MODEL CHECKING AND FINITE-MODEL THEORY IN THE DIFFERENT CONTEXTS OF COMPUTATIONAL LOGICS, KNOWLEDGE REPRESENTATION, AND DATABASE THEORY

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THE TRUTH,

THE WHOLE TRUTH,

AND NOTHING BUT THE TRUTH

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MATH AND SCIENCE

Galileo: "The book of nature is written in the language of mathematics"

Wigner, 1963: "On the unreasonable effectiveness of mathematics in the physical sciences"

question: why is mathematics so effective?

BACK TO BASICS

- All men are mortal
- Socrates is a man
- Therefore, Socrates is mortal

Forms: $a \in P$: membership

$P \subseteq Q$: containment

Reasoning:

- Model checking: $M \in models(\varphi)$
- Validity checking: $models(k) \subseteq models(\varphi)$

The Basic Reduction: TFAE

- $models(k) \subseteq models(\varphi)$
- $models(k) - models(\varphi) = \phi$
- $models(k) \cap models(\neg\varphi) = \phi$
- Not: $\exists M, M \in models(K), M \in models(\neg\varphi)$

LE

Let there Be M!

Three fundamental techniques:

1. Canonical model Property
2. Bounded model Property
3. Tree model Property

LOGIC AND COMPUTER SCIENCE

Halpern, Kolaitis, Papadimitriou, Vardi, Vianu, 1999: "On the Unreasonable effectiveness of logic in computer science" (AAAS Symp.)

IFCOLOG: International Federation of Computational Logicians (an umbrella organization, covering 10,000 members)

Question: Why is logic so effective?

Reminder: For its first 2,400 years logic was part of philosophy.

LOGIC AND AI

McCarty, 1958: Reasoning = Logic

- Knowledge representation:
  collection K of sentences
- Knowledge generation:
  Deduction: $K \vdash \varphi$

Difficulties:

- Knowledge representation requires powerful logics
- powerful logics make deduction hard
- classical deduction is monotonic

Claim: Very limited success, inspite of 40 years of research

Impact of Logic

Question: what concrete, significant impact did logic have on computing technology?

Answers:

- Propositional logic: circuits
- First-order logic: database queries

SQL

SQL: essentially 1st-order

- conceived as user-friendly
- used as an inter-DBMS language

Example: list pairs of persons such that each directed the other in a movie.

Movies(X, Y): X directed Y

- FOL: $Movies(X,Y) \wedge Movies(Y,X)$
- SQL:

```
Select m1.director
       m2.actor
From   Movies m1
       Movies m2
Where  m1.director = m2.actor
and    m1.actor = m2.director
```

Formulas as Queries

$\varphi(x_1, \ldots x_n)$: formula

$\mathcal{B}$: relational structure

$$\varphi(\mathcal{B}) = \{\langle a_1, \ldots a_n \rangle : \mathcal{B} \models \varphi(a_1, \ldots a_n)\}$$

Key points:

- Based on Tarski's notion of truth
- Codd elevated the status of formulas

MODEL CHECKING

The basic idea:

- Represent a finite-state program as a finite structure
- check that the structure is a model of the spec [CES, QS]

Success:

- [AK85]: "one of the most exciting development in the theory of program correctness"
- 1981-1995: the "push" period
- 1995- : the "pull" period

Formulas as Specs

φ : temporal formula

K : kripke structure

$\varphi(K) = \{ w : K, w \models \varphi \}$

key points:

- Based on kripke's notion of truth
- Pnueli proposed using temporal logic to specify ongoing behavior

The Search for the Truth

Truth: satisfaction in one structure

validity: satisfaction in all structures

claims:

- In database query evaluation and in temporal model checking the focus is on truth.
- Automated reasoning should include truth checking, in addition to validity checking.

VALIDITY IS IMPORTANT

Examples:

- Query optimization:

  Is $Q_1 \equiv Q_2$?

- Spec correctness:

  Is φ valid?

Observation: very different algorithms for truth and validity checking.

Question: Can we unify truth and validity checking?

TRANSITION SYSTEMS

$T = (S, S_0, \Lambda, O, \Pi)$ kripke structure

S: states

$S_0 \subseteq S$: initial states

$\Lambda \subseteq S^2$: transition relation

O: observables

$\Pi : S \to 2^O$: observation function

key insight: every discrete system can be modeled as a labeled graph.

Temporal Specs

Temporal specs: description of ongoing behavior

Examples:

- process 1 and process 2 can not enter the critical section at the same time (safety)
- Every request will be granted (liveness)
- Every continous request will be granted (liveness)
- Every repeated request will be granted (liveness)

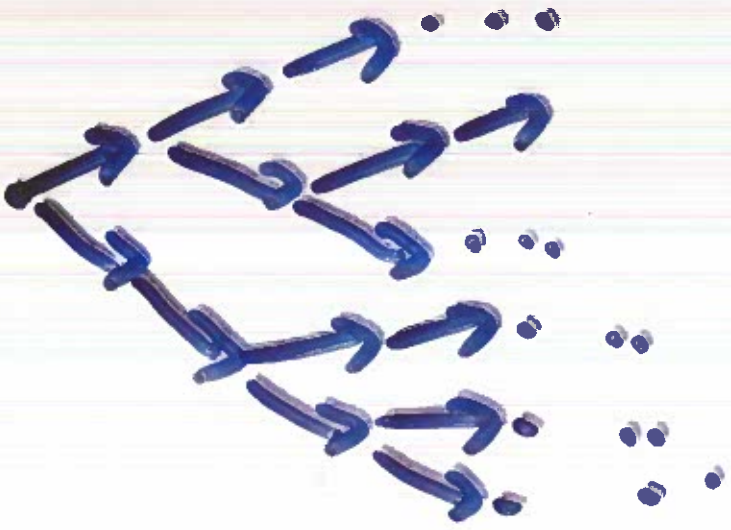
Linear vs. Branching

Linear time: a program generates a set of computations

Specs: describe computations

Branching time: a program generates a computation tree

Specs: describe computation trees



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PROGRAM EQUIVALENCE

P_1:

P_2:

Linear: $P_1 \equiv P_2$

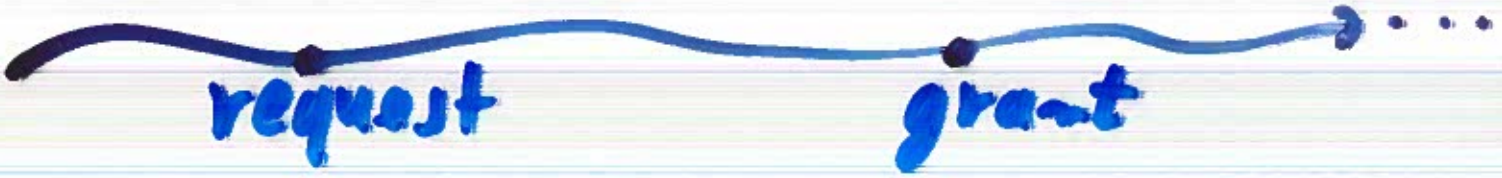
Branching: $P_1 \not\equiv P_2$

Temporal Logics

Linear Temporal Logic (LTL):

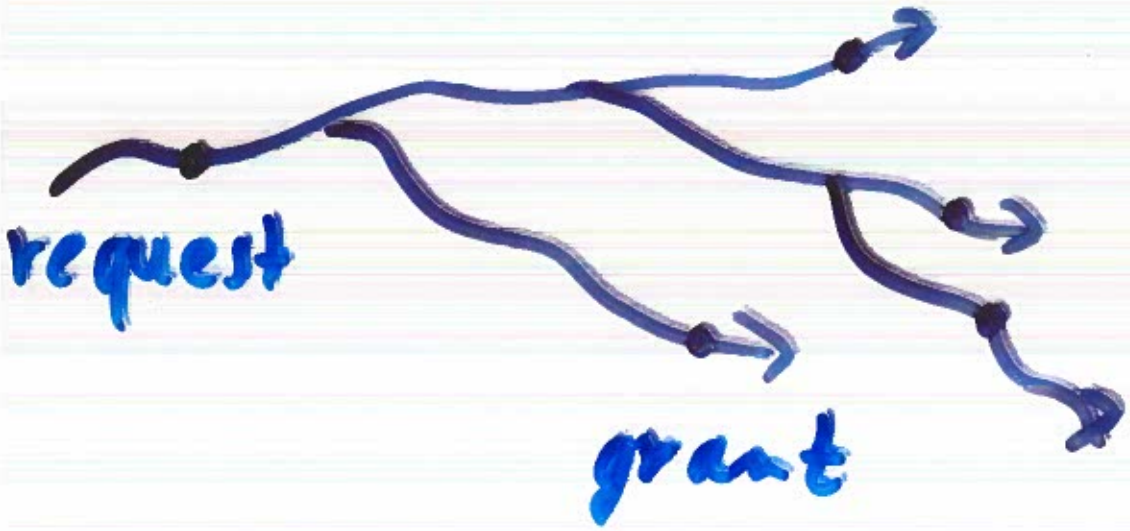
always (request → eventually grant)

Every request is eventually granted



Computation-Tree Logic (CTL):

∀ always (request → ∀ eventually grant)



TRUTH vs. VALIDITY

TRUTH: $\mathcal{M} \models \varphi$

↑ one structure

VALIDITY: $\models \varphi$

↑ all structures

Traditional focus:
Necessary truth = validity

Completeness Theorem:
$K \models \varphi$ iff $K \vdash \varphi$

TRUTH + VALIDITY

φ: spec $|\varphi| = m$

T: system $|T| = m$

Upper Time Bounds:

	LTL	CTL	
truth	$m \cdot 2^m$	$m \cdot m$	LP CES
validity	2^m	2^m	SC EH

Key Points

1. Low complexity in m
2. Different complexity in m for CTL.

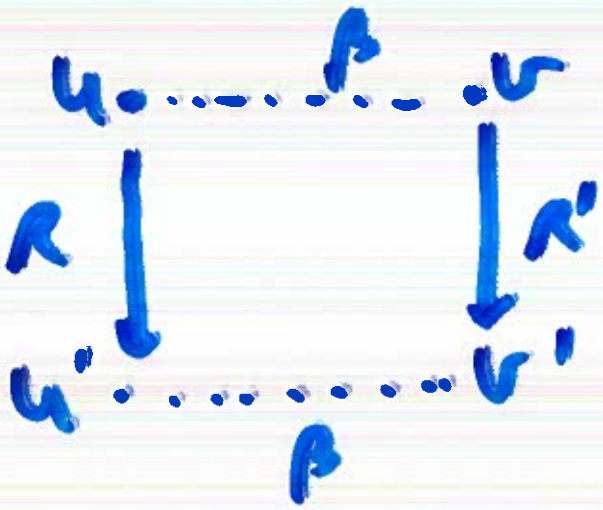
BISIMULATION

$T = (S, R, O, \pi)$

$T' = (S', R', O, \pi')$

Bisimilarity: $\beta \subseteq T \times T'$

$\beta(u, v) \Rightarrow$

- $\pi(u) = \pi'(v)$
- 

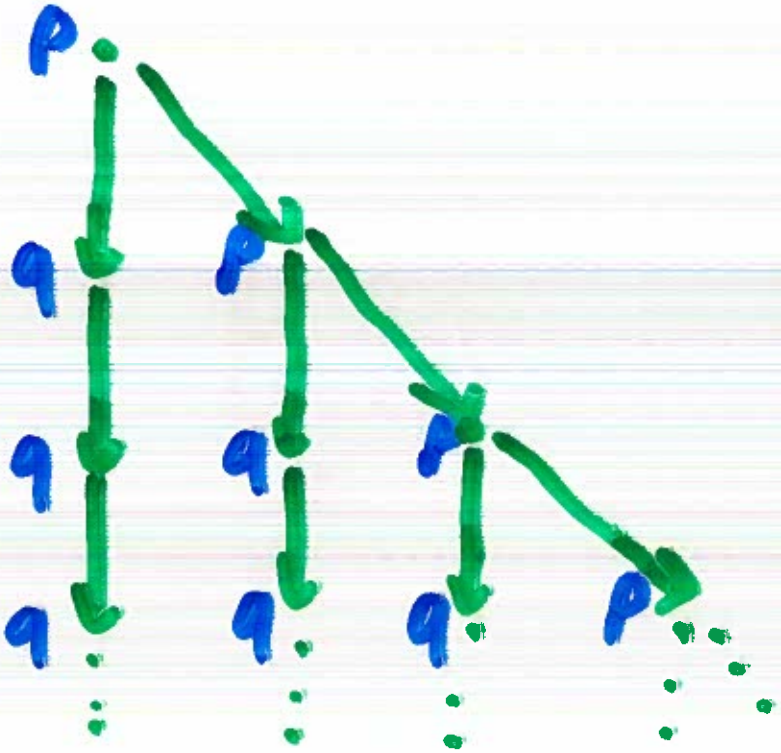
Fact: $\beta(u, v) \Rightarrow$

$T, u \models \varphi$ iff $T, v \models \varphi$

for $\varphi \in ML, LTL, CTL$

TREE MODEL PROPERTY

Fact: A transition system is bisimilar to its unfolding



Corollary:

- $T \Vdash \varphi$ iff $\mathrm{unfold}(T) \Vdash \varphi$.
- $\Vdash \varphi$ iff φ is true in all $|\varphi|$-ary trees.

Linear Temporal Logic

LTL: logic of temporal sequences

Main feature: time is implicit

- next φ: φ holds next
- eventually φ: φ holds eventually
- always φ: φ holds always
- φ until ψ: φ holds until ψ holds

Examples

- always($\neg cs_1 \vee \neg cs_2$):
  safety
- always(request → eventually grant)
  liveness
- always(request → request until grant)
  liveness
- always eventually request → eventually grant
  liveness

Semantics

Computation: $\sigma : \mathbb{N} \to 2^{\mathcal{O}}$

- $\sigma \models p$ if $p \in \sigma(0)$
- $\sigma \models \text{next } \psi$ if $\text{tail}(\sigma) \models \psi$
- $\sigma \models \text{always } \psi$ if $\text{tail}^i(\sigma) \models \psi$ for $i \geq 0$

$T = (S, S_0, R, \mathcal{O}, \Pi)$

Run: $w_0, w_1, w_2, \ldots$

- $w_0 \in S_0$
- $(w_i, w_{i+1}) \in R$

Computation: $\Pi(w_0), \Pi(w_1), \ldots$

- $L(T)$: computations of T
- $T \models \psi$: all computations of T satisfy ψ

BÜCHI AUTOMATA

$A = (\Sigma, S, S_0, \rho, F)$

- Σ : alphabet
- S : states
- $S_0 \subseteq S$: initial states
- $\rho \subseteq S \times \Sigma \times S$: transition relation
- F : accepting states

Input: $a_0, a_1, \ldots$

Run : $s_0, s_1, s_2 \ldots$

- $s_0 \in S_0$
- $(s_i, a_i, s_{i+1}) \in \rho$

Acceptance: F visited i.o.

$L(A)$: lang. accepted by A

Properties

- **Closure:** Let A_1, A_2 be Büchi automata w. m_1, m_2 states. There is a Büchi automata $A_1 \times A_2$ w. $2m_1m_2$ states s.t. $L(A_1 \times A_2) = L(A_1) \cap L(A_2)$

- **Emptiness:** There is a linear-time algorithm that checks whether $L(A) = \emptyset$.

LTL vs. AUTOMATA

Compilation Thm: Given an LTL formula φ, we can construct a Büchi automaton A_φ w. $2^{O(|\varphi|)}$ states such that σ satisfies φ iff A_φ accepts σ.

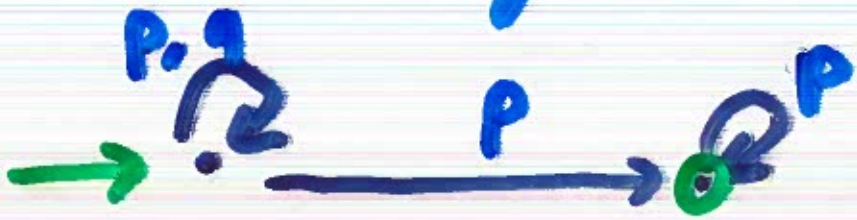
Application:

validity checking: φ is valid iff $L(A_{\neg\varphi}) = \emptyset$ - yields an algorithm of time $2^{O(|\varphi|)}$

Examples

- always eventually p:

- eventually always p:



TRUTH CHECKING

T.F.A.E:

- $T \models \varphi$
- $L(T) \subseteq L(A_\varphi)$
- $L(T) - L(A_\varphi) = \emptyset$
- $L(T) \cap L(A_{\neg\varphi}) = \emptyset$
- $L(T \times A_{\neg\varphi}) = \emptyset$

Application: Truth checking algorithm of time $|T| \cdot 2^{O(|\varphi|)}$

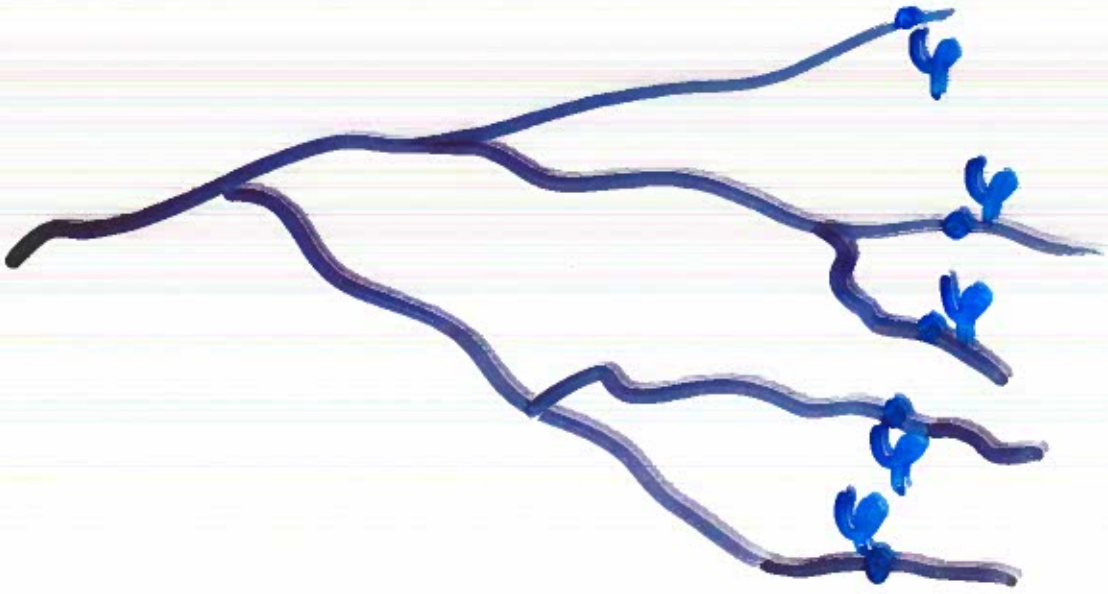
Bottom line: Büchi automata unify truth and validity checking for LTL.

COMPUTATION TREE LOGIC

CTL: logic of temporal trees

Main feature: quantification over possible futures

- $\forall$ always φ: φ holds always
- $\forall$ eventually φ: φ is inevitable
- $\exists$ always φ: always φ is possible
- $\exists$ eventually φ: φ is possible



BÜCHI TREE AUTOMATA

$A = (\Sigma, S, S_0, \rho, F)$

- Σ, S, S_0, F : same
- $\rho \subseteq S \times \Sigma \times S \times S$: transition relation

Input: $\tau : \{0,1\}^* \to \Sigma$

Run : $r : \{0,1\}^* \to S$

- $r(\varepsilon) \in S_0$
- $(r(x), \tau(x), r(x0), r(x1)) \in \rho$

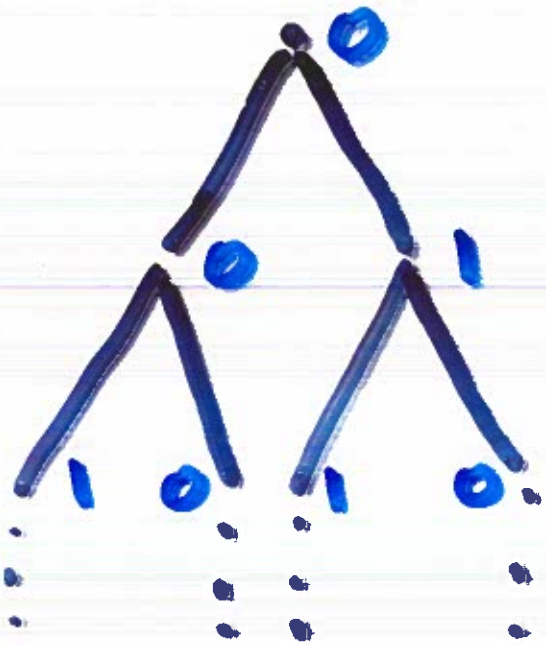
Acceptance: F visited i.o. along every branch

L(A): Tree language of A

n

$t \quad u$

- $n \in S_0$
- $(n, o, t, u) \in P$



PROPERTIES

- closure under intersection
- Quadratic-time emptiness checking algorithm (i.e., $L(A) = \emptyset$?)

CTL vs. AUTOMATA

Compilation Thm: Given a CTL formula φ, we can construct a Büchi tree automaton A_φ w. $2^{O(|\varphi|)}$ states such that τ satisfies φ iff A_φ accepts τ.

Application:

Validity checking: φ is valid iff $L(A_{\neg\varphi}) = \emptyset$ - yields an algorithm of time $2^{O(|\varphi|)}$

CTL TRUTH CHECKING

Problem: CTL truth checking is linear in $|\varphi|$, but $A\varphi$ is exponential in $|\varphi|$.

Solution: alternation

non determinism: $\exists$-choice

- $\exists$-choice: some run accepts
- $\forall$-choice: all runs accept

Alternation: $\exists + \forall$-choice

NONDETERMINISM VS. ALTERNATION

Nondeterminism:

- $(q, a) \longmapsto \{q_1, q_2, q_3\}$
- $\delta(q, a) = q_1 \vee q_2 \vee q_3$

Alternation:

$$\delta(q, a) = (q_1 \wedge q_2) \vee (q_3 \wedge q_4)$$

In general:

$$\delta : S \times \Sigma \to B^+(S)$$

positive formulas on S

LTL vs. AUTOMATA

Compilation Thm: Given an LTL formula φ, we can construct an alternating Büchi automaton A^a_φ w. $O(|\varphi|)$ states such that σ satisfies φ iff A^a_φ accepts σ.

Fact: There is an exponential translation from alt. Büchi automata to nondet. Büchi automata.

Application: LTL $\xrightarrow{\text{exp}}$ nondet. Büchi automata

Example

φ: always eventually p

φ': eventually p

$$A^a_\varphi = (\Sigma, S, S_0, \rho, F)$$

$$\Sigma = 2^{\{p\}}, \quad S = \{\varphi, \varphi'\}, \quad S_0 = F = \{\varphi\}$$

$$\rho(\varphi, p) = \varphi$$

$$\rho(\varphi, \bar{p}) = \varphi \wedge \varphi'$$

$$\rho(\varphi', p) = \text{true}$$

$$\rho(\varphi', \bar{p}) = \varphi'$$

ALT. TREE AUTOMATA

Non determinism:

- $(q, a) \mapsto \{\langle q_1, q_2\rangle, \langle q_3, q_4\rangle\}$
- $\rho(q, a) = [(1, q_1) \wedge (2, q_2)] \vee [(1, q_3) \wedge (2, q_4)]$

Alternation:

$\rho(q, a) = (1, q_1) \vee [(2, q_2) \wedge (2, q_3)]$

In general:

$\rho : S \times \Sigma \to B^+(\{1, 2\} \times S)$

CTL vs. AUTOMATA

Compilation Theorem: Given a CTL formula φ, we can construct an alt. Büchi tree automaton A^a_φ w. $O(|\varphi|)$ states such that τ satisfies φ iff A^a_φ accepts τ.

Fact: Emptiness problem for alt. Büchi tree auto. is EXPTIME-complete.

- good for validity checking
- bad for truth checking

TRUTH CHECKING

T. F. A. E:

- $T \models \varphi$
- $\text{Unfold}(T) \in L(A^q_\varphi)$
- $L(T \times A^q_\varphi) \neq \emptyset$

Crux: $T \times A^q_\varphi$ is on a 1-letter alphabet

Fact: There is a quadratic time alg. that checks emptiness of 1-letter aut.
Büchi: tree automata.

Application: Quadratic truth checking algorithm.

BETTER COMPLEXITY

Fact: If φ is a CTL formula, then A^a_φ is a weak alternating automaton [MS]

Fact: There is a linear-time algorithm that checks emptiness of 1-letter WAA.

Application: linear-time truth checking algorithm for CTL.

ultimate Unification

- Truth checking: 1-letter emptiness
- validity checking: 2-letter emptiness

Unification:

2-letter emptiness is solved via a (nontrivial) reduction to 1-letter emptiness

In Conclusion

1. Tree-model property is often more useful than finite-model property.

2. Tree automata are useful in automated reasoning.

3. Truth and validity checking can be unified.

Future work: other examples of such unification.

LTL vs. CTL

- eventually always p:

  "p is ultimately high"

- ∀ eventually ∀ always p:

  "p will stabilize high after a bounded number of steps"

Benefit: Nonmonotonicity

Domain — Relations

$\mathcal{M}_1 = (D_1, R^1_1, \ldots R^1_k)$

$\mathcal{M}_2 = (D_2, R^2_1, \ldots, R^2_k)$

$\mathcal{M}_2$ substructure of $\mathcal{M}_1$: $\mathcal{M}_2 \subseteq \mathcal{M}_1$

- $D_2 \subseteq D_1$
- $R^2_i = R^1_i | D_2$

Well-known:

- $\mathcal{M}_1 \models \varphi \not\Rightarrow \mathcal{M}_2 \models \varphi$
- $\mathcal{M}_2 \models \varphi \not\Rightarrow \mathcal{M}_1 \models \varphi$

Truth is not monotonic!

Benefit: Logical Omniscience

Hintikka, 1958: possible-worlds approach to epistemic reasoning -

A knows φ if φ holds in all worlds that A considers to be possible

Consequence: logical omniscience -

A knows all consequences of her knowledge, including all tautologies

Is this problematic?

Suppose A has 3 possible worlds, then knowing all tautologies is easy!

MODEL-CHECKING MANIFESTO

Halpern + Vardi, 1991:

Reasoning = Truth Checking =
Model Checking

Knowledge Representation:

semantic structure M

Knowledge generation:

Model checking $M \models \varphi$

Example: muddy children

Claim: Propositial satisfiability and model checking have so far been the only successful modes of automated reasoning

CONJUNCTIVE QUERIES

Select-Project-Join queries:

$FO(+, \wedge, \exists)$:

- Positive atomic formulas
- Conjunction
- Existential quantification

Example:

grandparent(x,y) :- PARENT(x,z), PARENT(z,y)

Key notion: $Q_1 \sqsubseteq Q_2$: for each db B,

$Q_1(B) \subseteq Q_2(B)$

Equivalently: $Q_1 \models Q_2$

Note: $Q_1 \equiv Q_2$ iff $Q_1 \sqsubseteq Q_2$ & $Q_2 \sqsubseteq Q_1$

CANONICAL MODEL PROPERTY

Chandra + Merlin, 1976:

Conjunctive Query ≈ Relational DB

$$CQ \longmapsto B^{Q}$$

Example: Grandparent(x, y) :- Parent(x, z), Parent(z, y)

B^{Q} :

Parent	
x	z
z	y

Thm: [CM'76]

$$Q_1 \models Q_2 \iff B^{Q_1} \models Q_2$$

$Q_1 \models Q_2$: logical implication

$B^{Q_1} \models Q_2$: model checking

canonical model: B^{Q_1}

BOUNDED MODEL PROPERTY

FOL^k: FOL with k variables

Fact: many description logics $\subseteq FO^2$

Validity checking:

FOL: undecidable

FOL^3: undecidable

FOL^2: co-NEXPTIME-complete

Theorem: [Grädel, Kolaitis, Vardi, 1997]
If an FO^2 formula is satisfiable, then it has a model of exponential size.

TREE-Model property

PA: Presburger Arithmetics = Theory of natural addition

$$\forall x \forall y \forall z ((x+y)+z = x+(z+y))$$

$$models(\varphi(x,y,z)) = \{(a,b,c) : \mathbb{N} \models \varphi(a,b,c)\}$$

$$Language(\varphi(x,y,z)) = \{ a_0 b_0 c_0 \, a_1 b_1 c_1 \ldots a_n b_n c_n : \mathbb{N} \models \varphi(a,b,c) \}$$

$a_0 a_1 \ldots a_n$: binary encoding of a
$b_0 b_1 \ldots b_n$: binary encoding of b
$c_0 c_1 \ldots c_n$: binary encoding of c

Theorem: [Cobham, 1965]

Language $(\varphi(x,y,z))$ is regular

Proof: Induction: constructive proof

$L(A_\varphi) = Language(\varphi)$

A_φ : automaton

Decidability

Algorithm for checking satisfiability:

1. Construct A_φ
2. check $L(A_\varphi) \neq \emptyset$

Nonemptiness Test:

$A = (\Sigma, S, S_0, \rho, F)$

Σ: alphabet

S: states

$S_0 \subseteq S$: initial states

$\rho : S \times \Sigma \to 2^S$: transition function

$F \subseteq S$: accepting states

$G_A = (S, E_A)$, $E_A = \{(s,t) : t \in \rho(s,a)\}_{a \in \Sigma}$

Fact: $L(A) \neq \emptyset$ iff there is a path from S_0 to F in G_A.

MEMBERSHIP TEST

Membership problem: $A = (\Sigma, S, S_0, \rho, F)$

$w \in \Sigma^*$

$w \stackrel{?}{\in} L(A)$

Let $w = a_0 a_1 \dots a_{n-1}$

Define: $A^w = (\{a\}, S \times \{0, \dots, n\}, S_0 \times \{0\}, \rho^w, F \times \{n\})$

$$\rho^w((s,i),a) = \{(t, i+1) : t \in \rho(s, a_i)\}$$

Intuitively: A^w is A "specialized" to w.

Lemma: $w \in L(A)$ iff $L(A^w) \neq \emptyset$

Moral: Emptiness testing and membership testing coincide for FSA.

Next: Temporal reasoning from this perspective.