COMP 481: Automata, Formal Languages, and Computability
Spring 2008
Homework Assignment #5 (Due date: 29 Feb 2008)

1. For each of the following languages, give a CFG that generates it.

   (a) $\{a^i b^j c^k : i < j \text{ or } i > k\}$.

   (b) $\{a^i b^j : i \leq j \leq 2i\}$.

   (c) $\{a^m b^n : m \geq n \text{ and } m - n \text{ is even}\}$.

   (d) $\{xc^n : x \in \{a, b\}^*,\ \#_a(x) + \#_b(x) \geq n\}$.

   (e) $\{w \in \{a, b\}^* : \forall x, y \in \{a, b\}^* \text{ such that } w = xy, \#_a(x) \geq \#_b(x)\}$.

2. Show that the following CFG is ambiguous.

$$S \to a|Sa|bSS|SSb|SbS$$

3. Find a CFG G' in CNF that generates $L(G) - \{\varepsilon\}$ for the following CFG G.

$$S \to AaA|CA|BaB$$
$$A \to aaBa|CDA|aa|DC$$
$$B \to bB|bAB|bb|aS$$
$$C \to Ca|bC|D$$
$$D \to bD|\varepsilon$$

4. For each of the following languages, give a CFG that generates it.

   (a) $\{a^i b^j c^k : i \neq j + k\}$.

   (b) $\{a^m b^n : 3m \leq 5n \leq 4m\}$.

   (c) $\{x \in \{0, 1\}^* : x \neq ww \text{ for any } w \in \{0, 1\}^*\}$.

5. Suppose Σ_1 and Σ_2 are two alphabets and the function $f : \Sigma_1^* \to \Sigma_2^*$ is a *homomorphism*; i.e., $f(xy) = f(x)f(y)$ for every $x, y \in \Sigma_1^*$. Show that if $L \subseteq \Sigma_1^*$ is a CFL, then $f(L) \subseteq \Sigma_2^*$ is also a CFL.