

COMP 481: Automata, Formal Languages, and Computability

Spring 2009

Homework Assignment #9 (Due date: 16 April 2009)

1. Show that P is closed under the Kleene star operation. That is, if $L \in P$, then $L^* \in P$. (Hint: Use dynamic programming. On input $y = y_1 \dots y_n$ for $y_i \in \Sigma$, build a table indicating for each $i \leq j$ whether the substring $y_i \dots y_j \in L^*$ for any $L \in P$.)
2. Show that, if $P=NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.
3. Show that, if $P=NP$, a polynomial time algorithm exists that produces a satisfying assignment when given a satisfiable Boolean formula.
4. If $G = (V, E)$ is a directed graph, a **feedback vertex set** is a subset of the nodes, $V' \subseteq V$, where every directed circuit in G contains at least one node from V' . If $G = (V, E)$ is an undirected graph, a **dominating set** is a subset of nodes, $V' \subseteq V$, where for every node $u \in V - V'$ there exists a node $v \in V'$ such that $(u, v) \in E$. The feedback vertex set and dominating set problems are defined as follows:

FEEDBACK-VERTEX-SET= $\{\langle G, k \rangle : G \text{ is a directed graph that has a feedback vertex set of size } \leq k\}$

DOMINATING-SET= $\{\langle G, k \rangle : G \text{ is an undirected graph that has a dominating set of size } \leq k\}$

Prove that both problems are NP-complete. (Hint: you may want to reduce from the vertex cover problem; see Section 28.6.5 in your textbook.)