

COMP 481: Automata, Formal Languages, and Computability

Spring 2009

Homework Assignment #1

Due date: 15 January 2009 (beginning of class)

1. Prove or disprove the following claims.

(a) For any language L , $L \subseteq L \cdot L$ iff $\varepsilon \in L$.

(b) For any alphabet Σ , $(\Sigma^*)^* = \Sigma^*$.

(c) For any language L , L^* is infinite.

2. For two languages L_1 over $\Sigma_1 = \{a\}$ and L_2 over $\Sigma_2 = \{b\}$, we define the following two operations:

• $L_1 \oplus L_2 = \{0^{m+n} : a^m \in L_1 \text{ and } b^n \in L_2\}$.

• $L_1 \ominus L_2 = \{0^{m-n} : a^m \in L_1 \text{ and } b^n \in L_2\}$.

(a) Let $A = \{aa\}$ and $B = \{bb\}$. Define formally the languages $A \oplus B$ and $A \ominus B$.

(b) If both A and B are finite, what are the cardinalities of $A \oplus B$ and $A \ominus B$? Justify your answer.

3. For a language L over $\Sigma = \{0\}$, we define the two operations:

• $\text{SplitP}(L) = \{a^m b^n : 0^{m+n} \in L\}$.

• $\text{SplitM}(L) = \{a^m b^n : 0^{m-n} \in L\}$.

(a) Let $L = \{00, 0000\}$. Define formally the languages $\text{SplitP}(L)$ and $\text{SplitM}(L)$.

(b) If L is finite, what are the cardinalities of $\text{SplitP}(L)$ and $\text{SplitM}(L)$? Justify your answer.

4. Draw the state diagram of a deterministic finite automaton for each of the following languages.

(a) $L_1 = \{w : \exists u \in \{0, 1\}^* \text{ s.t. } (w = 1u) \vee (w = 0), \text{ and } \text{val}(w) \equiv_5 3\}$. ($\text{val}(w)$, where w is a binary string, is value of w when viewed as a number. e.g., $\text{val}(1001) = 9$.)

(b) $L_2 = \{w \in \{0, 1\}^* : \exists u, v \in \{0, 1\}^*, w = u0101v\}$.

(c) $L_3 = \{w \in \{0, 1\}^* : \forall u, v \in \Sigma^* \text{ such that } w = uv, |\#_0(u) - \#_1(u)| \leq 2\}$.

(d) $L_4 = \{w \in \{a, b, c\}^* : \#_a(w) \equiv_5 0 \text{ and } \#_b(w) \equiv_5 1 \text{ and } \#_c(w) \equiv_5 2\}$.