COMP 481: Automata, Formal Languages, and Computability
Spring 2007
Homework Assignment #4 (Due date: 20 Feb 2008)

1. For each of the following languages, determine whether the language is regular or not, and prove your answer (you are not allowed to use the Myhill-Nerode Theorem).

   (a) $L_1 = \{a^i b^j c^j : i \geq 1 \text{ and } j \geq 0\} \cup \{b^j c^k : j, k \geq 0\}$.

   (b) L_2 is the language of all binary representations of non-negative integers which are the powers of 4.

   (c) $L_3 = \{a^{n!} : n \geq 0\}$.

   (d) $L_4 = \{xyx : x, y \in \{a, b\}^*\}$.

   (e) $L_5 = \{a^i b^j : \gcd(i, j) = 1\}$.

2. One of the following two languages is regular and the other not. For the regular language, show a regular grammar that generates it, and for the other language, prove that it is not regular using the Pumping Lemma.

   (a) $A = \{1^k y : y \in \{0, 1\}^*$ and y contains at most k 1s, for $k \geq 1\}$.

   (b) $B = \{1^k y : y \in \{0, 1\}^*$ and y contains at least k 1s, for $k \geq 1\}$.

3. Consider the language $C = \{a^i b^j c^k : i, j, k \geq 0$ and if $i = 1$ then $j = k\}$.

   (a) Show that C is no regular.

   (b) Show that C acts like a regular language in the Pumping Lemma. In other words, give a pumping length n and demonstrate that C satisfies the three conditions of the Pumping Lemma for this value of n.

   (c) Explain why parts (a) and (b) do not contradict the Pumping Lemma.

4. If D is any language, define

$$D_{\frac{1}{3}-\frac{1}{3}} = \{xz : \text{for some } y, |x| = |y| = |z| \text{ and } xyz \in D\}.$$

   Show that if D is regular, then $D_{\frac{1}{3}-\frac{1}{3}}$ is not necessarily regular.

5. Give a decision procedure for the following problem: given a finite automaton M, does M accept any string whose length is greater than or equal to 20?