

COMP 481: Automata, Formal Languages, and Computability  
Spring 2009  
Homework Assignment #6 (Due date: 17 March 2009)

1. Give a decision procedure for each of the following questions:
  - (a) Given a CFG  $G$ , is  $|L(G)| \geq 3$ ?
  - (b) Given a CFG  $G$ , does  $G$  generate any even length strings?
  - (c) Given a regular grammar  $G$ , is  $L(G)$  context-free?
2. Describe a TM that decides each of the following languages.
  - (a)  $\{a^i b^j : i < j\}$ .
  - (b)  $\{www : w \in \{a, b\}^*\}$ .
  - (c)  $\{x \in \{a, b, c\}^* : \#_a(x) = \#_b(x) = \#_c(x)\}$ .
3.
  - (a) Describe a TM that computes the sum of two numbers represented in binary.
  - (b) Describe a TM that converts binary numbers to their unary representation.
4. Suppose  $L_1, \dots, L_k$  form a partition of  $\Sigma^*$ . Prove that if  $L_i \in RE$ , for all  $1 \leq i \leq k$ , then  $L_i \in R$  as well.
5. Prove that D is closed under
  - (a) union
  - (b) concatenation
  - (c) Kleene start
  - (d) intersection
6. Prove that SD is closed under
  - (a) union
  - (b) concatenation
  - (c) Kleene start
  - (d) intersection