Random 3-SAT: The Plot Thickens

C. Coarfa, D. Demopoulos, A. San Miguel Aguirre, D. Subramanian, M. Y. Vardi
Rice University

The questions

- How does the average complexity of random 3-SAT as function of order for a given density, vary with density?
- Is there a phase transition where the average complexity shifts from polynomial to exponential in the order?
- If there are such phase transitions, are they solver-dependent or -independent?

What is already known

- "Easy-hard-easy" pattern is widely publicized.
  - High density region is not "easy": For density region above 5.2 known exponential lower bound proven in (Chvatal &Szemeredi 88).
  - Various experimental results showing linear time behavior for low densities, but no systematic investigation. (Crawford&Auton 93, Kirkpatrick&Selman 94)
- Finite-size scaling
  - Claims to provide a comprehensive account of the running time behavior for over the density-order quadrant centered around the crossover point, but claim is overstated.

Experimental methodology

- Explore the density-order quadrant systematically
  - determine median running time as a function of order for a fixed density,
  - for different algorithm classes.

GRASP

- Complete solver based on the Davis-Longemann-Loveland (DLL) method.
- Search augmented with conflict analysis procedure that enables backjumping.
- Effective SAT solver for realistic SAT problems, has been fielded in FPGA layout.
- Available at vinci.inesc.pt/~jpms/grasp/ and from SATLIB

CPLEX

- Commercial MIP solver.
- Branch and bound technique using LP relaxations that can be complemented by dynamic generation of cutting planes (a technique more powerful than resolution).
- Complete SAT solver.
- Fielded in the context of large MIP problems in helicopter design, membrane filtration design, etc.

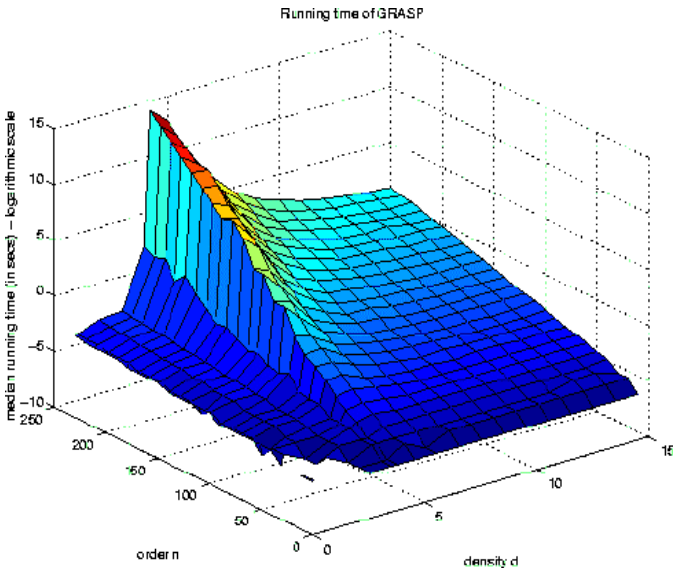
CUDD

- CUDD constructs a compact symbolic representation (ordered binary decision diagram) of the set of all satisfying assignments of a Boolean formula, and checks whether it is non-empty.
- Complete SAT solver.
- Used heavily in the semiconductor industry.
- Available from bessie.colorado.edu/~fabio/CUDD

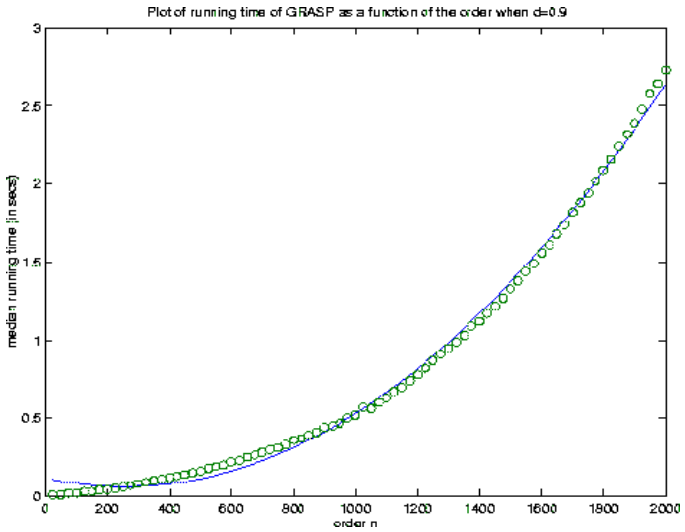
Why is CUDD in this lineup?

- Isn't the problem of finding all solutions to a 3-SAT instance different from that of finding a single solution?
  - Yes, but CUDD isn't enumerating solutions, it is constructing a compact representation of all solutions.
  - BDD techniques are incomparable with DP methods and sometimes dominate them -- CUDD is therefore a valuable 'alternative' algorithm to consider for our experimental analysis.

N-d-runtime surface for GRASP

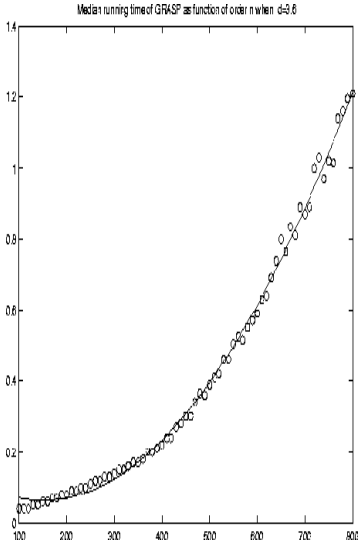


Low density behavior for GRASP

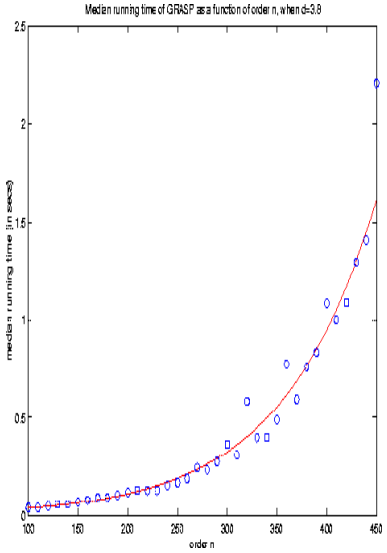


Quadratic in order for density = 0.9

Phase transition for GRASP

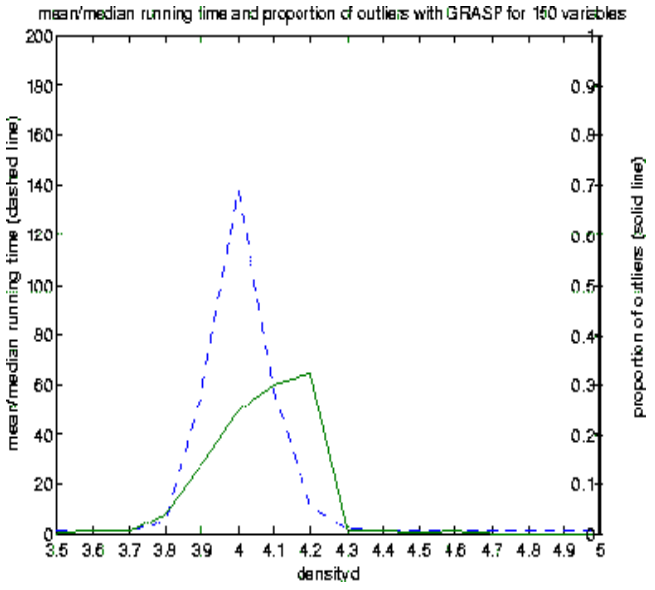


Quadratic at d = 3.6



Exponential at d = 3.8

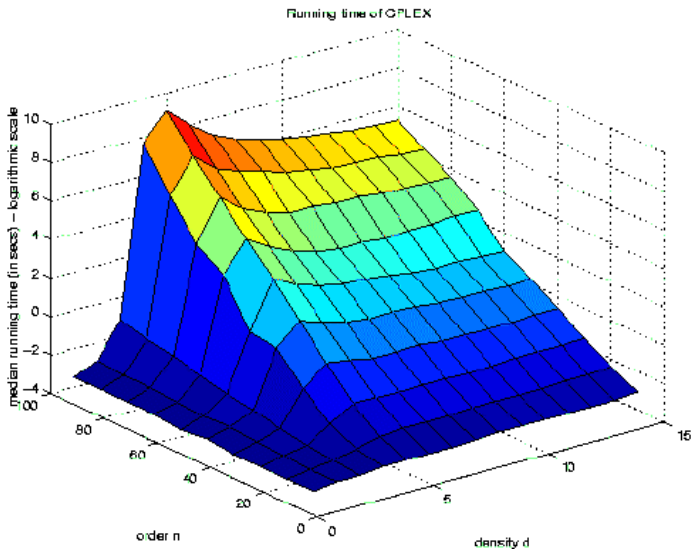
Heavy tailed phase transition for GRASP



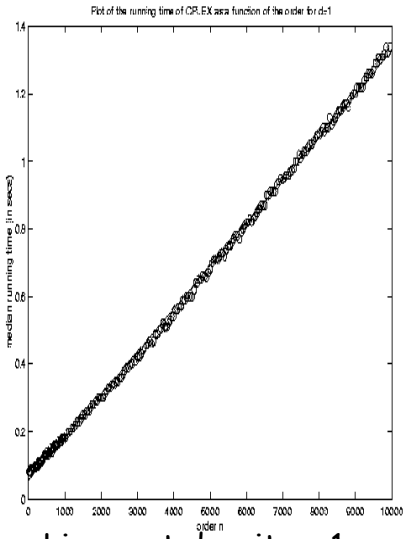
GRASP Summary

- For densities below 3.6, GRASP is quadratic in the order.
- For densities above 3.8, GRASP is exponential in the order.
- Between densities 3.7 to 4.3 prevalence of outliers , and marked increase in mean to median runtime ratios which peaks around density 4.
- From density 3.8 to 4.26, the exponent increases with density, after 4.26 it decreases with density (slower than 1/d).

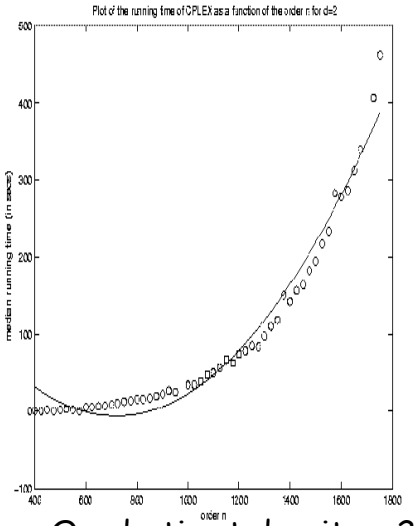
N-d-runtime surface for CPLEX



Low density behavior for CPLEX

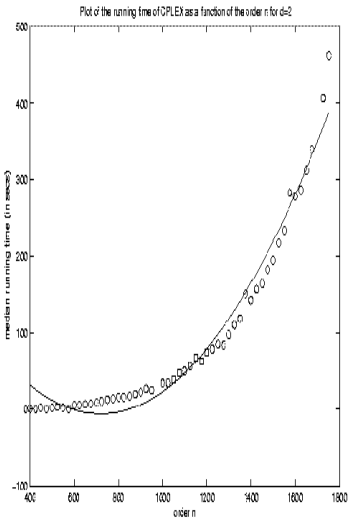


Linear at density = 1

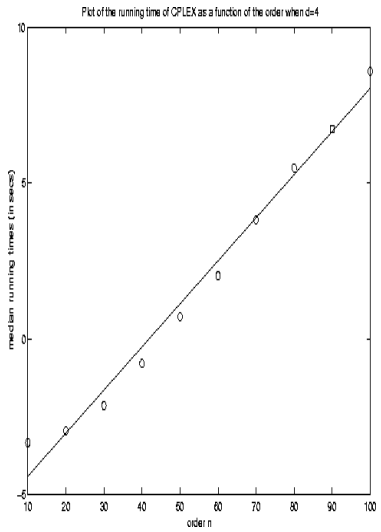


Quadratic at density = 2

Phase transition for CPLEX

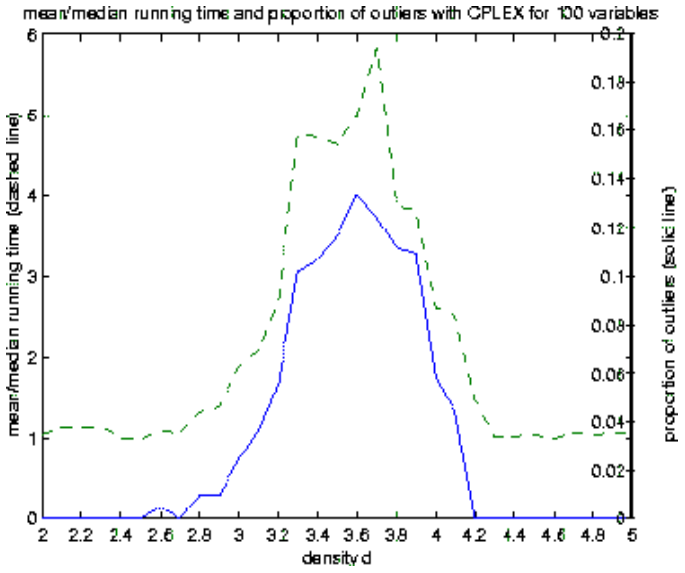


Quadratic at density = 2



Exponential at density = 4

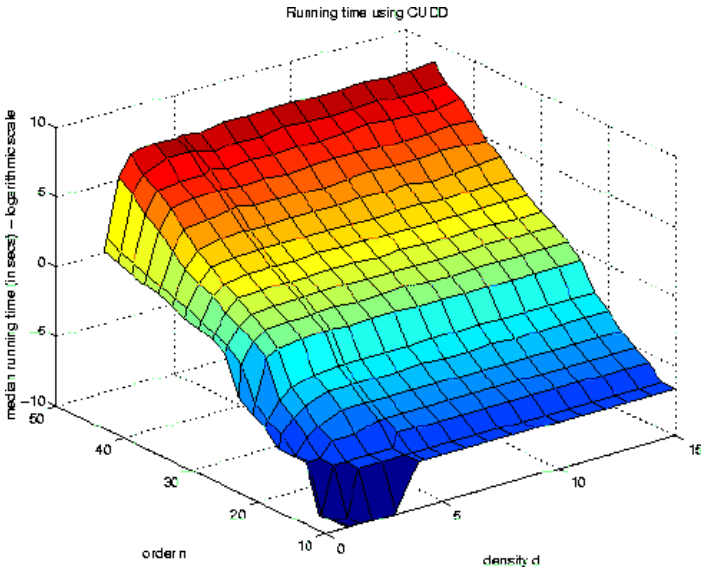
Heavy tailed phase transition for CPLEX



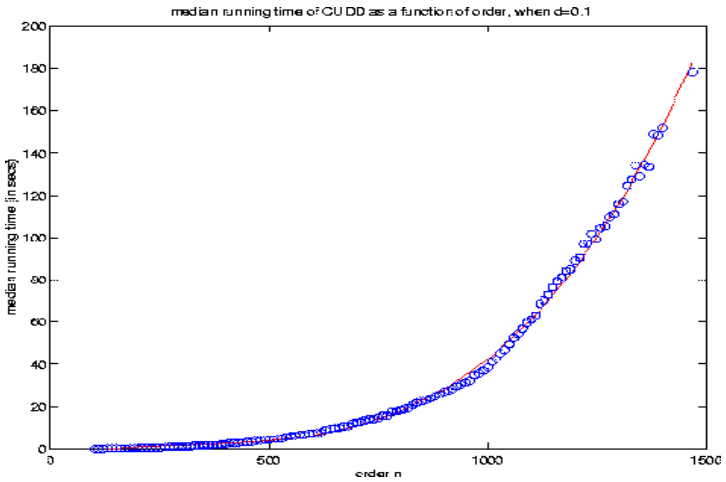
CPLEX Summary

- For densities below 1.7, CPLEX is linear in the order.
- For densities from 2 to 3.5, CPLEX is quadratic in the order.
- Between densities 4.0 and above, CPLEX is exponential in the order.
- Between densities 3.0 to 4.2, prevalence of outliers , and marked increase in mean to median runtime ratios which peaks around density 3.6.

N-d-runtime surface for CUDD

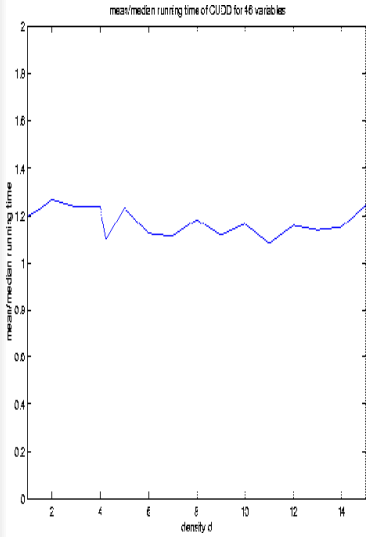


Very low density behavior for CUDD

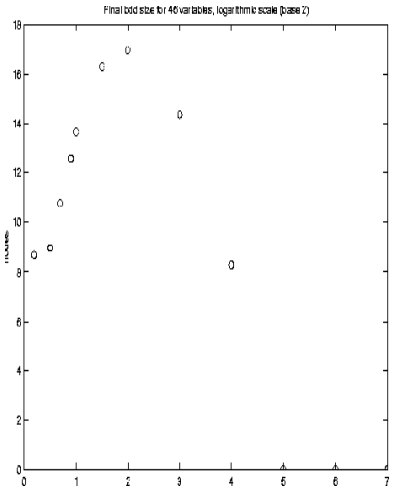


Cubic in order at density = 0.1

Outlier and size analysis for CUDD



Mean to median ratio vs density



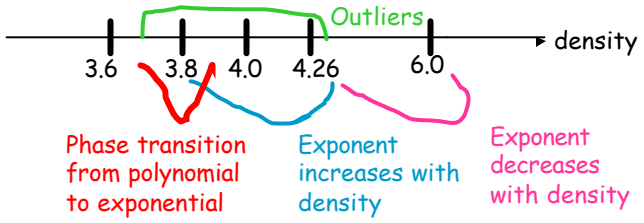
Peak in BDD size at density = 2

CUDD Summary

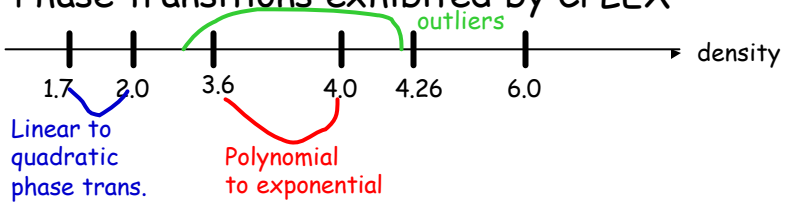
- For density 0.1, CUDD is cubic in the order.
- For densities 0.5 and above, CUDD is exponential in the order.
- For density 2 and above, the exponent is independent of density.
- Peak in ROBDD size at density 2, suggesting a phase transition there.

Experimental results summary

- Phase transitions exhibited by GRASP

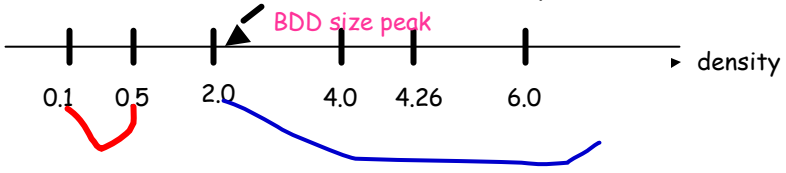


- Phase transitions exhibited by CPLEX



Experimental results summary

- Phase transitions exhibited by CUDD



Polynomial to exponential phase transition

Exponent independent of density

So, what?

- The density region between 0 and the crossover point is very eventful.
- What you see in this region depends on the instrument you use to observe the problem.
- Our understanding of the region to the left of the crossover point is limited.

Open questions

- What is the nature of the transition from polynomial to exponential?
- Can we analytically derive polynomial to exponential transition regions/points from algorithm itself, for the class of complete solvers studied here?
- Why does polynomial to exponential transition come much before crossover point, and why is it so sensitive to solver?