Comp 411
Principles of Programming Languages
Lecture 9
Meta-interpreters III

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Major Challenge

LC does not include a recursive binding operation (like Scheme \texttt{letrec} or \texttt{local}). How would we define \texttt{eval} for such a construct?

• Key problem: the closure structure for a recursive \texttt{lambda} must include an environment that refers to itself!

• In imperative Java, how would we construct such an environment. Hint: how did we build “circular” data structures in Comp 211/212? Imperativity is \textit{brute force}. But it works. We will use it in Project 3 and thereafter.
Minor Challenge

How could we define an environment that refers to itself in *functional* Scheme?

- Key problem: observe that in `let` and `lambda` the expression defining the value of a variable cannot refer to itself.
- Solution: does functional Scheme contain a recursive binding construct?
- How can we use this construct to define a recursive environment?
- What environment representation must we use?
A Bigger Challenge

Assume that we want to write LC in a purely functional language without a recursive binding construct (say functional Scheme without `define` and `letrec`)?

• Key problem: must expand `letrec` into `lambda`

• No simple solution to this problem. We need to invoke syntactic magic or (equivalently) develop some sophisticated mathematical machinery.
Key Intuitions

• Computation is incremental—not monolithic
• Slogan: general computation is successive approximation (typically in response to successive demands for more information).
• Familiar example: a program mapping a potentially infinite input stream of characters to a potentially infinite output stream of characters. Generalization: infinite trees mapped to infinite trees.
Key Mathematical Concepts

Domains of computations (like streams, trees):

- partially ordered set (po)
- finitary basis (set finite approximations)
  - countable
  - closed under LUBs on finite bounded subsets
- chain
- chain-complete
- complete partial order (cpo)
- “home-plate” cpo (not domain; finite elements not a finitary basis)
- bottom (⊥)
- flat domain (monolithic set of values formulated as domain)
  - integers, booleans, strings, conventional finite lists, ASTs
Key Mathematical Concepts

Computable functions:

- monotonic (universal)
- continuous (universal)
- strict (typical)
Examples

Domains

• flat domains
• strict function spaces on flat domains
• lazy trees of boolean (of $D$ where $D$ is flat)
• factorial functional