COMP 481: Automata, Formal Languages, and Computability
Spring 2009
Homework Assignment #2 (Due date: 22 January 2009)

1. For each of the following languages, show an NFA that accepts it.

   (a) $\{0^p10^q : p, q \geq 1, p \equiv_3 q\}$.

   (b) $\{w \in \{\sigma_1, \sigma_2, \ldots, \sigma_n\}^* : w$ begins and ends with the same character$\}$. (Write this one formally; do not draw the state diagram)

2. Convert the NFA in Exercise 9(a) in the textbook (Page 123) into a DFA.

3. (a) In homework assignment #1 we defined the following operation on a language L over $\Sigma = \{0\}$:
$$\mathrm{SplitP}(L) = \{a^m b^n : 0^{m+n} \in L\}.$$
   Prove that if L is regular, then $\mathrm{SplitP}(L)$ is also regular.

   (b) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that
$$A_{\frac{1}{2}-} = \{x : \text{for some } y, |x| = |y| \text{ and } xy \in A\}.$$
   Show that if A is regular, then so is $A_{\frac{1}{2}-}$.

4. Let $C_n = \{x : x$ is a binary number that is a multiple of $n\}$. Show that for each $n \geq 1$, the language C_n is regular.

5. This problem shows that, for certain languages, when converting an NFA to a DFA, we cannot avoid an exponential explosion in the number of states. Consider the following problem:
$$L = \{w \in \{a, b\}^* : \ |w| = n \ and \ w_{n-m+1} = a\}$$
   where w_i denotes the i^{th} symbol of w and $1 \leq m \leq n$ (m is a constant).

   (a) Describe in English the language L.

   (b) Describe an NFA with $m + 1$ states that recognizes L.

   (c) Describe a DFA with 2^m states that recognizes L.

   (d) Argue that any DFA for language L would have at least 2^m states, thus showing that 2^m is the minimum number of states needed for a DFA that recognizes language L.

6. Let Σ and Δ be two alphabets. Consider a function $h : \Sigma \to \Delta^*$. Extend h to a function from Σ^* to Δ^* as follows.

   $h(\varepsilon) = \varepsilon$.

   $h(w\sigma) = h(w)h(\sigma)$ for any $w \in \Sigma^*$ and $\sigma \in \Sigma$.

Any function $h : \Sigma^* \to \Delta^*$ defined in this way from a function $h : \Sigma \to \Delta^*$ is called a **homomorphism**. Let h be a homomorphism from Σ^* to Δ^*.

(a) Show that if $L \subseteq \Sigma^*$ is accepted by an FA, then so is $h(L)$. ($h(L) = \{h(w) : w \in L\}$.)

(b) Show that if L is accepted by an FA, then so is $\{w \in \Sigma^* : h(w) \in L\}$.