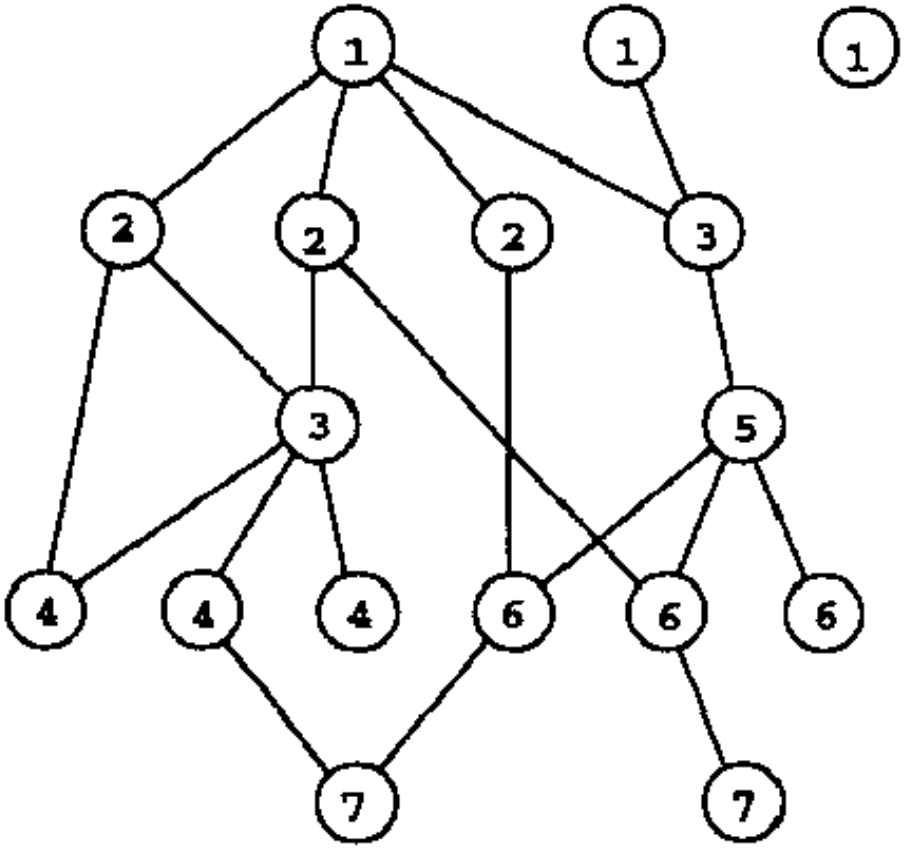

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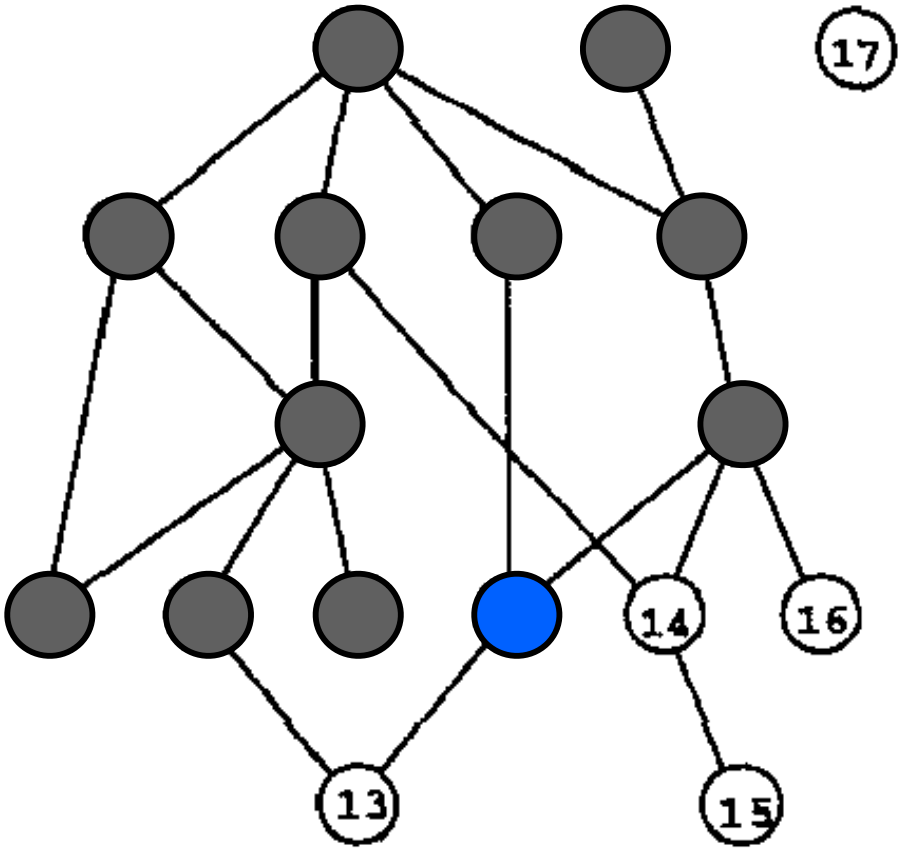

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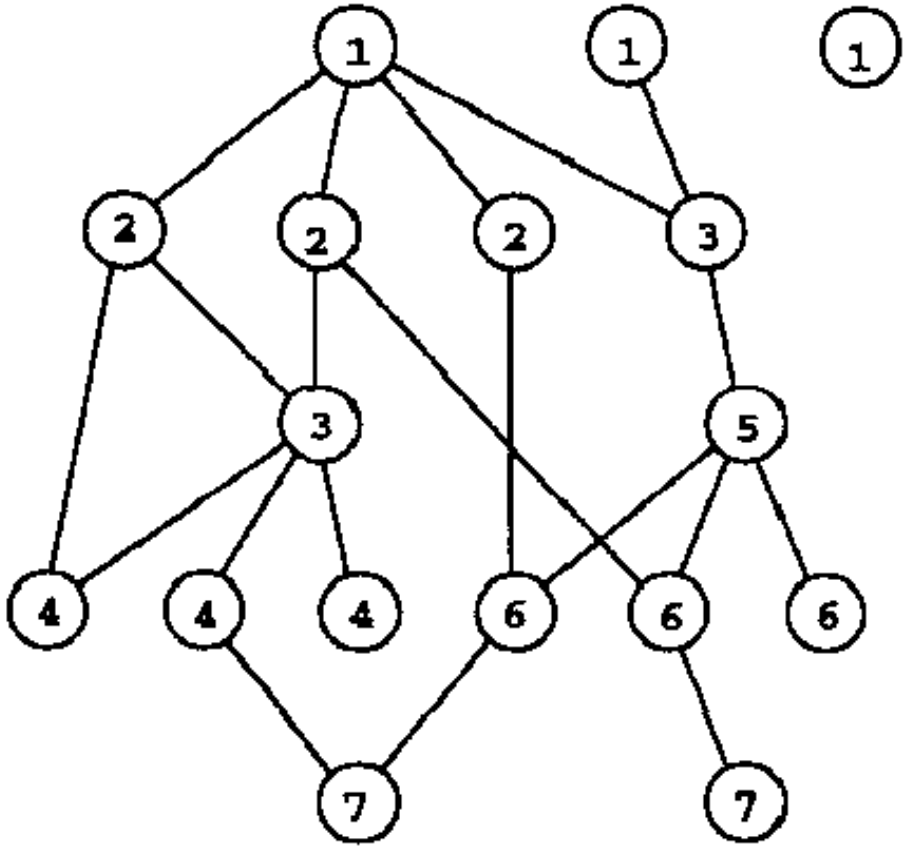

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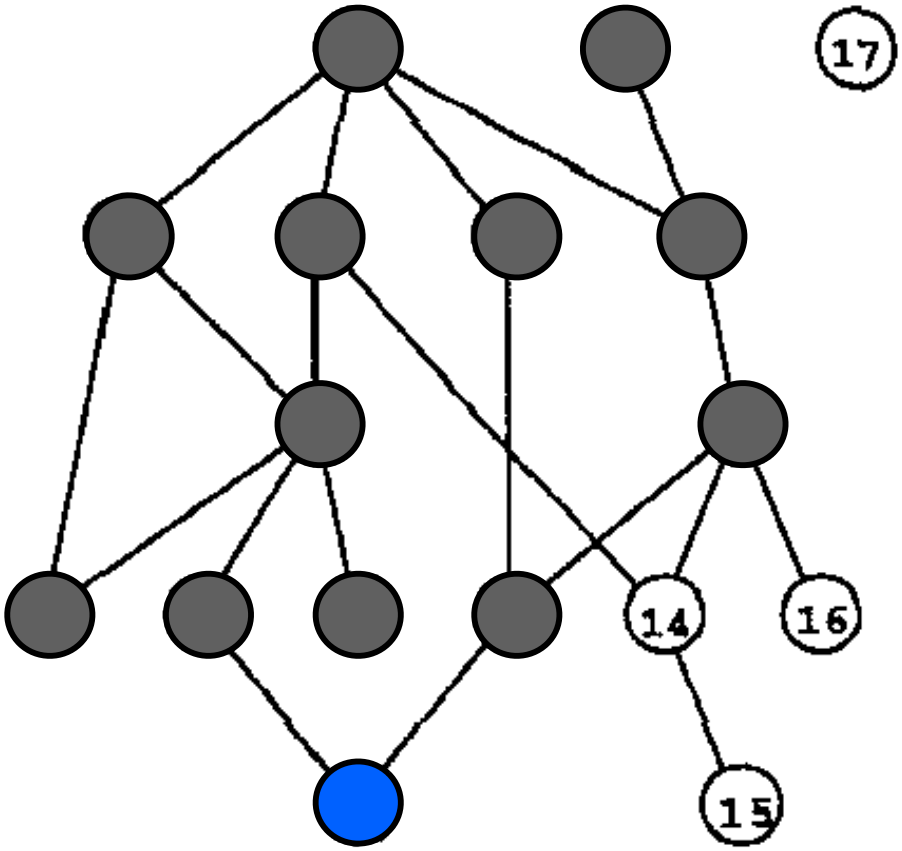

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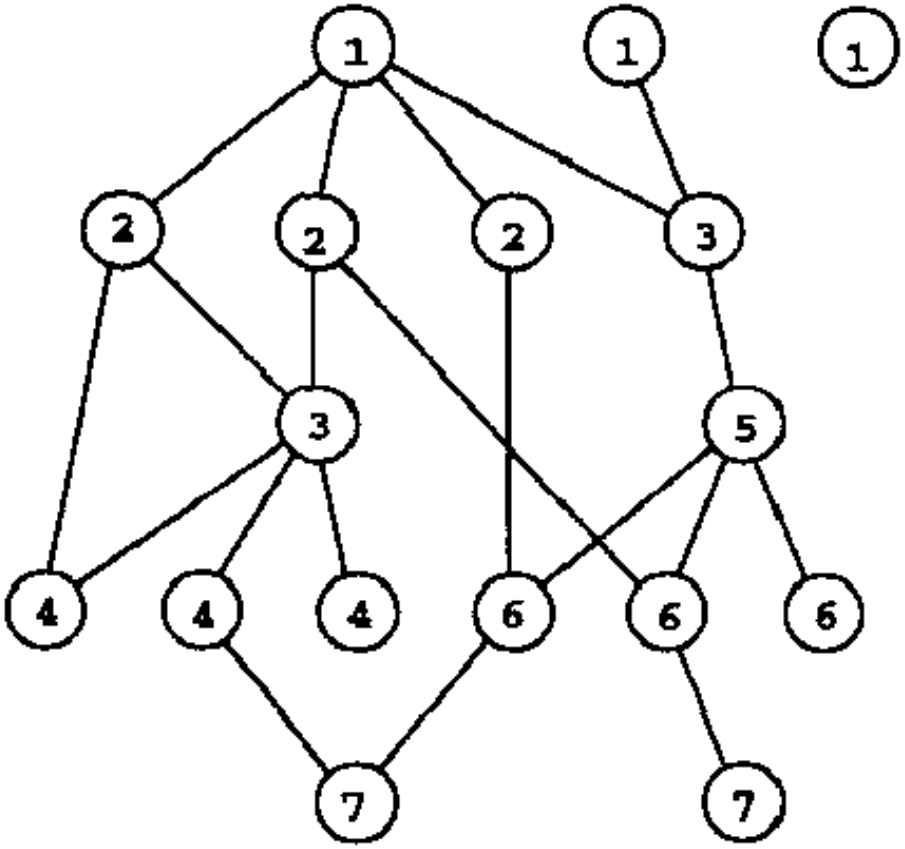

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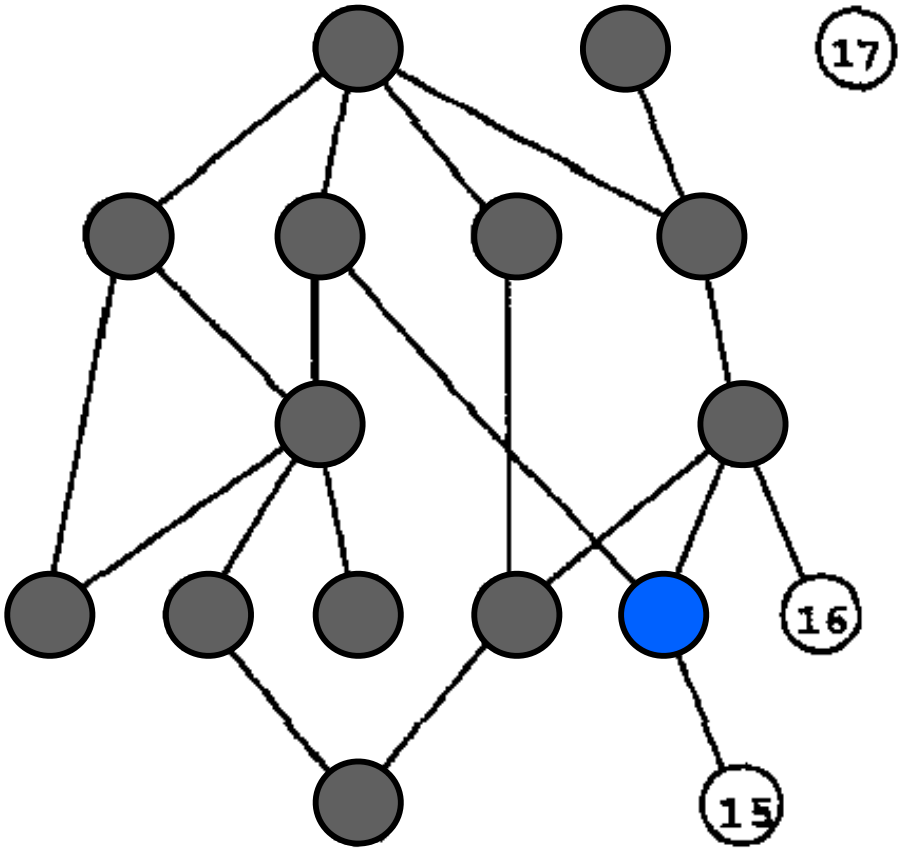

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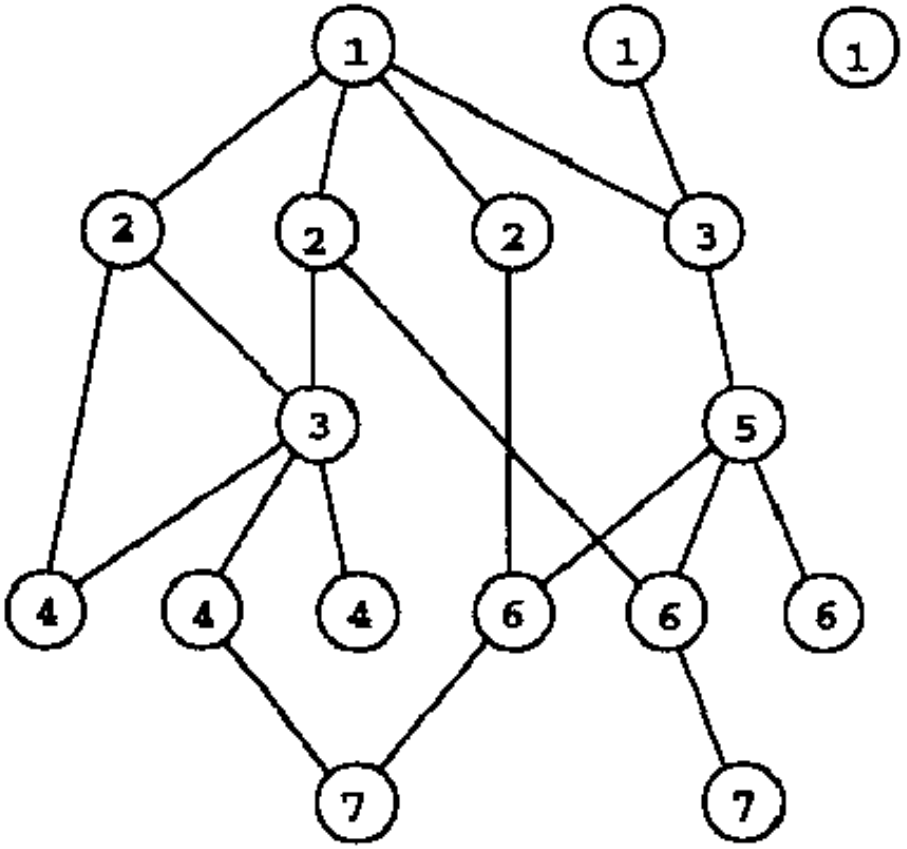

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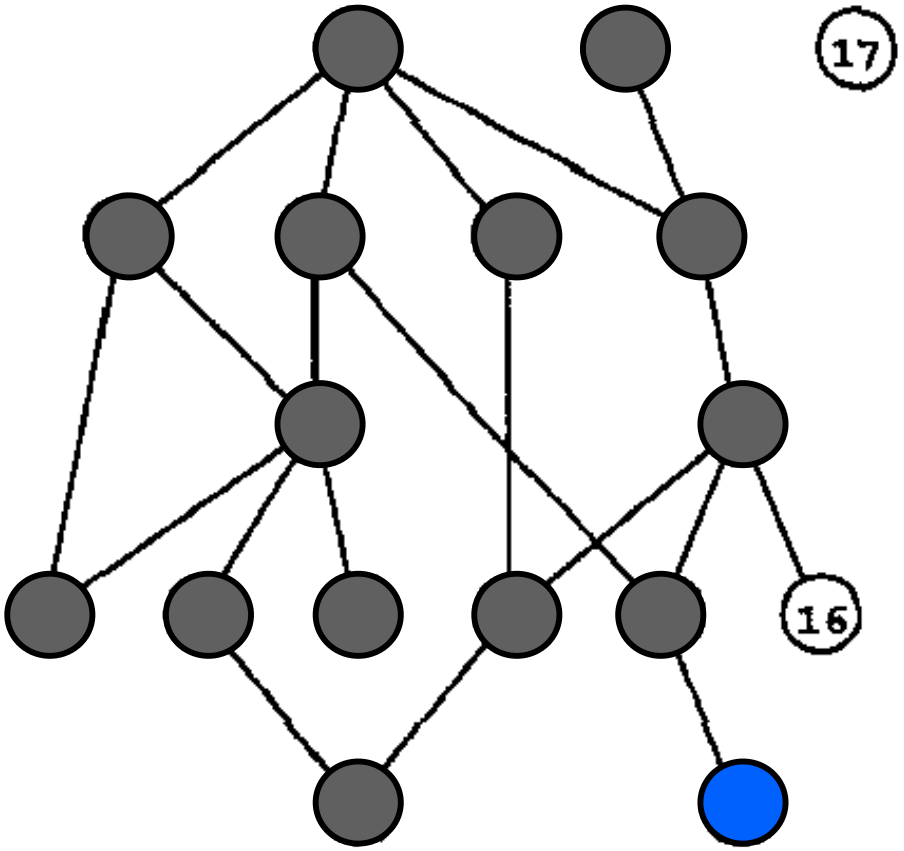

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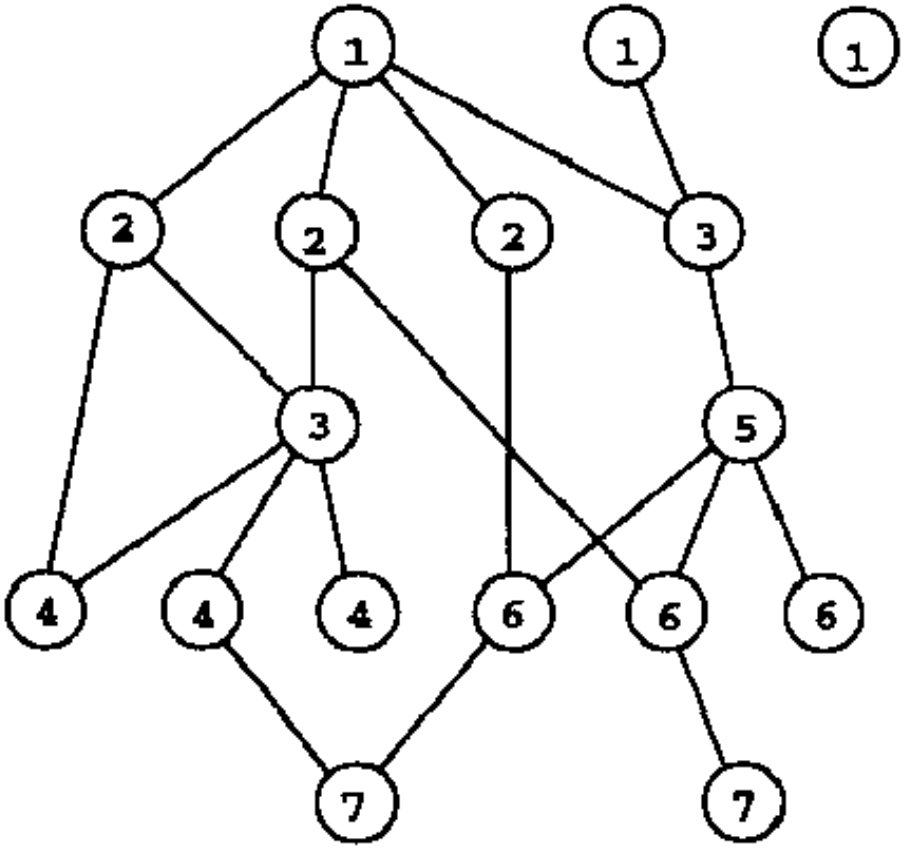

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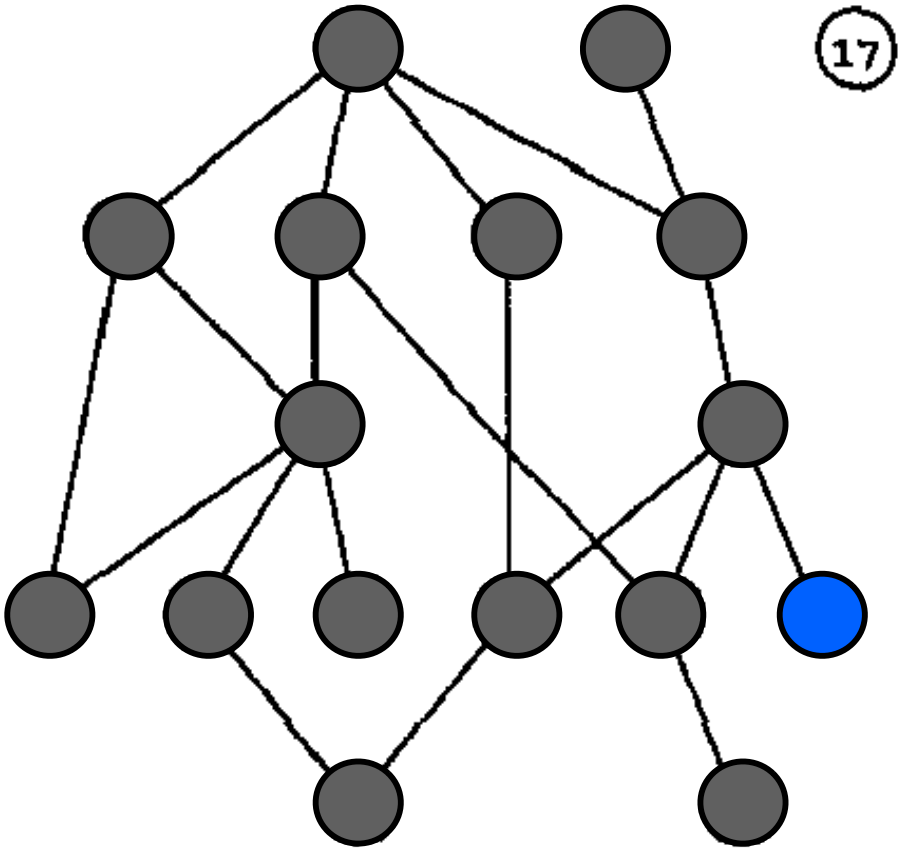

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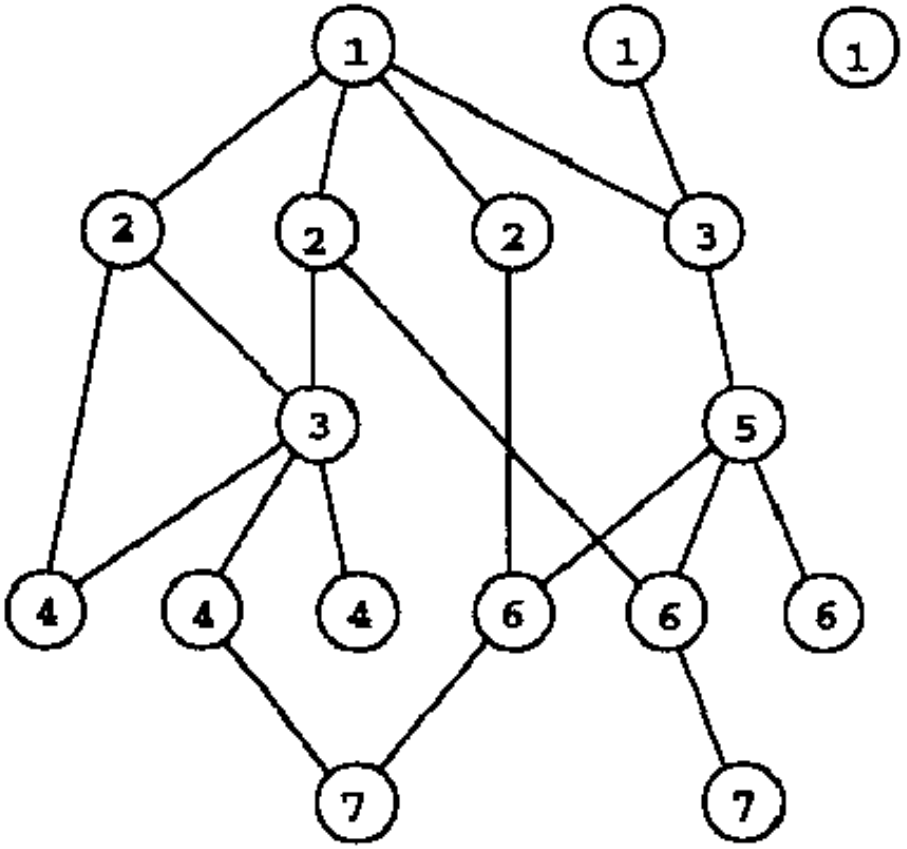

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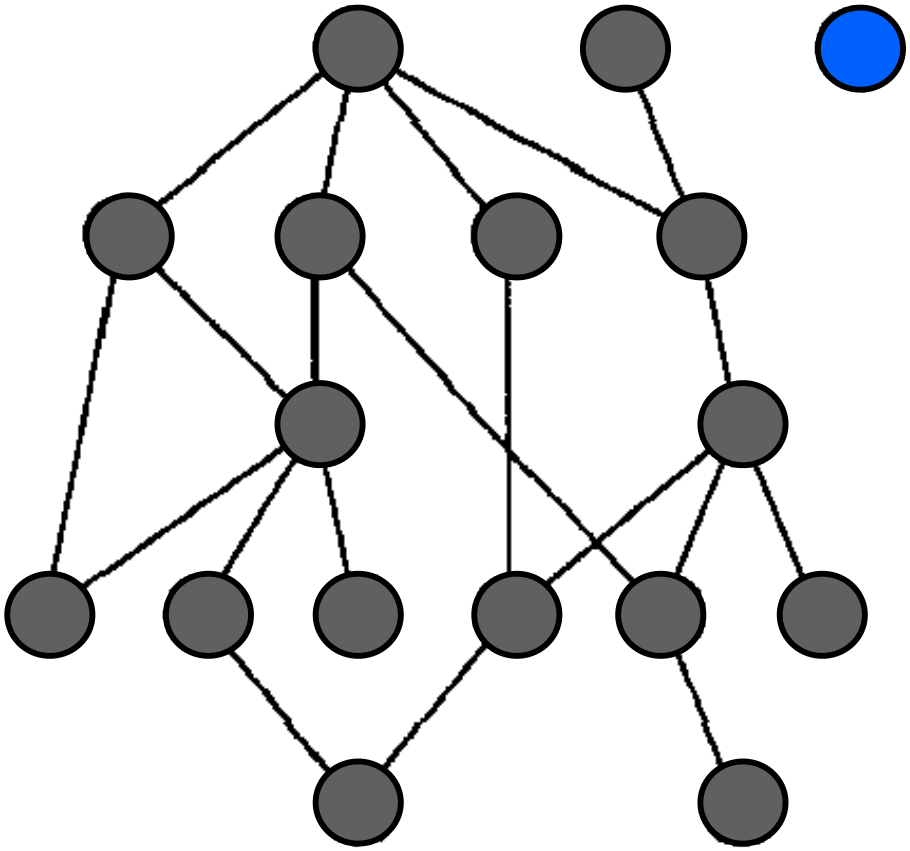

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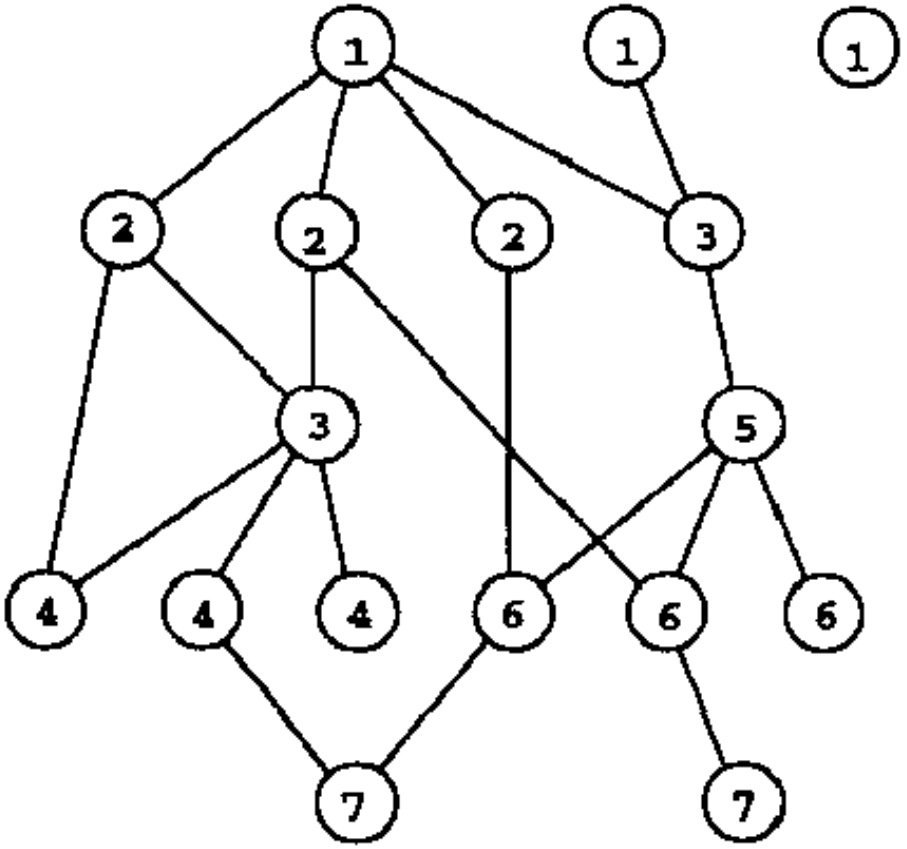

PDF Schedule

- Let R be a vector containing only the root node
- Schedule the first min(P, |R|) nodes from R with the i^{th} node in R assigned to processor i
- Replace each newly scheduled node in by its ready children, in left to right order, in place in R

DAG Scheduling

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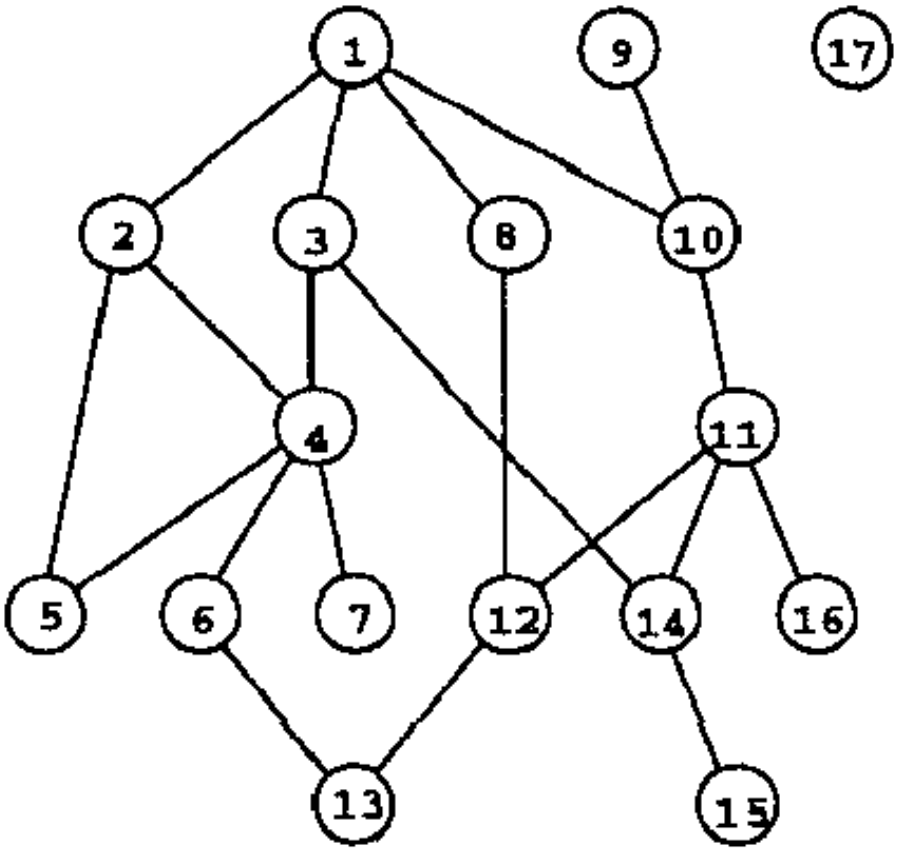

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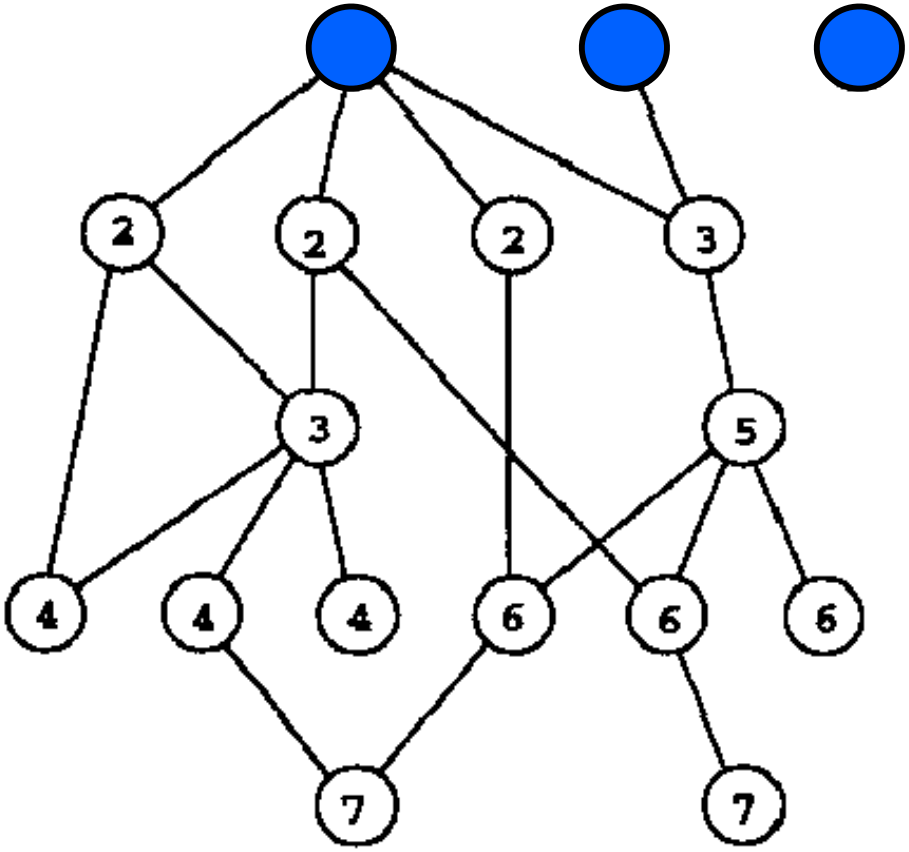

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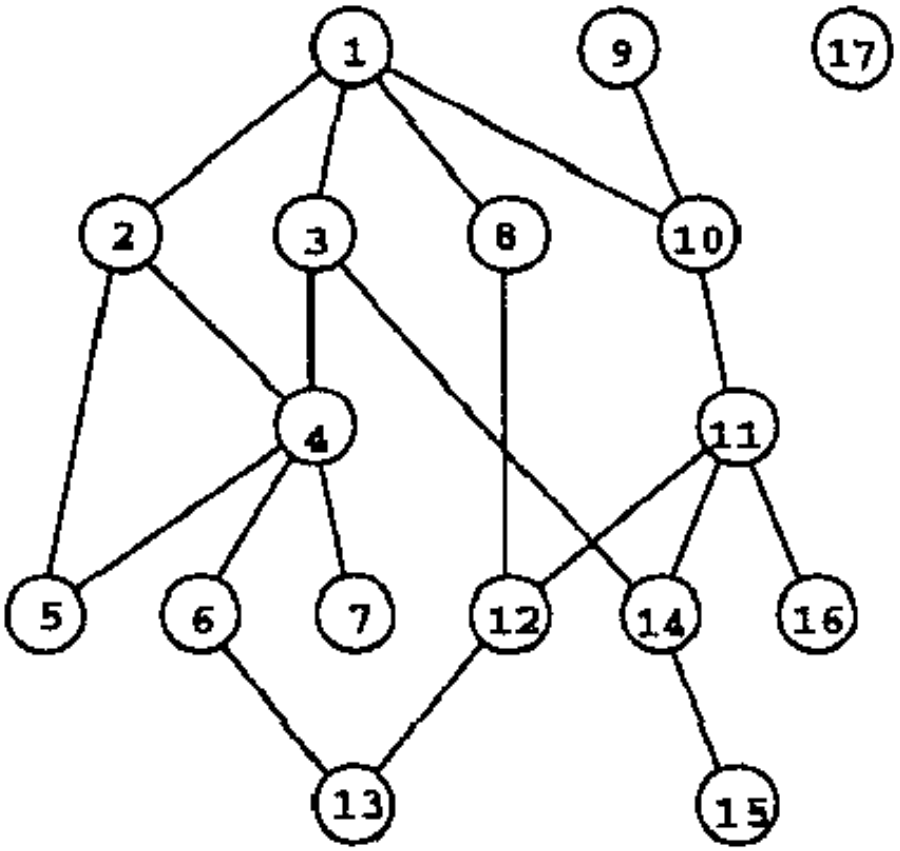

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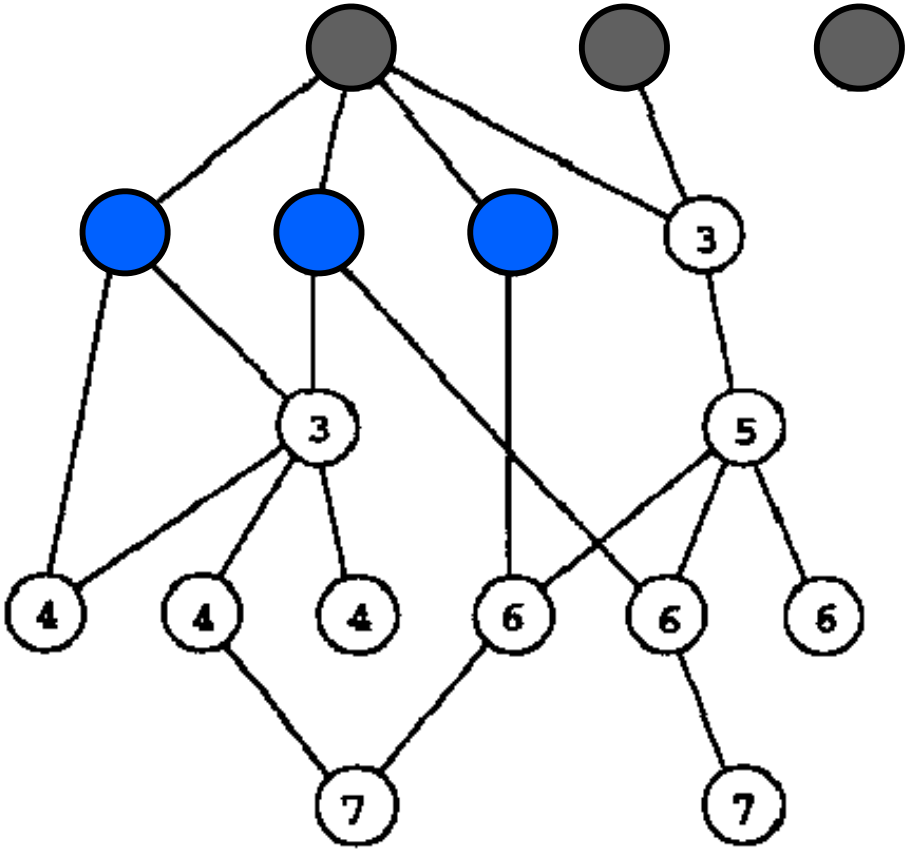
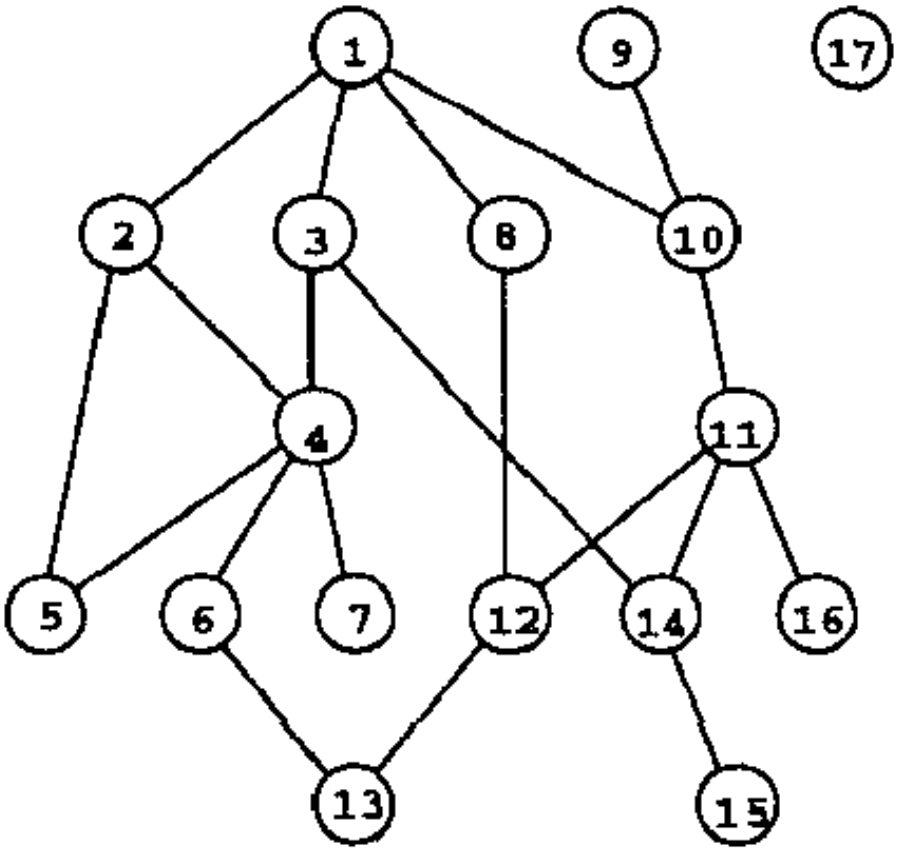
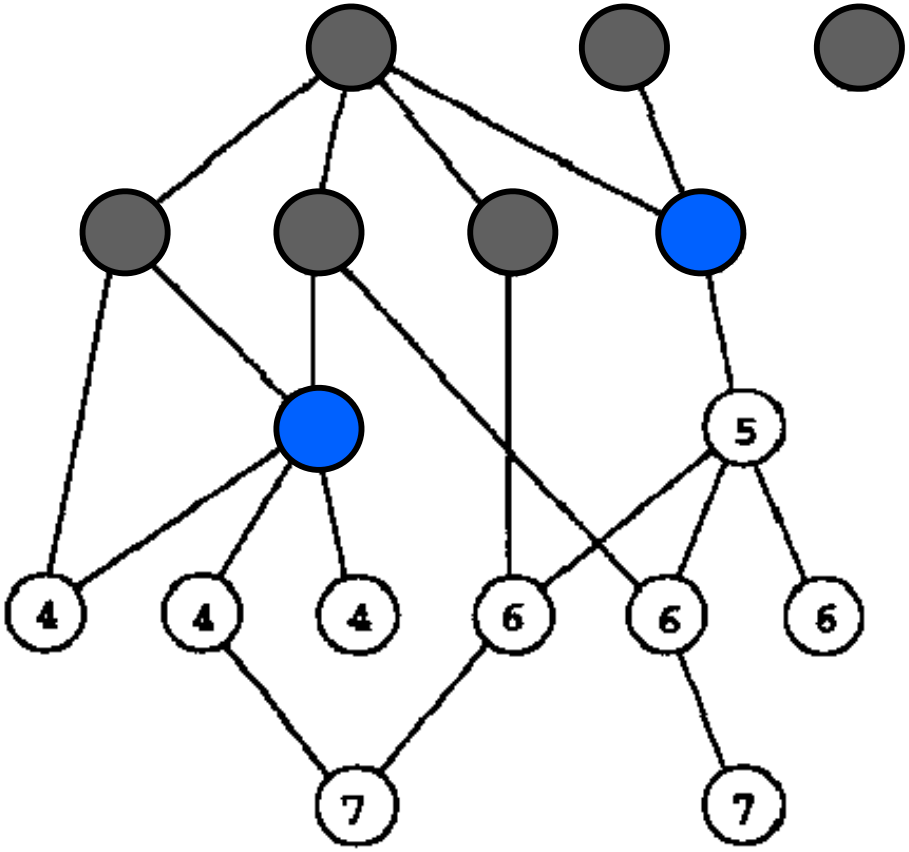


DAG Scheduling

1 DF Schedule

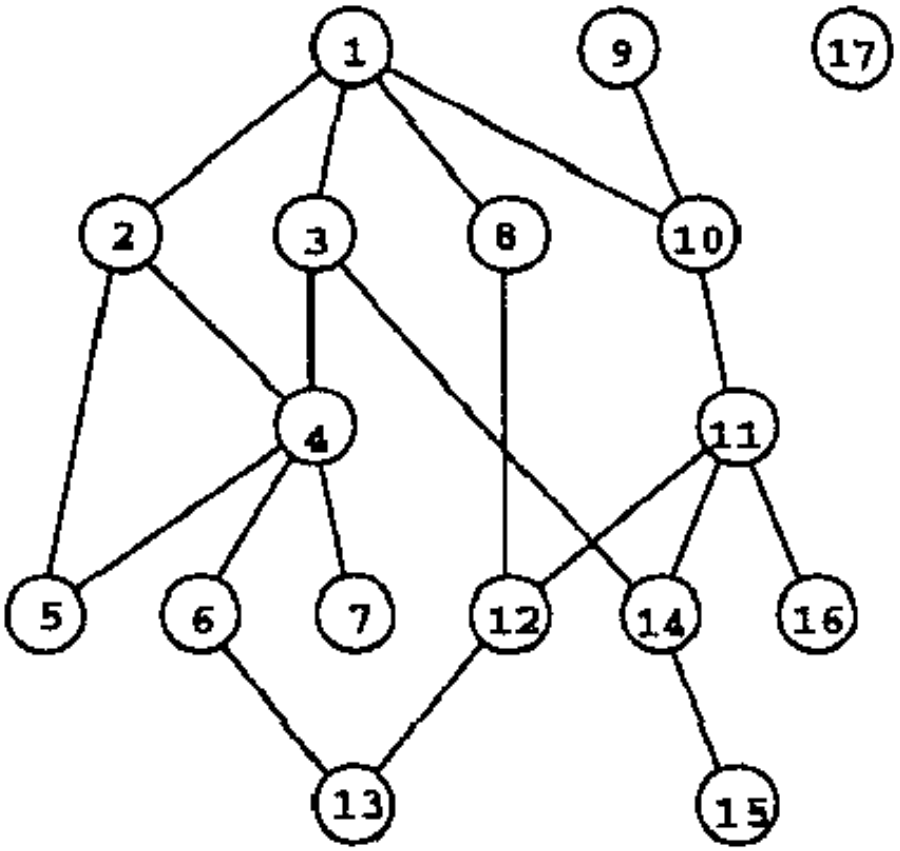


PDF Schedule for p=3

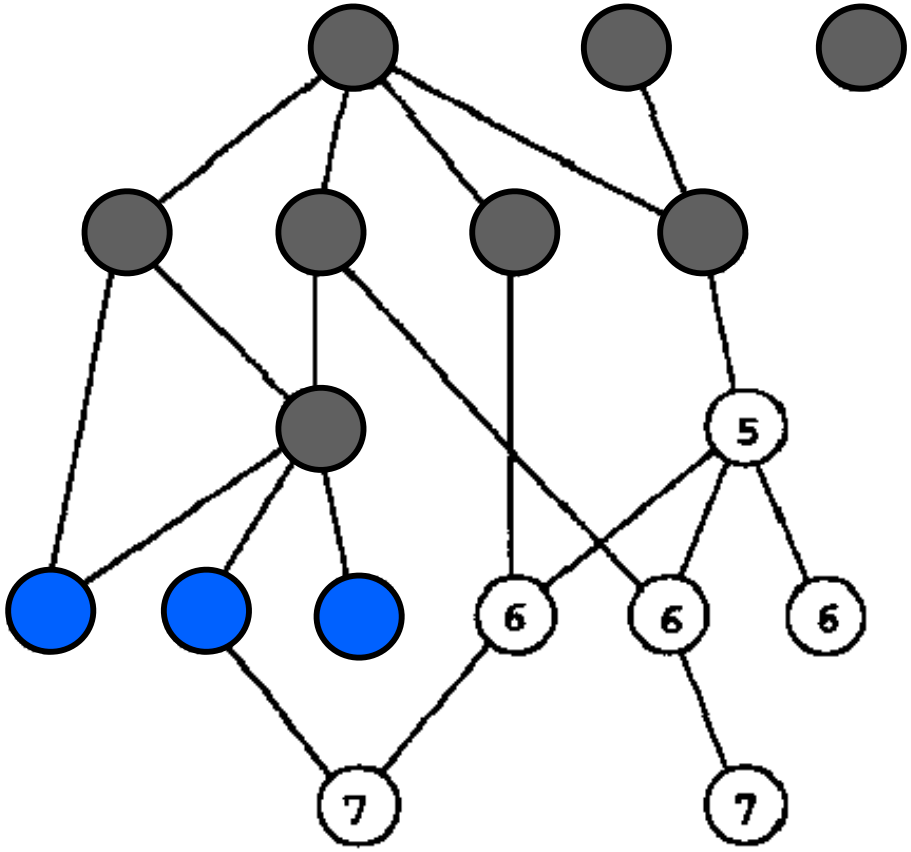


DAG Scheduling

1 DF Schedule



PDF Schedule for p=3



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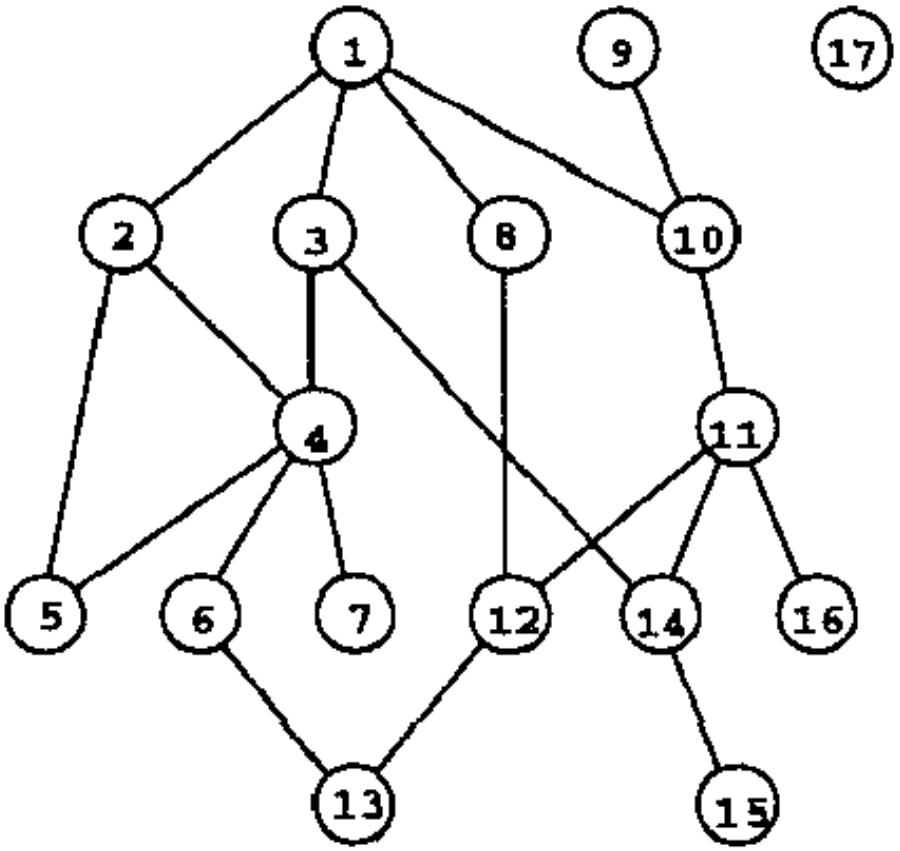
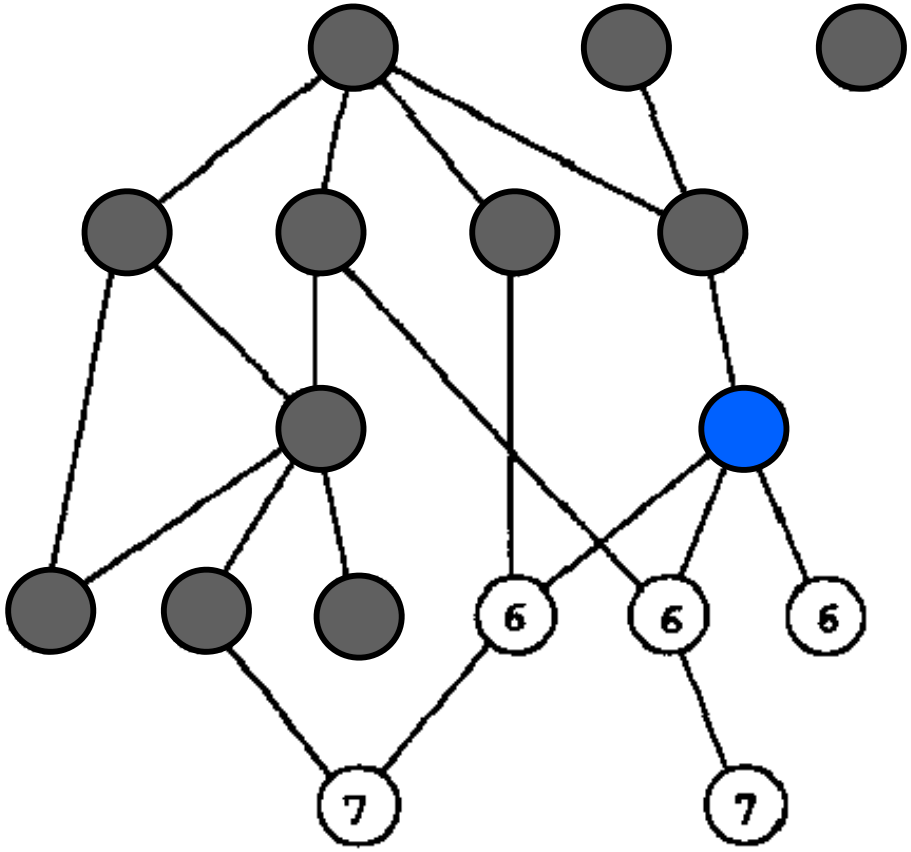


PDF Schedule for p=3



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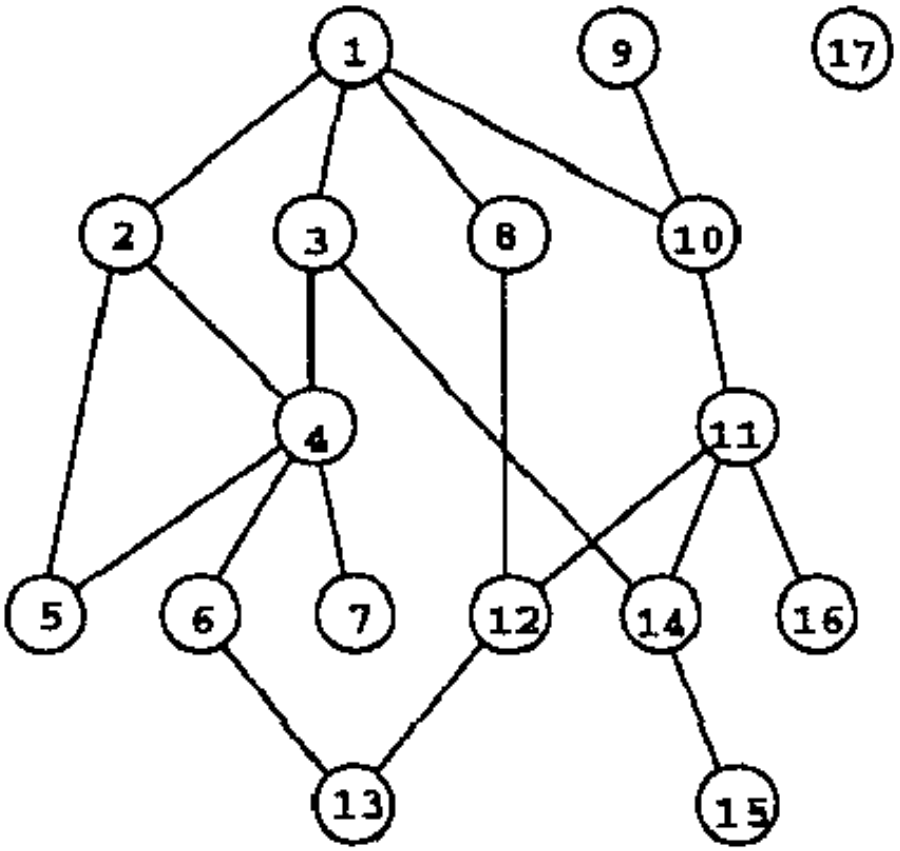
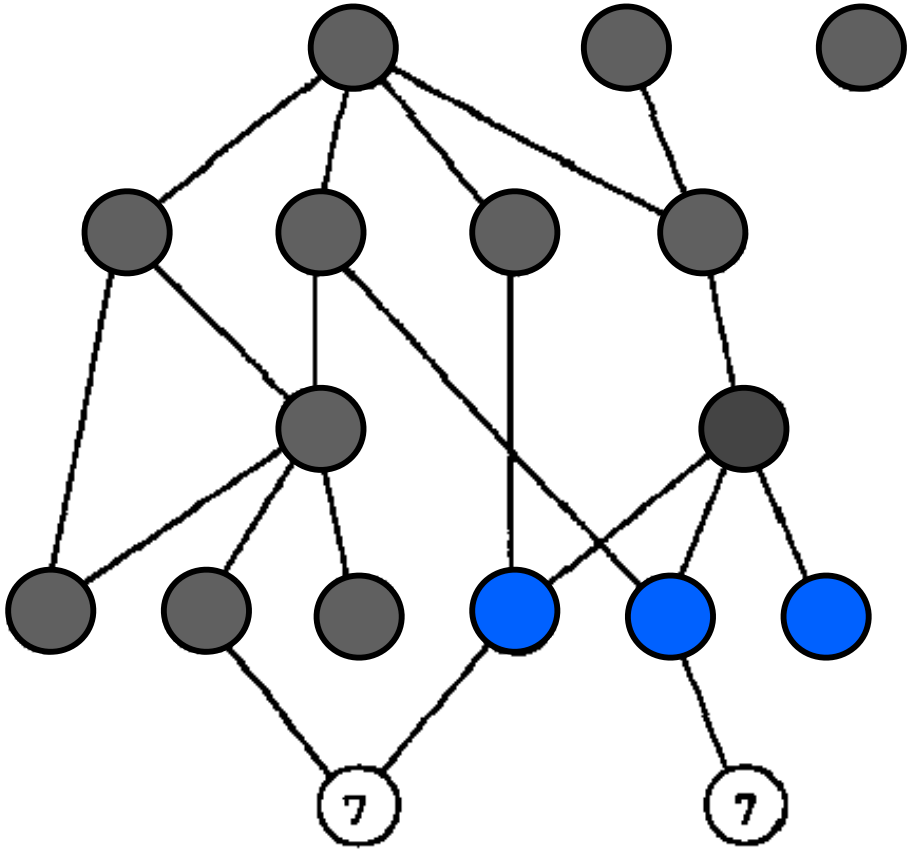


PDF Schedule for p=3



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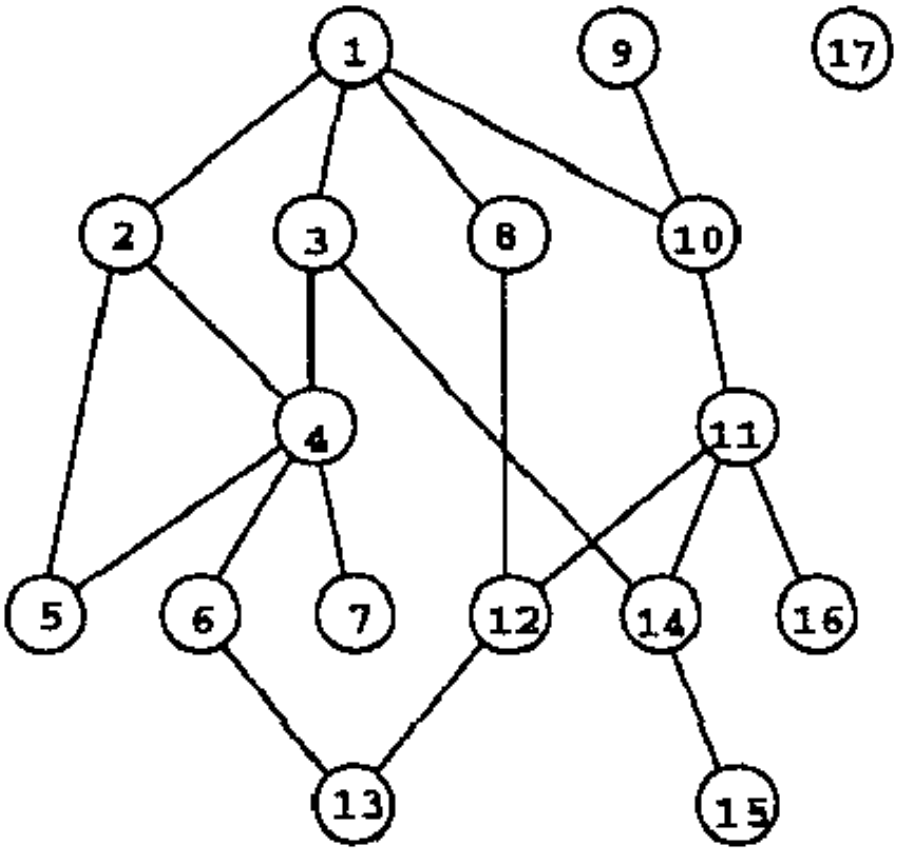

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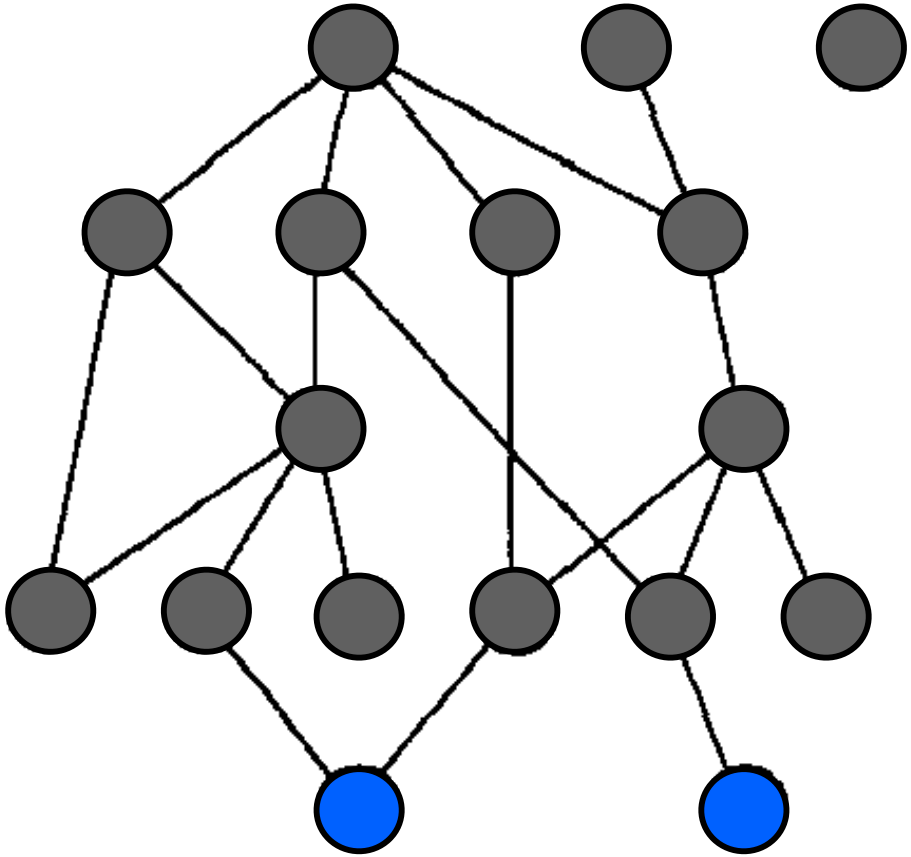
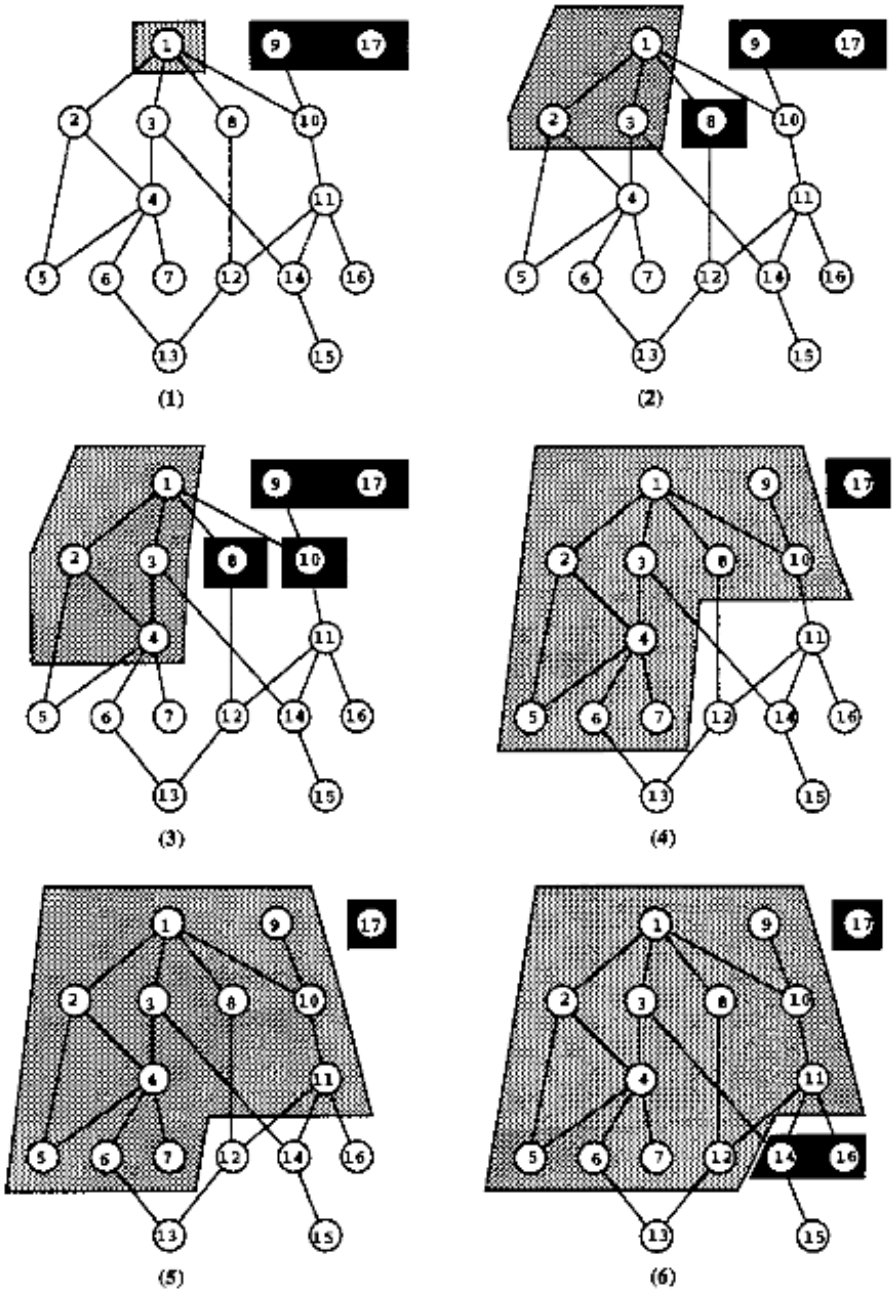


7 steps

PDF Schedule Properties

- Each time step completes work on the critical path
- Only (P - 1) processors available for “other work”
- At most (P - 1) * d nodes are “premature”
  - —ahead of 1DF order
- Premature node bound leads to cache miss bound



PDF Schedules vs. Work Stealing

- PDF schedule uses only $S_1 + O(DP)$ space
  - —S_1 is the space required to execute the computation serially
  - —D is depth of the computation
- Bound is asymptotically lower than the previous bound of S_1P for work stealing when $D = o(S_1)$, which is true for parallel computations that have a sufficient degree of parallelism
  - —Example: a simple algorithm to multiply two n x n matrices has depth $D = \Theta(\log n)$ and serial space $S_1 = \Theta(n^2)$, giving space bounds of $O(n^2 + P \log n)$ instead of $O(n^2 P)$
- How?
  - —have parallel computation follow an order as close as possible to the serial execution

Coordinating a PDF Schedule

- **Uses constant # of prefix-sums per scheduling round**
- **Time complexity $O(T_1/P + d * \lg P)$**

- **Reducing scheduling overhead**
  - **—maintaining R can exceed space bound**
    - **maintain parents instead of kids ... see paper for details**
  - **—reducing time overhead**
    - **approach**
      - **schedule multiple tasks per processor per round**
        - **use a parallel DFT with wider width than P**
    - **impact**
      - **increases additive term in time and space complexities**
      - **ensures work is within constant factor of optimal**

Ideal Cache Miss Bounds

- Ideal cache = fully associative, optimal replacement policy
- For ideal cache, if $C_p \geq (C_1 + (P - 1) * D)$, then $M_p \leq M_1$
  - —size of shared cache size for P threads only needs to be P*d larger than the cache for a single thread for same miss rate
- Intuition
  - —at most (P - 1) * d nodes can be executed prematurely
  - —cache is (P - 1) * d blocks larger
    - premature nodes have space for them in the cache
  - —cache is ideal, won't cause extra evictions until cache is full
  - —there is a 1-1 correspondence between bad and good nodes
    - bad node incurs a hit in 1DF schedule but a miss in PDF schedule
    - good node incurs a miss in 1DF schedule but a hit in PDF schedule

LRU Cache Miss Bounds

- Tarjan and Sleator show that an ideal cache of size C_1 can be simulated with an LRU cache of size $2 * C_1$
- Use this result to extend previous bound to LRU caches
  - —for an LRU cache, if $C_p \geq 2 * (C_1 + (P - 1) * d)$, then $M_p \leq M_1$

Value of PDF Schedules

- Shows that PDF schedules can manage shared cache effectively for multiple cores
  - —provides upper and lower bounds on cache misses
- For computations based on nested data parallelism, the size of memory and cache need not grow linearly with number of cores and threads in manycore designs

Space-efficient Implementation of Nested Parallelism

- **Problem: PDF scheduler tightly controls space, but has too many expensive scheduling steps**
- **Goal: allow threads to execute non-preemptively and asynchronously to improve locality and reduce scheduling overhead**
- **Approach: allocate a pool of a constant K units of memory to each thread when it starts up, and allow a thread to execute non-preemptively until it runs out of memory from that pool (and reaches an instruction that requires more memory), or until it suspends**
  - **—instead of preallocating a pool of K units of memory for each thread, assign it a counter that keeps track of its net memory allocation. We call this runtime, user-defined constant K the memory threshold for the scheduler**

AsyncDF Scheduler Sketch

- When the scheduler forks (creates) child threads, insert them into R (priority Q of ready threads) immediately to the left of their parent thread
  - —maintain the invariant that the threads in R are always in the order of increasing 1DF-numbers of their leading nodes
- At every scheduling step, P ready threads whose leading nodes have the smallest 1DF-numbers are moved to Q_{out} (FIFO of ready threads removed from R)
  - —every time a ready thread is picked from Q_{out} and scheduled on a worker processor, it may allocate space from a global pool in its first action
  - —a thread must preempt itself before any subsequent action that requires more space
- Child threads are forked only when they are to be added to Q_{out}, that is, when they are among the leftmost P ready threads in R

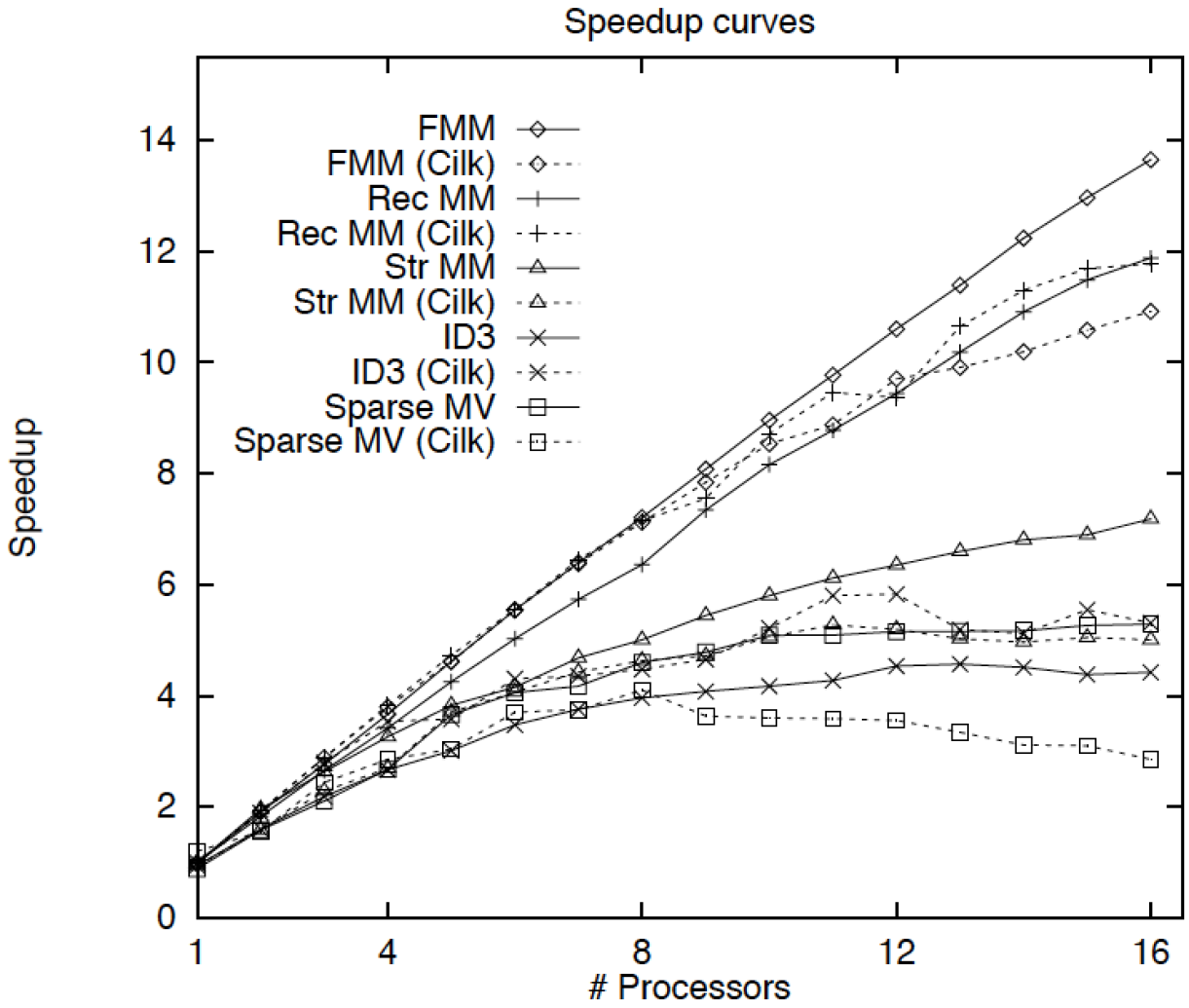
AsyncDF Scheduler Result

Theorem

Let S_1 be the space required by a 1DF-schedule for a computation with work W and depth D, and let S_a be the total space allocated in the computation.

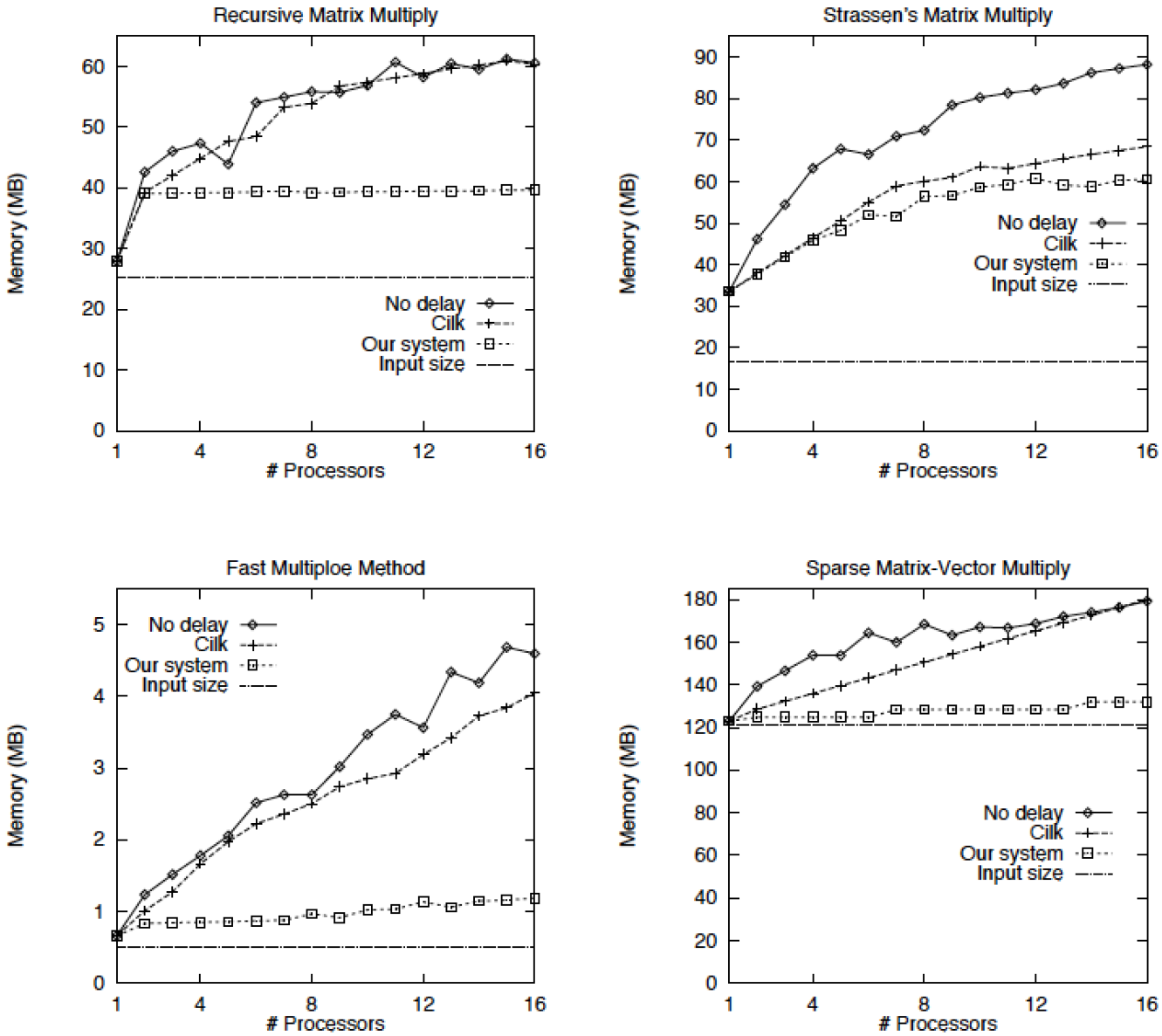
The parallelized scheduler with a memory threshold of K units, generates a schedule on P processors that requires $S_1 + O(KDP \log P)$ space and $O(W/P + S_a/PK + D \log P)$ time to execute

AsyncDF vs. Work Stealing: Speedup



Girija J. Narlikar and Guy E. Blelloch. 1999. Space-efficient scheduling of nested parallelism. *ACM TOPLAS.* 21, 1 (January 1999), 138-173

AsyncDF vs. Work Stealing: Space



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Summary

- PDF schedules have the same number of asymptotic time steps as work stealing schedulers, though there they incur log P scheduling overhead at each step
- PDF schedules use asymptotically smaller space than work stealing
- PDF scheduling overhead can be reduced by letting processors operate asynchronously within a bounded amount of memory
- PDF space bounds can be maintained by having asynchronous processors coordinate when they hit a local space bound to ensure that the collection of processors achieve the desired global asymptotic bound

Differences

- Data storage
  - —Blumofe and Leiserson allows for only stack allocation
  - —PDF scheduling results allow for arbitrary heap allocation/ deallocation, both in the case that it is explicitly handled by the programmer and when it is implicitly handled by a garbage collector.
- Scheduling
  - —Blumofe and Leiserson use distributed “work stealing”. As long as each processor has tasks to execute, no overheads are incurred.
  - —PDF scheduling is a centralized, parallel approach
- Communication
  - —Blumofe and Leiserson’s work-stealing scheduler is shown to provide good bounds on the total amount of interprocessor communication for the task model they consider.

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