First-order Logic Inference rules

The following are in addition to those of propositional logic[1] (.ps)[2].
Our first-order inference rules

Abbreviation	Name	If you know...	...then you can infer
∀Intro	∀-introduction	$\phi[x/y]$	$\forall x.\phi$
y not free in a dependent premise.			
y not introduced in a dependent use of ∃Elim.			
∀Elim	∀-elimination	$\forall x.\phi$	$\phi[x/t]$
t is a term (that is free to replace in ϕ).			
∃Intro	∃-introduction	$\phi[x/t]$	$\exists x.\phi$
t is a term (that is free to replace in ϕ).			
∃Elim	exists-elimination	$\exists x.\phi$	$\phi[x/c]$
c is a new constant in the proof.			
c does not occur in the proof's conclusion.			

As usual, we use ϕ as a meta-variable to range over first-order WFFs. Similarly, t is a meta-variable for first-order terms, and c is a meta-variable for domain constants.

As a detail in ∀Elim and ∃Intro, the term t must be **free to replace** the variable x in ϕ. This means that it is *not* the case that both t contains a variable quantified in ϕ, and x occurs free within than quantifier. In short, the bound variable names should be kept distinct from the free variable names. The restriction in ∃Elim on c being new is similar.

Also in ∀Elim and ∃Intro, note that if t is a constant, then this also implies that the domain is non-empty. (Empty domains aren't very interesting, but they must be considered.)

[1] http://cnx.rice.edu/modules/m10529/latest/
[2] http://www.teachLogic.org/Base/Printables/inference-rules.ps