COMP 409/509
Assignment No. 1

Due date: Sept. 13, 2022

Note:

1. The submitted assignment needs to be typeset in LaTeX.
2. **Teamwork**: You are expected to complete this assignment in pairs. By signing your name on the assignment you are asserting that the work submitted was done collaboratively, and you have adhered to Rice University's Honor Code. At the very least, team work entails:
   - discussing each problem and agreeing on a sketch of the solution,
   - reviewing the written solutions for technical correctness and typographical errors,
   - doing a joint codewalk over all written programs,
   - dividing labor equally (roughly), and
   - being able to explain every answer as if it is your own answer.
3. Note on the submission the total number of hours put by each partner separately and by the two partners together.

Syntax of Propositional Logic:

1. Show that there are no formulas of length 2,3, or 6, but that every other length is possible.
2. Show that between every left parenthesis and right parenthesis in a formula φ there is at least one occurrence of a propositional connective.
3. Let φ be a formula, let p be the number of parentheses (left and right) in φ and let l be the length (number of symbols) of φ. What are the minimum and maximum values of p/l?
4. Let φ be a formula, let c be the number of occurrences of binary connectives ($\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$) in φ, and let s be the number of occurrences of atomic propositions in φ. Show that $s = c + 1$.
5. Give examples of formulas α and β and expressions γ and δ such that $(\alpha \wedge \beta)$ and $(\gamma \wedge \delta)$ are identical, but α is different than γ.
6. The BNF for formulas in *prefix notation* (also called *Polish notation*) is

$$Form := AP|\neg Form| \bigcirc FormForm,$$

   where $\bigcirc$ is a binary propositional connective. Note that this syntax does not use parentheses. For example, the prefix notation for $((p \wedge q) \rightarrow p)$ is $\rightarrow \wedge pqp$. Formulate and prove a unique-readability theorem for formulas in prefix notation.

7. We say that θ is a *subformula* of φ if

   - θ is φ,
   - φ is $(\neg\psi)$ and θ is a subformulas of ψ, or
   - φ is $(\psi_1 \circ \psi_2)$ and θ is either a subformulas of ψ_1 or a subformula of ψ_2.

   Let α and β be expressions. We say that β is a *subexpression* of α if $\alpha = \gamma\beta\delta$, for some expressions γ and δ. Prove that if α and β are formulas and β is a subexpression of α, then β is a subformula of α.

8. Well Formedness:

   - Describe an algorithm that checks whether a given expression is a formula.
   - Analyze the time and space requirements of your algorithm.
   - Implement the algorithm in your favorite programming language and use the program to check which of the following expressions is a formula (attach a listing of the code).

     (a) $((\neg(A \vee B)) \wedge C)$
     (b) $(A \wedge B) \vee C$
     (c) $A \rightarrow (B \wedge C)$
     (d) $((A \leftrightarrow B) \rightarrow (\neg A))$
     (e) $((\neg A) \rightarrow B \vee C)$
     (f) $(((C \vee B \wedge A) \leftrightarrow D)$
     (g) $((\vee A) \wedge (\neg B))$
     (h) $(A \wedge (B \wedge C)))$