Comp411 Notes - 21 Feb 2005
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1 Problem 13.5.2

Seth Nielson asked if the class could go over the solution to problem 13.5.2, which was assigned as part of the homework that was due Monday, Febuary 14. As taken from the text, the problem reads:

> Can you find a context Γ, a store μ, and two different store typings Σ_1 and Σ_2 such that both Γ, $\Sigma_1 \vdash \mu$ and Γ, $\Sigma_2 \vdash \mu$?

In other words, the problem is asking if we can have two different store typings that prove the same store, under the same environment. The solution to this problem comes about after thinking about unicity of types. We have yet to read about unicity of types with respect to a type store, so this may be a hint that the answer is yes. A counter example exists in which two different typing stores prove the same store:

Γ, $\{ \ell_1 : \text{nat} \rightarrow \text{nat}\} \vdash \{ \ell_1 : \lambda \text{x} : \text{nat}.(!\ell_1\ 17)\}$

Γ, $\{ \ell_1 : \text{nat} \rightarrow \text{bool}\} \vdash \{ \ell_1 : \lambda \text{x} : \text{nat}.(!\ell_1\ 17)\}$

2 Motivation for Subtypes

Why should we study subtypes? Say we have a function that expects a record type, for instance `f : {x:int} → int`. Furthermore, say we have some record `a`, and `a :{x:int}`, so we can say `f a`. Suppose, also, that we have some record `b`, such that `b : {x:int,y:bool}`, so we could say `f b`. Is the statement `f b` well typed? Yes! the rest of the fields in the record `b` are not important. Think of it as `f([{x:int,y:int}])b`, where the operation before the record `b` is a coersion.

This raises an interesting point: is there a general way to go from $\{\text{x}_i\text{:t}_i, \text{x}_j\text{:t}_j\}$ to $\{x_i{:}t_i\}$? In other words, is there a way to describe a valid syntactic coersion of A to B for any A and B (A $\Rightarrow$ B)? Here is a hint: can we have a function from A $\Rightarrow$ B? In general, it is any context that will allow you to use A instead of B.

For example, say A $\times$ B $\Rightarrow$ A is a valid coersion. We can have some relation $\cdot \vdash \lambda$x:A $\times$ B. fst x : A $\times$ B $\rightarrow$ A to show A $\times$ B can be used where A is expected. Ultimately what we want is some unique term (morphism) that takes us from a type A to a type B.