First-order Logic Equivalences

The following equivalences are in addition to those of propositional logic[1] (.ps)[2].
First-order Logic Equivalences

Complementation of Quantifiers	$\forall x.\ \neg\phi \equiv \neg\ \exists x.\phi$	$\exists x.\ \neg\phi \equiv \neg\ \forall x.\phi$
Interchanging Quantifiers	$\forall x.\ \forall y.\phi \equiv \forall y.\ \forall x.\phi$	$\exists x.\ \exists y.\phi \equiv \exists y.\ \exists x.\phi$
Distribution of Quantifiers	$\forall x.(\phi \wedge \psi) \equiv (\forall x.\phi \wedge \forall x.\psi)$	$\exists x.(\phi \vee \psi) \equiv (\exists x.\phi \vee \exists x.\psi)$
The following identities each assume that ψ does not have any free occurrences of variable x.		
Simplification of Quantifiers	$\forall x.\psi \equiv \psi$	$\exists x.\psi \equiv \psi$
Distribution of Quantifiers	$\forall x.(\phi \wedge \psi) \equiv (\forall x.\phi \wedge \psi)$	$\exists x.(\phi \wedge \psi) \equiv (\exists x.\phi \wedge \psi)$
	$\forall x.(\phi \vee \psi) \equiv (\forall x.\phi \vee \psi)$	$\exists x.(\phi \vee \psi) \equiv (\exists x.\phi \vee \psi)$
	$\forall x.(\phi \rightarrow \psi) \equiv (\exists x.\phi \rightarrow \psi)$	$\exists x.(\phi \rightarrow \psi) \equiv (\forall x.\phi \rightarrow \psi)$
	$\forall x.(\psi \rightarrow \phi) \equiv (\psi \rightarrow \forall x.\phi)$	$\exists x.(\psi \rightarrow \phi) \equiv (\psi \rightarrow \exists x.\phi)$

[1] http://cnx.rice.edu/modules/m10540/latest/
[2] http://www.teachLogic.org/Base/Printables/algebra-laws.ps