COMP 481: Automata, Formal Languages, and Computability
Spring 2009
Homework Assignment #3 (Due date: 3 February 2009)

1. Determine the equivalence classes of the relation $\equiv_L$ (equivalently, $\approx_L$)for the following two languages. If the language is regular, build an FSM for it from the equivalence classes.

   (a) $L_a = \{ww : w \in \{0,1\}^*\}$.

   (b) $L_b = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$.

   (c) $L_c = \{w \in \{a,b,c\}^* : \exists u, v \in \{a,b,c\}^* \ s.t.,\ w = ubbbv\}$.

2. Using the algorithm minDFSM (described in Section 5.7.2 in the textbook), minimize the FSM in Figure 1.

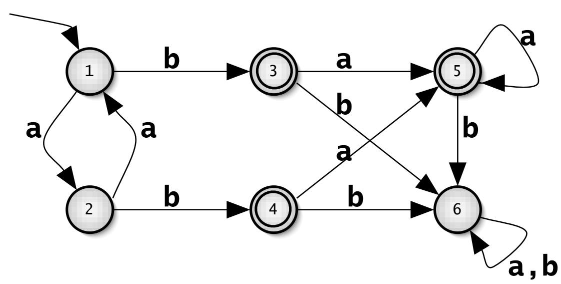

Figure 1: The FSM for problem 2.

3. Let M be a DFA with n states. Prove the following two claims.

   (a) $L(M) \neq \emptyset$ if and only if there there exists a string w, where $|w| < n$, such that $w \in L(M)$.

   (b) $L(M)$ is infinite if and only if M accepts a string w, where $n \leq |w| < 2n$.

4. For each of the following languages, determine whether the language is regular or not, and prove your answer (you are not allowed to use the Myhill-Nerode Theorem).

   (a) $L_1 = \{a^i b^j c^j : i \geq 1 \text{ and } j \geq 0\} \cup \{b^j c^k : j, k \geq 0\}$.

   (b) L_2 is the language of all binary representations of non-negative integers which are the powers of 4.

   (c) $L_3 = \{a^{n!} : n \geq 0\}$.

   (d) $L_4 = \{xyx : x, y \in \{a,b\}^*\}$.

   (e) $L_5 = \{a^i b^j : \gcd(i,j) = 1\}$.