Deduction and Refutation:

1. Use Hilbert’s Deductive System to prove that \( \vdash (p \rightarrow p) \).

2. Convert the following formulas to clausal form (CNF):
   - \( ((p \lor q) \rightarrow (s \lor t)) \)
   - \( (\neg (p \land (q \land \neg s))) \)
   - \( (\neg ((p \land q) \lor ((q \lor r) \lor (p \land r)))) \)

3. Show that the following set of clauses is refutable:
   \( \{ p \lor \neg q \lor r, q \lor r, \neg p \lor r, q \lor \neg r, \neg q \} \).

4. Use resolution to show that the following formulas are not satisfiable:
   - \( (((p \leftrightarrow (q \rightarrow r)) \land ((p \leftrightarrow q) \land (q \leftrightarrow \neg r))) \)
   - \( (\neg (((p \rightarrow q) \rightarrow \neg q)) \rightarrow (\neg q)) \)

5. Use resolution to show that the formula \( ( (\neg r \lor (p \land q)) \rightarrow ((r \rightarrow p) \land (r \rightarrow q)) ) \) is valid.

6. Use resolution to prove the soundness of Meredith’s Axiom: \( (((p \rightarrow q) \rightarrow (\neg r \rightarrow \neg s)) \rightarrow (r \rightarrow u) \rightarrow ((u \rightarrow p) \rightarrow (s \rightarrow p)) \)

7. Show that if a set \( C \) of clauses is refutable then it has a refutation whose length is at most exponential in \( |C| \).

8. A clause is Krom if it contains at most two literals, i.e., it is a 2-clause. A Krom formula is a 2-CNF formula, i.e., set of Krom clauses. Use resolution to develop a polynomial-time algorithm for satisfiability of Krom formulas.

9. Consider the \( n \times n \) discrete grid. A curve from \((0,0)\) to \((n,n)\) is a set of grid points that includes \((0,0)\) and \((n,n)\) and each point \((i,j)\) on the curve has as next point either \((i+1,j)\) or \((i,j+1)\) but not both. Similarly, a curve from \((0,n)\) to \((n,0)\) is a set of grid points that includes \((0,n)\) and \((n,0)\) and each point \((i,j)\) on the curve has as next point either \((i+1,j)\) or \((i,j-1)\) but not both. A curve from \((0,0)\) to \((n,n)\) and from \((0,n)\) to \((n,0)\) must meet. Express this in propositional logic, and prove it for \( n = 3 \) using resolution.