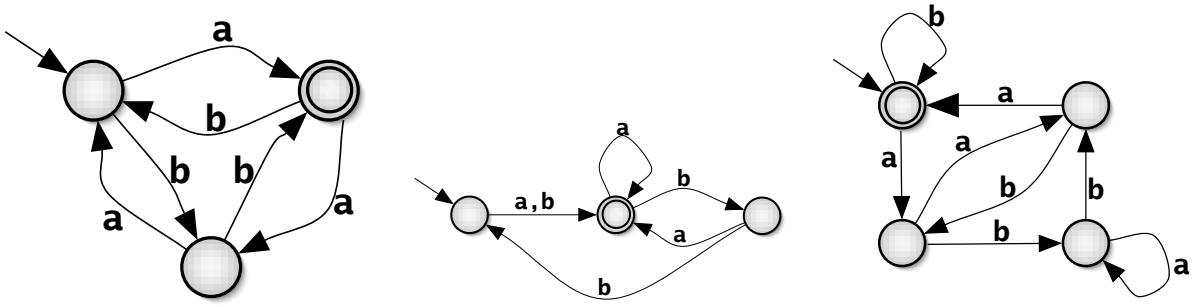
COMP 481: Automata, Formal Languages, and Computability
Spring 2009
Homework Assignment #4 (Due date: 12 Feb 2009)

1. Using the algorithm in Section 6.2.2 in the textbook, convert each FSM M_i ($i = 1, 2, 3$) in Figure 1 into an equivalent regular expression E_i.



M_1 M_2 M_3

Figure 1: The FSMs for problem 1.

2. One of the following two languages is regular and the other not. For the regular language, show a regular grammar that generates it, and for the other language, prove that it is not regular using the Pumping Lemma.

   (a) $A = \{1^k y : y \in \{0,1\}^*$ and y contains at most k 1s, for $k \geq 1\}$.

   (b) $B = \{1^k y : y \in \{0,1\}^*$ and y contains at least k 1s, for $k \geq 1\}$.

3. Consider the language $C = \{a^i b^j c^k : i, j, k \geq 0$ and if $i = 1$ then $j = k\}$.

   (a) Show that C is no regular.

   (b) Show that C acts like a regular language in the Pumping Lemma. In other words, give a pumping length n and demonstrate that C satisfies the three conditions of the Pumping Lemma for this value of n.

   (c) Explain why parts (a) and (b) do not contradict the Pumping Lemma.

4. If D is any language, define

$$D_{\frac{1}{3}-\frac{1}{3}} = \{xz : \text{for some } y, |x| = |y| = |z| \text{ and } xyz \in D\}.$$

   Show that if D is regular, then $D_{\frac{1}{3}-\frac{1}{3}}$ is not necessarily regular.

5. Give a decision procedure for the following problem: given a finite automaton M, does M accept any string whose length is greater than or equal to 20?

6. For each of the following languages, give a CFG that generates it.

   (a) $\{a^i b^j c^k : i < j \text{ or } i > k\}$.

   (b) $\{a^i b^j : i \leq j \leq 2i\}$.

   (c) $\{a^m b^n : m \geq n \text{ and } m - n \text{ is even}\}$.

   (d) $\{xc^n : x \in \{a, b\}^*,\ \#_a(x) + \#_b(x) \geq n\}$.

   (e) $\{w \in \{a, b\}^* : \forall x, y \in \{a, b\}^* \text{ such that } w = xy, \#_a(x) \geq \#_b(x)\}$.

7. Show that the following CFG is ambiguous.

$$S \to a|Sa|bSS|SSb|SbS$$

8. Find a CFG G' in CNF that generates $L(G) - \{\varepsilon\}$ for the following CFG G.

$$S \to AaA|CA|BaB$$
$$A \to aaBa|CDA|aa|DC$$
$$B \to bB|bAB|bb|aS$$
$$C \to Ca|bC|D$$
$$D \to bD|\varepsilon$$

9. For each of the following languages, give a CFG that generates it.

   (a) $\{a^i b^j c^k : i \neq j + k\}$.

   (b) $\{a^m b^n : 3m \leq 5n \leq 4m\}$.

   (c) $\{x \in \{0, 1\}^* : x \neq ww \text{ for any } w \in \{0, 1\}^*\}$.

10. Suppose Σ_1 and Σ_2 are two alphabets and the function $f : \Sigma_1^* \to \Sigma_2^*$ is a *homomorphism*; i.e., $f(xy) = f(x)f(y)$ for every $x, y \in \Sigma_1^*$. Show that if $L \subseteq \Sigma_1^*$ is a CFL, then $f(L) \subseteq \Sigma_2^*$ is also a CFL.