

COMP 481: Automata, Formal Languages, and Computability

Spring 2009

Homework Assignment #5 (Due date: 19 Feb 2009)

1. Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

(a) $\{w : w \text{ contains at least three 1s}\}$.

(b) $\{w : \text{the length of } w \text{ is odd}\}$.

(c) The empty set

2. Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$$S \rightarrow TT|U$$

$$T \rightarrow 0T|T0|\#$$

$$U \rightarrow 0U00|\#$$

(a) Describe $L(G)$ in English.

(b) Prove that $L(G)$ is not regular.

3. Find an unambiguous CFG for the language of all algebraic expressions involving parentheses, the identifier x , and the four binary operators $+$, $-$, \times , and \div .

4. For each of the following languages, describe a PDA that recognizes it.

(a) $\{a^n x : n \geq 0, x \in \{a, b\}^*, \text{ and } |x| \leq n\}$.

(b) $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } (i = j \vee j = k)\}$.

(c) $\{x \in \{a, b, c\}^* : \#_a(x) < \#_b(x) \text{ or } \#_a(x) < \#_c(x)\}$.

5. Suppose $L \subseteq \Sigma^*$ is accepted by a PDA M , and for some fixed k , and every $x \in \Sigma^*$, no sequence of moves made by M on input x causes the stack to have more than k elements. Show that L is regular.

6. For each of the following languages, describe a DPDA that recognizes it.

(a) $\{x \in \{a, b\}^* : \#_a(x) \neq \#_b(x)\}$.

(b) $\{a^n b^{n+m} a^m : m, n \geq 0\}$.

7. For each of the following languages, decide whether the language is context-free and prove your answer.

(a) $\{x \in \{a, b\}^* : \#_a(x) = (\#_b(x))^2\}$.

- (b) $\{xayb : x, y \in \{a, b\}^* \text{ and } |x| = |y|\}$.
- (c) the set of non-balanced strings of parentheses.
- (d) $\{xyx : x, y \in \{a, b\}^* \text{ and } |x| \geq 1\}$.
- (e) $\{x \in \{a, b, c\}^* : \#_a(x) = \max\{\#_b(x), \#_c(x)\}\}$.
- (f) $\{0, 1\}^* - \{(0^m 1^n)^m : m, n \geq 1\}$.

8. Construct a PDA that accepts the language

$$L = \{x \in \{a, b\}^* : 2\#_a(x) \neq 3\#_b(x)\}.$$

9. Show that if L is regular, then

$$L' = \{xz : \exists y \text{ s.t. } |x| = |y| = |z| \text{ and } xyz \in L\}$$

is context-free.

10. Consider the language $B = L(G)$, where G is the grammar given in Problem 2 above. The pumping lemma for context-free languages states the existence of a pumping length p for B . What is the minimum value of p that works in the pumping lemma? Justify your answer.
11. Let G be a CFG in Chomsky normal form that contains b variables. Show that, if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.
12. Give an example of a language that is not context free but that acts like a CFL in the pumping lemma. Prove that your example works. (hint: think of the analogous example we saw for regular languages.)