Scheduling Multithreaded Computations by Work-Stealing [Blumofe and Leiserson, 1999]

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Abstract

- work-stealing scheduling method: idle processors steal threads from busy processors
- contribution: efficient randomized work-stealing algorithm for fully strict computations

Overview

challenge: efficiently executing a dynamic multithreaded computation on a MIMD computer

- parallelism not known in advance
  - dynamically growing and shrinking as computation unfolds
  - static scheduling: ill-suited
- threads depend on each other

scheduler goals:

- ensuring an appropriate number of threads are active at each step (keeping all processors busy)
- limiting memory usage of active threads

Contents

Progess

Scheduling paradigms

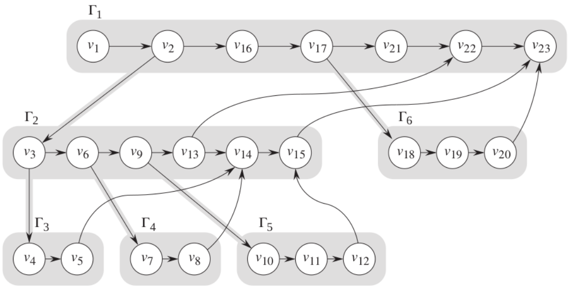
work-sharing:

- scheduler migrates threads to underutilized processors (even if processors are busy)
- more thread migration

work-stealing:

- idle processors steal threads
- less thread migration

Fully strict computations



fully strict (well-structured) computations include:

- backtrack search
- divide-and-conquer
- data flow

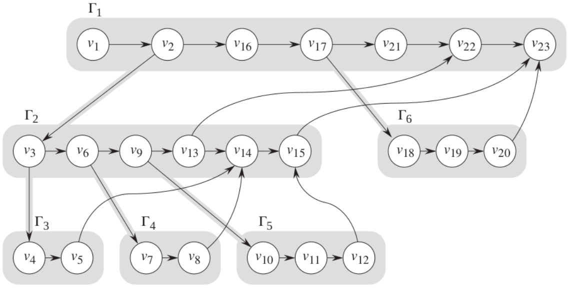
Contribution

efficient randomized work-stealing algorithm for scheduling fully strict multithreaded computations:

- expected time: $T_1/P + O(T_\infty)$
  - T_1: serial time
  - P: number of processors
  - T_∞: time with ∞ processors
- space: $S_1 P$
  - S_1: serial space

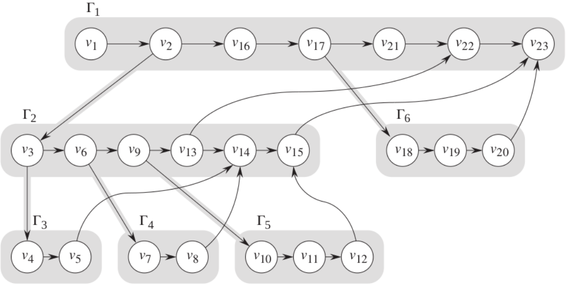
Progess

Multithreaded computation: continue-edges



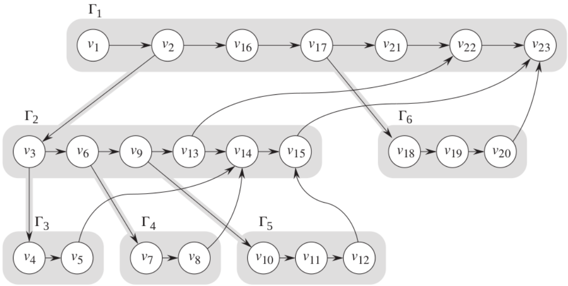
- v_1: **instruction**
- (v_1, v_2): **continue-edge** (horizontal)
- Γ_6: **thread**
  - **activation frame**
    - **alive**
    - **dead**

Multithreaded computation: spawn-edges

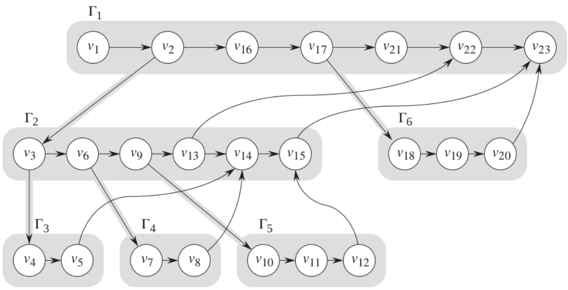


- (v_2, v_3): **spawn-edge** (shaded)
- **spawn-tree**:
  - Γ_1: root thread
  - Γ_3: leaf thread

Multithreaded computation: join-edges



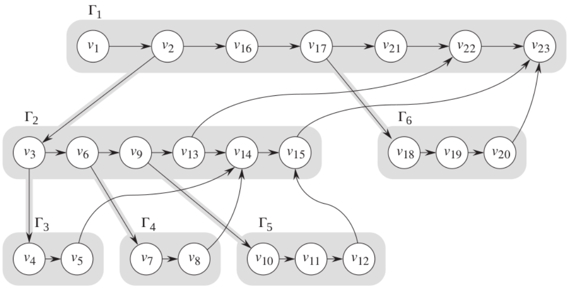
- (v_5, v_{14}): **join-edge** (curved)
- thread Γ_2:
  - **ready** after v_2
  - **stalled** at v_{14} (**join-depenency**)
    - **enabled** by v_5 and v_8 (**resolved** join-depenency)



2-processor **execution schedule**

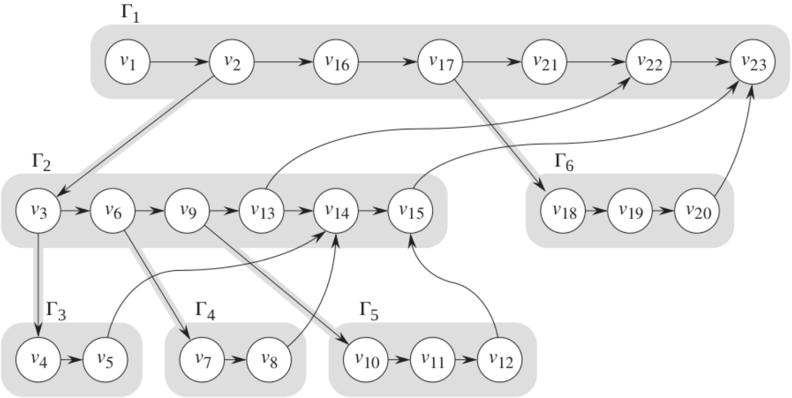
step	thread pool		processor activity p_1		p_2	
1			Γ_1:	v_1		
2				v_2		
3			Γ_2:	v_3	Γ_1:	v_{16}
4		Γ_2	Γ_3:	v_4		v_{17}
5	Γ_1	Γ_2		v_5	Γ_6:	v_{18}
6	Γ_1		Γ_2:	v_6		v_{19}
7	Γ_1	Γ_2	Γ_4:	v_7		v_{20}
8		Γ_2		v_8	Γ_1:	v_{21}
9	Γ_1		Γ_2:	v_9		
10	Γ_1		Γ_5:	v_{10}	Γ_2:	v_{13}
11	Γ_1			v_{11}		v_{14}
12		Γ_2		v_{12}	Γ_1:	v_{22}
13	Γ_1		Γ_2:	v_{15}		
14			Γ_1:	v_{23}		

Multithreaded computation: (full) strictness



- **strict**: each join-edge ends at an ancestor
- **fully strict** (well-structured): each join-edge ends at the parent

Multithreaded computation: work (T_1), span (T_∞)



- **work**: number of instructions (23)
- **span (critical-path length)**: number of instructions in longest path (10)

Execution time

notations:

- P: number of processors
- X: P-processor execution schedule
- $T(X)$: execution time of X
- $T_P = \min_X T(X)$: least execution time with P processors over all execution schedules X

observations:

1. $T_1 =$ work (number of instructions)
2. $T_\infty =$ span (length of longest path)
3. $T_P \geq T_1/P$
4. $T_P \geq T_\infty$

Greedy execution schedule

greedy P-processor execution schedule:

- if at least P instructions are ready, P instructions are executed (**complete step**)
- otherwise, all ready instructions are executed (**incomplete step**)

Theorem (1)

If a P-processor execution schedule X is greedy, then $T(X) \leq T_1/P + T_\infty$.

Proof.

$$\begin{aligned} T(X) &= \#CompleteSteps &&+ \#IncompleteSteps \\ &\leq T_1/P &&+ T_\infty \end{aligned}$$

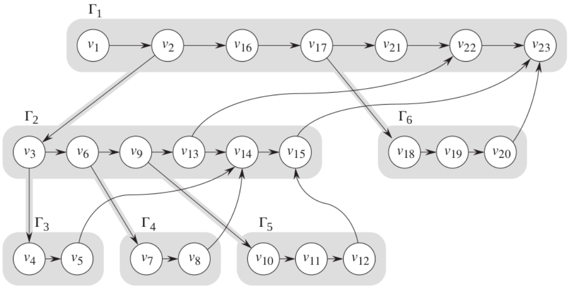
□

Linear speedup

P-processor execution schedule X achieves **linear speedup** when $T(X) = O(T_1/P)$

- if X is greedy:
  - linear speedup is achieved when **parallelism** $T_1/T_\infty = \Omega(P)$
    - using Theorem 1: $T(X) \leq T_1/P + T_\infty$

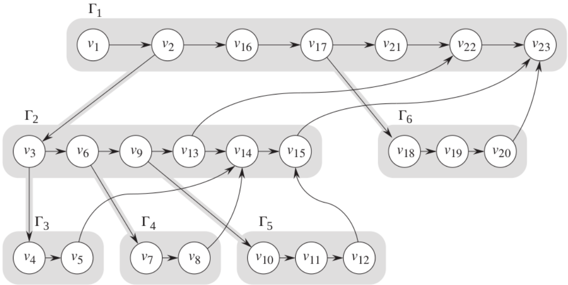
Linear space expansion



- **stack depth of thread**: sum of sizes of activation frames of the thread and its ancestors
- **stack depth of computation**: max stack depth across all threads in the computation
- S_1: space usage with 1 processor (equal to stack depth of computation)
- $S(X)$: space usage of P-processor execution schedule X
- X exhibits **linear space expansion** if $S(X) = O(S_1 P)$

Progess

Busy-leaves property



spawn-subtree at time step t: alive threads of spawn-tree

- given execution schedule X:
  - at time step t, leaf thread Γ in the spawn-subtree is **busy** if some processor in X is working on Γ
  - X has **busy-leaves property** if: at every time step, all leaf threads in the spawn-subtree are busy

Busy-leaves property implying linear space expansion

Lemma (2)

If a P-processor execution schedule X has busy-leaves property, then $S(X) \leq S_1 P$.

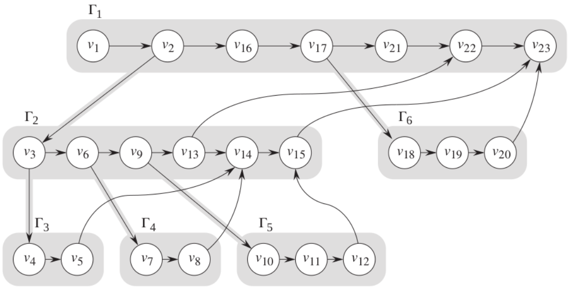
- *$S(X)$: space usage of X*
- *S_1: serial space usage (stack depth of computation)*

Proof.

1. by busy-leaves property: at every time step, the spawn-subtree has at most P leaf threads
2. for each such leaf thread, the space used by the thread and its ancestors is S_1
3. at every time step, the total space used by all threads is $S_1 P$

□

Busy-leaves property implied by strict computation



in a strict computation:

- after a thread Γ is spawned and until Γ dies, the subcomputation rooted at Γ can be finished by 1 processor
- no leaf thread can stall

observation: if a computation is strict, then it has an execution schedule with busy-leaves property

Busy-leaves algorithm: linear speedup and linear space expansion

given a strict computation, the **busy-leaves algorithm** finds a P-processor execution schedule X such that:

- X is greedy
  - $T(X) \leq T_1/P + T_\infty$ (Theorem 1)
    - excluding algorithm's time to find schedule X
- X has busy-leaves property
  - $S(X) \leq S_1 P$ (Lemma 2)

Busy-leaves algorithm: overview

- online algorithm:
  - only using information from the subcomputation revealed so far
  - no knowledge of:
    - instructions not yet executed
    - threads not yet spawned
- global pool of alive threads
  - processors take ready threads from this pool
  - processors return stalled threads to this pool

Busy-leaves algorithm: part 1

- root thread is put in global thread pool
- for each step:
  - each idle processor attempts to take a ready thread from the global thread pool
  - each busy processor executes the next instruction in a thread, until the thread:
    1. spawns
    2. stalls
    3. dies

Busy-leaves algorithm: part 2

each busy processor p executes the next instruction in a thread Γ_a, until:

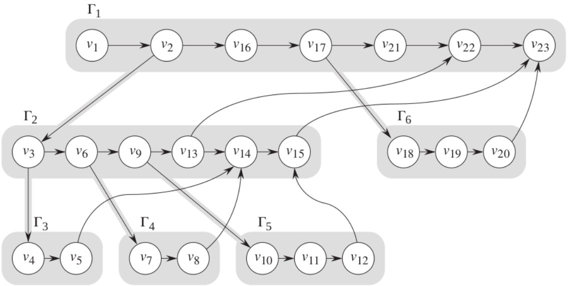
1. thread Γ_a spawns a child thread:
   - p returns Γ_a to the thread pool
   - p works on the child thread in the next step
2. thread Γ_a stalls:
   - p returns Γ_a to the thread pool
   - p becomes idle in the next step
3. thread Γ_a dies:
   - Γ_a's parent is some thread Γ_b
   - if Γ_b has no alive child and no processor is working on Γ_b, then p takes Γ_b from the thread pool and works on Γ_b in the next step
   - otherwise, p becomes idle in the next step

Busy-leaves algorithm: example

			processor activity			
step	thread pool		p_1		p_2	
1			$\mathbf{\Gamma}_1$:	v_1		
2				v_2		
3			$\mathbf{\Gamma}_2$:	v_3	$\mathbf{\Gamma}_1$:	v_{16}
4		$\mathbf{\Gamma}_2$	$\mathbf{\Gamma}_3$:	v_4		v_{17}
5	$\mathbf{\Gamma}_1$	$\mathbf{\Gamma}_2$		v_5	$\mathbf{\Gamma}_6$:	v_{18}
6	$\mathbf{\Gamma}_1$		$\mathbf{\Gamma}_2$:	v_6		v_{19}
7	$\mathbf{\Gamma}_1$	$\mathbf{\Gamma}_2$	$\mathbf{\Gamma}_4$:	v_7		v_{20}
8		$\mathbf{\Gamma}_2$		v_8	$\mathbf{\Gamma}_1$:	v_{21}
9	Γ_1		$\mathbf{\Gamma}_2$:	v_9		
10	Γ_1		$\mathbf{\Gamma}_5$:	v_{10}	$\mathbf{\Gamma}_2$:	v_{13}
11	$\mathbf{\Gamma}_1$			v_{11}		v_{14}
12		Γ_2		v_{12}	$\mathbf{\Gamma}_1$:	v_{22}
13	Γ_1		$\mathbf{\Gamma}_2$:	v_{15}		
14			$\mathbf{\Gamma}_1$:	v_{23}		

thread pool:

- **ready threads are in boldface**
- stalled threads are not



Busy-leaves algorithm: linear speedup and linear space expansion, revisited

for every strict computation, the busy-leaves algorithm computes a P-processor execution schedule X such that:

- X uses time $T(X) \leq T_1/P + T_\infty$
  - T_1: work
  - T_∞: span (critical-path length)

  (X is greedy)
- X uses space $S(X) \leq S_1 P$
  - S_1: serial space

  (X has busy-leaves property)

Progess

Ready deque

each processor p maintains a **ready deque** of threads

- other processors steal threads from the **top** of p's ready deque
- p inserts threads to the **bottom** of p's ready deque
- p removes threads from the **bottom** of p's ready deque

Work-stealing algorithm

each processor p works on a thread Γ_a, until:

1. Γ_a spawns some thread Γ_b:
   p: inserts Γ_a at the bottom of p's ready deque, and starts working on Γ_b
2. Γ_a stalls:
   - if p's ready deque has some thread Γ_b:
     p: removes Γ_b from p's ready deque, and starts working on Γ_b
   - otherwise:
     p: steals the top-most thread Γ_b of a randomly chosen processor, and starts working on Γ_b
3. Γ_a dies: same as when Γ_a stalls
4. Γ_a enables some thread Γ_b: Γ_b becomes the bottom-most thread in p's ready deque

Space usage

for every fully strict computation, the work-stealing algorithm needs at most $S_1 P$ space

- S_1: serial space
- P: number of processors

(the work-stealing algorithm find execution schedules with busy-leaves property)

Progess

Atomic-access model

atomic-access model:

- parallel computer with P processors
- concurrent accesses to the same data are serially queued by an adversary
  - the adversary tries to maximize the **total delay**
    (sum of numbers of outstanding access requests over all steps)

Total delay proportional to number of access requests

Lemma (6)

The total delay incurred by M random access requests made by P processors is:

1. $O(M + P \ln P - P \ln \epsilon)$, *with probability at least* $1 - \epsilon$, *for every* $0 < \epsilon < 1$
2. *at most M (expected)*

very rough proof sketch:

1. tracking the **delay** of an access request
   (number of steps in which the request is waiting to be serviced)
2. linearity of expectation

Progess

Time usage

for every fully strict computation with work T_1 and span T_∞, the work-stealing algorithm has time usage:

- $T_1/P + O(T_\infty + \ln P - \ln \epsilon)$, with probability at least $1 - \epsilon$, for every $0 < \epsilon < 1$
- $T_1/P + O(T_\infty)$ (expected)

very rough proof sketch:

- summand T_1/P: T_1 instructions executed in parallel by P processors
- summand $O(T_\infty)$: scheduling overhead
  (time for steal attempts to wait before being satisfied)
  - overhead is high if many steal attempts are made
  - a large number of steal attempts can occur only with low probability

Progess

Cilk

C-based language `Cilk`:

- runtime system employs work-stealing algorithm
- guaranteed performance to user applications
  - with high probability, linear speedup is achieved ($T_P = O(T_1/P)$), if **parallel slackness** $T_1/(PT_\infty)$ is large
- applications:
  - protein folding
  - graphic rendering
  - backtrack search
  - chess

References

Robert D Blumofe and Charles E Leiserson. Scheduling multithreaded computations by work stealing. *Journal of the ACM (JACM)*, 46(5):720–748, 1999.

John Mellor-Crummey. Personal Communication, 2019.

Summary

efficient randomized work-stealing algorithm for scheduling fully strict multithreaded computations:

- expected time: $T_1/P + O(T_\infty)$
  - T_1: serial time
  - P: number of processors
  - T_∞: time with ∞ processors
- space: $S_1 P$
  - S_1: serial space