SAT-based conformant planning

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Goal

See whether the Planning as Satisfiability idea can be extended to Expressive Action Languages allowing for

1. Concurrency, Constraints, Nondeterminism, environmental changes, . . . : [Giunchiglia, KR'00]
2. in an effective way: [Ferraris&Giunchiglia, AAAI'00], [Castellini, Giunchiglia, Tacchella, ECP'01].

Talk overview

1. Conformant Planning via SAT
2. $\mathcal{C}$-plan overview
3. Optimization 1: Pruning possible plans,
4. Optimization 2: Introducing backjumping and learning
5. Experimental Analysis
6. Conclusions

Conformant planning

A planning problem is a triple $\langle I, D, G \rangle$ in which

- I is a formula representing the possible initial states,
- D is a formal representation describing how actions affect the world, and
- G is a formula representing the goal states.

In conformant planning, the problem is to find a *sequence* of actions which, if executed sequentially starting from any initial state, is ensured to lead to a goal state.

$\Rightarrow$ if D is deterministic and I is a singleton, then

conformant planning = classical planning ...

... however, I am not making any assumption about I, D, G, and focus is on correct, complete, optimal approaches.

A conformant planning problem

There are $\#B$ bombs. Each bomb may or may not be armed. There are also $\#C$ containers, and if a bomb is placed in a container, then it is no longer dangerous, even if it is armed.

Starting from an initial state in which the set of armed bombs is unknown, the goal is to reach a “safe” state, i.e., a state in which which each bomb is no longer dangerous.

The idea

Consider a planning problem $\pi = \langle I, D, G \rangle$. Let tr_i^D be a propositional formula corresponding to the transition relation of D.

We may divide the problem of finding a "valid" plan for π into two parts:

1. *generate* "possible" plans of length n by satisfying

$$I_0 \wedge \wedge_{i=0}^{n-1} tr_i^D \wedge G_n.$$

2. *test* whether a generated plan $\alpha^1; \ldots; \alpha^n$ is "valid" by checking that

$$I_0 \wedge \wedge_{i=0}^{n-1} \alpha_i^{i+1} \wedge \wedge_{i=0}^{n-1} tr_i^D \models G_n.$$

The idea

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If the transition relation is not total then the “test” does not work!

Conformant planning via SAT

Consider a planning problem $\pi = \langle I, D, G \rangle$. Let tr_i^D be a propositional formula corresponding to the transition relation of D.

We may divide the problem of finding a "valid" plan for π into two parts:

1. *generate* "possible" plans of length n by satisfying

$$I_0 \wedge \wedge_{i=0}^{n-1} \mathit{tr}_i^D \wedge G_n.$$

2. *test* whether a generated plan $\alpha^1; \ldots; \alpha^n$ is "valid" by checking that

$$I_0 \wedge \neg Z_0 \wedge \wedge_{i=0}^{n-1} \alpha_i^{i+1} \wedge \wedge_{i=0}^{n-1} \mathit{trt}_i^D \models G_n \wedge \neg Z_n.$$

   where trt_i^D is defined as

$$(\mathit{tr}_i^D \wedge \neg Z_i \wedge \neg Z_{i+1}) \vee ((Z_i \vee \neg \mathit{Poss}_i^D) \wedge Z_{i+1}),$$

   (Z_i is a new fluent, Poss_i^D is true for an action α in a state σ if α is executable in σ).

Algorithm: $\mathcal{C}$-SAT

$P := I_0 \wedge \wedge_{i=0}^{n-1} tr_i^D \wedge G_n;$ $\quad V := I_0 \wedge \neg Z_0 \wedge \wedge_{i=0}^{n-1} trt_i^D \wedge \neg(G_n \wedge \neg Z_n);$

function $\mathcal{C}$-SAT() **return** $\mathcal{C}$-SAT_GEN$_{\text{DLL}}$(*cnf*(P), $\{\}$).

function $\mathcal{C}$-SAT_GEN$_{\text{DLL}}$(φ, μ)

if $\varphi = \{\}$ **then return** $\mathcal{C}$-SAT_TEST(μ);

if $\{\} \in \varphi$ **then return** *False*;

if $\{$ a unit clause $\{L\}$ occurs in φ $\}$ **then return** $\mathcal{C}$-SAT_GEN$_{\text{DLL}}$(*assign*(L, φ),$\mu \cup \{L\}$);

L := $\{$ a literal occurring in φ $\}$;

return $\mathcal{C}$-SAT_GEN$_{\text{DLL}}$(*assign*(L, φ),$\mu \cup \{L\}$) **or** $\mathcal{C}$-SAT_GEN$_{\text{DLL}}$(*assign*$(\overline{L}, \varphi)$,$\mu \cup \{\overline{L}\}$).

function $\mathcal{C}$-SAT_TEST(μ)

α := $\{$the "partial" plan in $\mu\}$;

foreach $\{$"total" plan $\alpha^1; \ldots; \alpha^n$ which extends $\alpha\}$

if not SAT($\wedge_{i=0}^{n-1} \alpha_i^{i+1} \wedge V$) **then exit with** $\alpha^1; \ldots; \alpha^n$;

return *False*.

Algorithm: $\mathcal{C}$-SAT

P := $I_0 \wedge \wedge_{i=0}^{n-1} tr_i^D \wedge G_n$; V := $I_0 \wedge \neg Z_0 \wedge \wedge_{i=0}^{n-1} trt_i^D \wedge \neg(G_n \wedge \neg Z_n)$;

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- L := $\{$ a literal occurring in φ $\}$;
- **return** $\mathcal{C}$-SAT_GEN$_{\mathrm{DLL}}$(*assign*(L, φ),$\mu \cup \{L\}$) **or** $\mathcal{C}$-SAT_GEN$_{\mathrm{DLL}}$(*assign*$(\overline{L}, \varphi)$,$\mu \cup \{\overline{L}\}$).

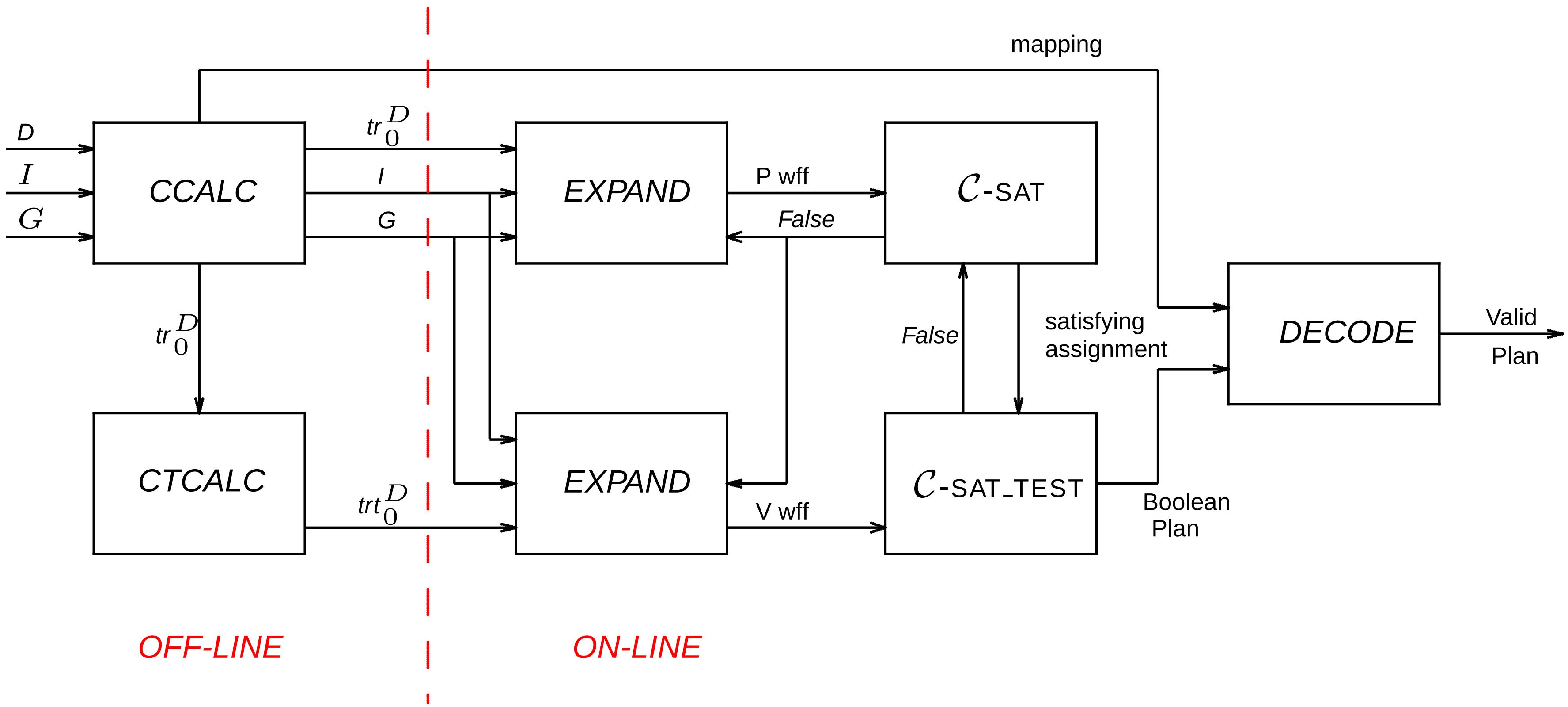
function $\mathcal{C}$-SAT_TEST(μ)

- α := $\{$the "partial" plan in $\mu\}$;
- **foreach** $\{$"total" plan $\alpha^1; \ldots; \alpha^n$ which extends $\alpha\}$
  - **if not** SAT($\wedge_{i=0}^{n-1} \alpha_i^{i+1} \wedge V$) **then exit with** $\alpha^1; \ldots; \alpha^n$;
- **return** *False*.

For any fixed n,

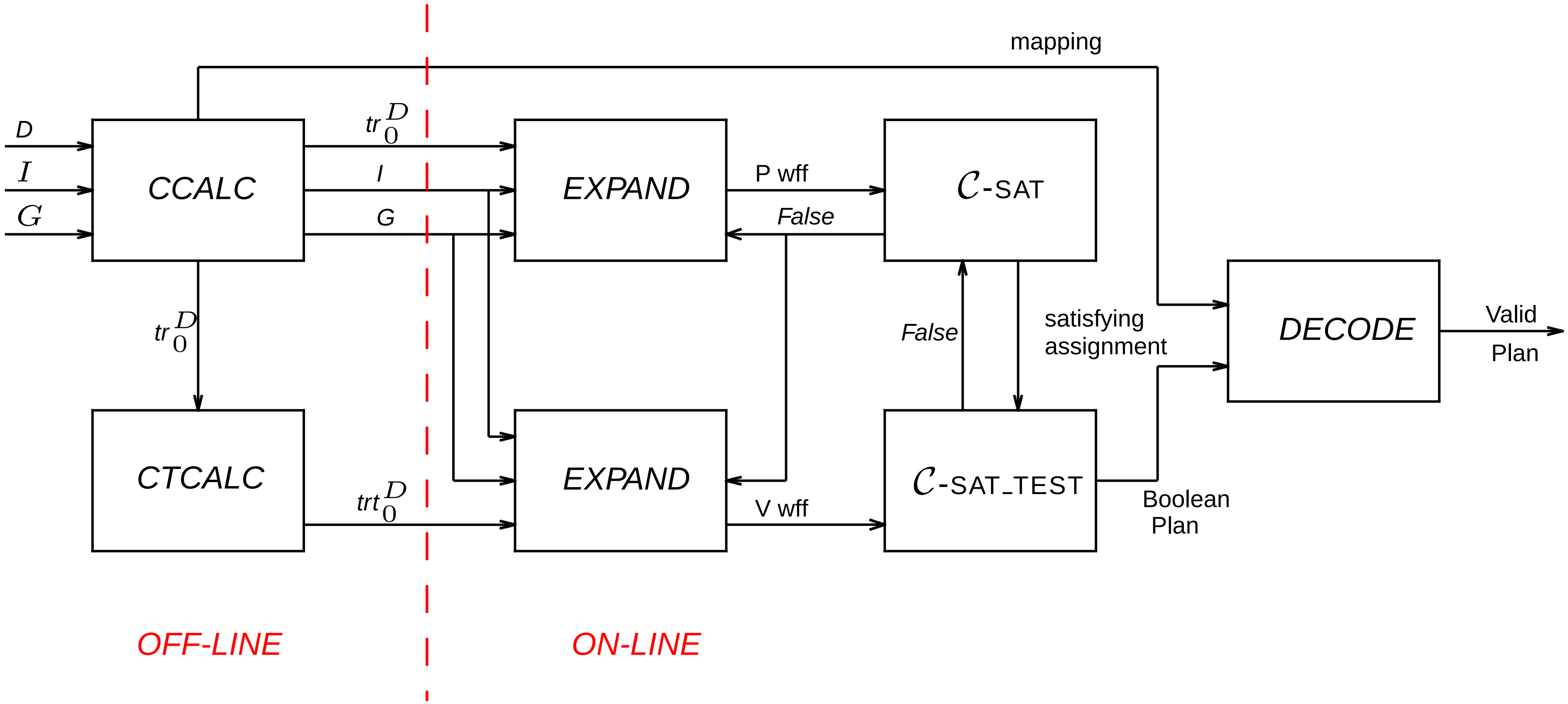
$\mathcal{C}$-SAT is correct and complete

$\mathcal{C}$-plan overview



- *CCALC* has been developed by Norman McCain,
- $\mathcal{C}$-plan first version has been implemented by Paolo Ferraris.

$\mathcal{C}$-plan overview



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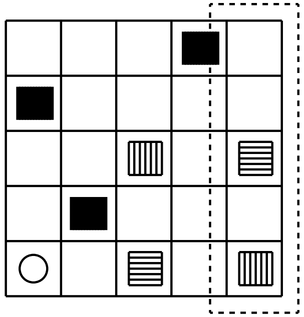
$\mathcal{C}$-plan is correct and complete

Example: Simple robot navigation problem

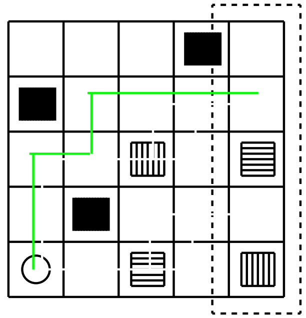


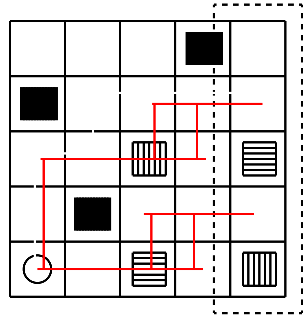
A robot (the circle) has to reach a position inside the dashed box.

Black locations are occupied by objects.

Either locations $\{(3,1),(5,3)\}$ or $\{(3,3),(5,1)\}$ are occupied.

$\mathcal{C}$-plan finds a valid plan but after generating many possible plans





Problems:

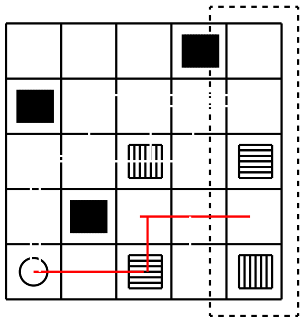
1. For each possible configuration of the obstacles, all possible plans are considered, and
2. For each fixed configuration of the obstacles, nothing is learned from previous attempts

Problem 1: all possible plans are generated

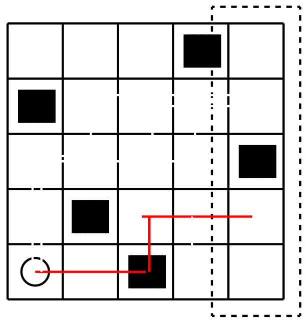
Solution: for plan generation, eliminate uncertainty in the planning problem

As soon as a possible plan is generated and rejected, the corresponding configuration of obstacles is discarded:

From



to



The new configuration is then considered for generating future possible plans.

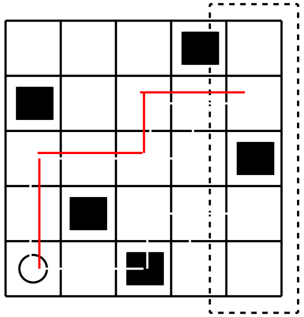
The system is still correct and complete.

Problem 2: nothing is learned from previus failures

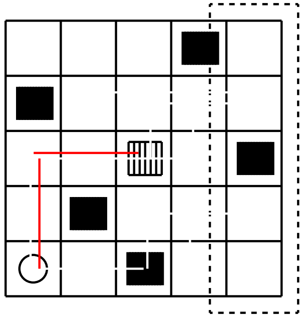
Solution: compute the reasons for failures, and introduce backjumping and learning

For a given configuration of objects, “learn” the reason for failure

From



to



Reasons are dynamically learnt and forgot, as in SAT.

The system is still correct and complete.

Comparative Analysis

Systems:

- Bonet's and Geffner's GPT [*AIPS'2000*]
- Cimatti's and Roveri's CMBP [*ECP'1999, JAIR'2000*]

Test cases:

- Purely parallel
- Purely sequential, low uncertainty
- Purely sequential, high uncertainty

Working environment:

- Pentium III, 850MHz, 512MBRAM running Linux SUSE 7.0
- Timeout at 1200s
- Systems stopped when memory requirements $>$ 512MB

Purely parallel

	GPT	CMBP		Cplan				
$\#B$-$\#C$	Total	#s	Total	#s	#pp	Last	Tot.search	Total
2-1	0.03	2	0.00	1	1	0.00	0.00	0.00
4-1	0.03	4	0.01	1	1	0.00	0.00	0.00
6-1	0.04	6	0.02	1	1	0.00	0.00	0.00
8-1	0.15	8	0.08	1	1	0.00	0.00	0.00
10-1	0.27	10	0.61	1	1	0.00	0.00	0.00
15-1	17.05	15	42.47	1	1	0.00	0.00	0.00
20-1	MEM	—	MEM	1	1	0.00	0.00	0.00

Purely Sequential, Low Uncertainty

	GPT	CMBP		Cplan				
$\#B$-$\#C$	Total	#s	Total	#s	#pp	Last	Tot.search	Total
2-1	0.10	3	0.00	3	6	0.00	0.00	0.01
2-5	0.04	2	0.01	1	1	0.00	0.00	0.00
2-10	0.05	2	0.03	1	1	0.00	0.00	0.00
4-1	0.04	7	0.00	7	540	0.12	0.15	0.65
4-5	0.23	4	0.79	1	1	0.00	0.00	0.00
4-10	2.23	4	11.30	1	1	0.00	0.00	0.01
6-1	0.09	11	0.04	11	52561	15.39	49.39	221.55
6-5	3.29	7	16.80	3	98346	56.92	57.34	419.53
6-10	74.15	—	MEM	1	1	0.00	0.00	0.01
8-1	0.41	15	0.20	—	—	—	—	TIME
8-5	32.07	11	112.48	—	—	—	—	TIME
8-10	MEM	—	MEM	1	1	0.00	0.00	0.01
10-1	2.67	19	1.55	—	—	—	—	TIME
10-5	MEM	15	974.45	—	—	—	—	TIME
10-10	MEM	—	MEM	1	1	0.00	0.00	0.04

Purely Sequential, Low Uncertainty

$\#B$-$\#C$	GPT	CMBP		Cplan				
	Total	#s	Total	#s	#pp	Last	Tot.search	Total
2-1	0.10	3	0.00	3	6	0.00	0.00	0.01
2-5	0.04	2	0.01	1	1	0.00	0.00	0.00
2-10	0.05	2	0.03	1	1	0.00	0.00	0.00
4-1	0.04	7	0.00	7	540	0.12	0.15	0.65
4-5	0.23	4	0.79	1	1	0.00	0.00	0.00
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8-5	32.07	11	112.48	—	—	—	—	TIME
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2-10	0.24	2	0.02	1	1	0.00	0.00	0.02
4-1	0.17	7	0.01	7	15	0.01	0.02	0.02
4-5	0.06	4	0.54	1	1	0.01	0.00	0.01
4-10	0.38	4	7.13	1	1	0.02	0.00	0.02
6-1	0.08	11	0.03	11	117	0.25	1.39	2.01
6-5	0.33	7	10.71	3	48	0.62	0.66	1.36
6-10	7.14	—	MEM	1	1	0.00	0.00	0.00
8-1	0.06	15	0.17	15	1195	12.23	147.25	184.29
8-5	2.02	11	90.57	3	2681	14.84	15.60	317.13
8-10	MEM	—	MEM	1	1	0.00	0.00	12.68
10-1	0.21	19	1.02	—	—	—	—	TIME
10-5	12.51	15	591.33	—	—	—	—	TIME
10-10	MEM	—	MEM	1	1	0.00	0.00	0.06

Purely Sequential, High Uncertainty

$\#B$-$\#C$	GPT	CMBP		Cplan				
	Total	#s	Total	#s	#pp	Last	Tot.search	Total
2-1	0.03	3	0.00	3	3	0.00	0.00	0.00
2-5	0.04	2	0.00	1	1	0.00	0.00	0.00
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4-5	0.06	4	0.54	1	1	0.01	0.00	0.01
4-10	0.38	4	7.13	1	1	0.02	0.00	0.02
6-1	0.08	11	0.03	11	117	0.25	1.39	2.01
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10-5	12.51	15	591.33	—	—	—	—	TIME
10-10	MEM	—	MEM	1	1	0.00	0.00	0.06

Summary

- SAT-based (conformant) planning is very flexible. It is easy to define procedures for planning in the presence of, e.g., for
  - concurrency,
  - constraints, and
  - nondeterminism
- In the presence of an uncertain initial state and/or nondeterminism, $\mathcal{C}$-PLAN
  - employs a "generate" and "test" approach
  - incorporates some optimizations, but many other are possible to improve performances, e.g., taking into account domain specific features, or by giving up optimality and/or completeness.
  - range of applicability is different from GPT's and CMBP's
  - performances are not directly correlated with the "degree of uncertainty" of the domain.

Other Symbolic approaches

- David Smith’s alternative “generate and test, anytime” approach,
- Jussi Rintanen’s QBF-based approach,
- Alessandro Cimatti’s BDD-based approach.

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