View-Based Query Processing for Regular Path Queries with Inverse

Diego Calvanese
Giuseppe De Giacomo
Maurizio Lenzerini
Dipartimento di Informatica e Sistemistica
Università di Roma “La Sapienza”
Via Salaria 113, I-00198 Roma, Italy
lastname@dis.uniroma1.it

Moshe Y. Vardi
Department of Computer Science
Rice University, P.O. Box 1892
Houston, TX 77251-1892, U.S.A.
vardi@cs.rice.edu

ABSTRACT

View-based query processing is the problem of computing the answer to a query based on a set of materialized views, rather than on the raw data in the database. The problem comes in two different forms, called query rewriting and query answering, respectively. In the first form, we are given a query and a set of view definitions, and the goal is to reformulate the query into an expression that refers only to the views. In the second form, besides the query and the view definitions, we are also given the extensions of the views and a tuple, and the goal is to check whether the knowledge on the view extensions logically implies that the tuple satisfies the query.

In this paper we address the problem of view-based query processing in the context of semistructured data, in particular for the case of regular-path queries extended with the inverse operator. Several authors point out that the inverse operator is one of the fundamental extensions for making regular-path queries useful in real settings. We present a novel technique based on the use of two-way finite-state automata. Our approach demonstrates the power of this kind of automata in dealing with the inverse operator, allowing us to show that both query rewriting and query answering with the inverse operator has the same computational complexity as for the case of standard regular-path queries.

1. INTRODUCTION

View-based query processing is the problem of computing the answer to a query based on a set of materialized views, rather than on the raw data in the database. It represents an abstraction for various data management problems arising in several contexts, including query optimization, query answering with incomplete information, data warehousing, and data integration.

There are two approaches to view-based query processing, called query rewriting and query answering, respectively. In the former approach, we are given a query $Q$ and a set of view definitions, and the goal is to reformulate the query into an expression that refers only to the views, and provides the answer to $Q$. The rewriting is usually expressed in the same language used for both the query $Q$ and the views, and is called exact if it is equivalent to $Q$. In the latter approach, besides $Q$ and the view definitions, we are also given the extensions of the views. The goal is to compute the set of tuples $t$ such that the knowledge on the view extensions logically implies that $t$ is an answer to $Q$, i.e., $t$ is in the answer to $Q$ in all the databases that are consistent with the views.

Notice the difference between the two approaches. In query rewriting, query processing is divided in two steps, where the first re-expresses the query in terms of a given query language over the alphabet of the view names, and the second evaluates the rewriting over the view extensions. Obviously, it may happen that no rewriting in the target language exists that is equivalent to the original query. In this case, we are interested in computing a so-called maximally contained rewriting, i.e., an expression that captures the original query in the best way.

In query answering, we do not pose any limit on how queries are processed, and the only goal is to exploit all possible information, in particular the view extensions, to compute the answer to the query. Since the view extensions provide partial knowledge on the database, the problem is a special case of query answering with incomplete information.

A large number of results have been reported for both problems in the last years. Several papers investigate query rewriting for the case of conjunctive queries (with or without arithmetic comparisons) [25, 28], disjunctive views [5], queries with aggregates [30; 15], recursive queries and non-recursive views [16], queries expressed in Description Logics [6], regular-path queries [10], and in the presence of integrity constraints [22; 17]. Rewriting techniques for query optimization are described, for example, in [13; 4; 31], and
In [19; 26; 27] for the case of path queries in semistructured data.

A comprehensive framework for view-based query answering, as well as several interesting results, is presented in [21]. The framework considers various assumptions for interpreting the view extensions with respect to the corresponding definitions (closed, open, and exact view assumptions). In [2], an analysis of the data complexity of the problem under the different assumptions is carried out for the case where the views and the queries are expressed in terms of various languages (conjunctive queries, Datalog, first-order queries, etc.). In [11], a further distinction is proposed for characterizing the domain of the database (open vs. closed domain assumption), and the problem is studied for the case of regular-path queries. In [9], upper bound results are reported for view-based query answering in Description Logics.

In this paper, we address view-based query processing in the context of semistructured data. Semistructured data are modeled by labeled directed graphs, and have been introduced with the aim of capturing data in the web. The main difficulty arising in this context is that languages for querying semistructured data enable expressing regular-path queries [3; 8; 18]. A regular-path query asks for all pairs of nodes in the database connected by a path conforming to a regular expression, and therefore may contain a restricted form of recursion. Note that when the query contains unrestricted recursion, both view-based query rewriting and view-based query answering become undecidable, even when the views are not recursive [16].

To the best of our knowledge, the only known decidability results for view-based query processing in the case where the query and the views may contain recursion are reported in [10; 11].

In [10], a method for rewriting a regular-path query in terms of other regular-path queries is proposed. The method computes the rewriting in 2EXPTIME, and is able to check whether the computed rewriting is exact in 2EXPSPACE. It is also shown that checking whether there is a nonempty rewriting is EXPSPACE-complete, and verifying the existence of an exact rewriting is 2EXPSPACE-complete.

In [11], algorithms and complexity results for view-based query answering for regular-path queries are presented. With respect to data complexity, the problem is co-NP-complete under all assumptions, whereas with respect to combined complexity it is co-NP-complete under the closed domain assumption, and PSPACE-complete under the open domain assumption.

While regular-path queries represent the core of any query language for semistructured data, their expressive power is limited. Several authors point out that extensions are required for making them useful in real settings (see for example [7; 8; 26]). One fundamental step in this direction is to enrich regular-path queries with the inverse operator. The inverse operator is essential for expressing navigations in the database that traverse the edges both backward and forward [14].

The existing approaches to view-based query processing for regular-path queries do not easily extend to the case with the inverse operator. The goal of this paper is to present a novel technique for this case, based on the use of two-way finite-state automata. Unlike standard finite-state automata, two-way automata are equipped with a head that can move back and forth on the input string. A transition of this kind of automata indicates not only the new state but also whether the head should move left, right, or stay in place.

Generally speaking, the method presented here works by searching for counterexamples. In the case of rewriting, a counterexample demonstrates that a certain rewriting is not good for the query, whereas in the case of answering, the counterexample demonstrates that a given tuple is not an answer to the query. The basic idea underlying our techniques is to encode candidate counterexamples as words over a suitable alphabet, and then use two-way automata to check that a candidate is indeed a counterexample. The ability of two-way automata to move back and forth on words is essential in capturing the computation done when processing a regular-path query with inverse over a database, and therefore in checking candidate counterexamples. Indeed, the power of two-way automata in dealing with the inverse operator allows us to derive the following results:

- View-based query rewriting can be done with the same computational complexity as for the case without inverse. Indeed, we describe a method based on two-way automata that computes the maximal contained rewriting in 2EXPTIME, and checks whether the rewriting is exact in 2EXPSPACE.

- We provide algorithms for view-based query answering under different assumptions. With respect to data complexity, the algorithm works in co-NP in all cases. With respect to combined complexity, the algorithm works in co-NP under the closed domain assumption, and in PSPACE under the open domain assumption. Once again, by exploiting two-way automata, we are able to show that the complexity of the problem does not increase by adding the inverse operator.

Besides the specific results, our method provides the basis for using two-way automata in reasoning about complex queries. Indeed, the idea of encoding candidate counterexamples as words, and then checking candidates with two-way automata, can be adapted to reasoning about queries of more general forms and in other logical formalisms (e.g., temporal and dynamic logics with a backward modality). A first step in this direction is presented in [12], where the technique is used to show that containment of conjunctive regular-path queries with inverse is EXPSPACE-complete.

The paper is organized as follows. Section 2 defines regular-path queries with inverse, and Section 3 introduces the notion of two-way automata. Sections 4 and 5 present the results for query rewriting and query answering, respectively. Section 6 concludes the paper.
2. REGULAR-PATH QUERIES WITH INVERSE

Following the usual approach in semistructured data, we define a (semi-structured) database (DB) as a graph whose nodes represent objects, and whose edges are labeled by elements from a given alphabet \( \Sigma' \), which we assume to be finite \([7; 1]\). Different nodes represent different objects, and an edge from node \( x \) to node \( y \) labeled by \( p \) denoted \( x \sim p y \), represents the fact that relation \( p \) holds between the object represented by \( x \) and the object represented by \( y \).

The basic querying mechanism on a DB is that of regular-path queries (RPQs). An RPQ \( E \) is expressed as a regular expression or a finite-state automaton over \( \Sigma' \), and computes the set of pairs of nodes of the DB connected by a path that conforms (see formal definition below) to the regular language \( L(E) \) defined by \( E \).

As we said in the introduction, we consider queries that extend regular-path queries with the inverse operator. Formally, let \( \Sigma = \Sigma' \cup \{p^- | p \in \Sigma'\} \) be the alphabet including a new symbol \( p^- \) for each \( p \) in \( \Sigma' \). Intuitively, \( p^- \) denotes the inverse of the binary relation \( p \). If \( r \in \Sigma \), then we use \( r^- \) to mean the inverse of \( r \), i.e., if \( r \) is a relation, then \( r^- \) is its inverse, and if \( r \) is \( p^- \), then \( r^- \) is \( p \).

**Regular-path queries with inverse (RPQIs)** are expressed by means of regular expressions or finite-state automata over \( \Sigma \). Thus, in contrast with RPQs, RPQIs may use also the inverse \( p^- \) of \( p \), for each \( p \in \Sigma' \).

When evaluated over a DB \( B \), an RPQI \( E \) computes the set \( \text{ans}(E; B) \) of pairs of nodes connected by a semipath that conforms to the regular language \( L(E) \) defined by \( E \). A semipath in \( B \) from \( x \) to \( y \) (labeled with \( r_1 \ldots r_q \)) is a sequence of the form \( (y_1, r_1, y_2, \ldots, y_q, r_q, y_{q+1}) \), where \( q \geq 0 \), \( y_1 = x \), \( y_{q+1} = y \), and for each \( y_i, r_i, y_{i+1}, r_{i+1} \in \Sigma \), and, if \( r_i = p \) then \( y_i \sim p y_{i+1} \), and if \( r_i = p^- \) then \( y_{i+1} \sim p y_i \). We say that a semipath \( (y_1, r_1, \ldots, r_q, y_{q+1}) \) conforms to \( E \) if \( r_1 \ldots r_q \in L(E) \). A semipath is said to be **simple** if no node appears more than once in the corresponding sequence.

A word \( w = r_1 \ldots r_q \) over \( \Sigma \) satisfies \( E \) iff \((x, y) \in \text{ans}(E; B_w)\), where \( B_w \) is constituted by a single semipath \((x, r_1, \ldots, r_q, y)\) labeled with \( w \). Note that \( w \) may satisfy \( E \) even if \( w \notin L(E) \), since we may obtain the semipath from \( x \) to \( y \) conforming to \( E \) by moving back and forth on the nodes of \( B_w \).

**Example 1.** Let us consider a DB of software modules, where the relation **hasSubmodule** connects \( m_1 \) to \( m_2 \) if \( m_2 \) is a module defined in \( m_1 \), and the relation **containsVar** connects \( m \) to \( v \) if \( v \) is a variable defined in module \( m \). The RPQI

\[
(\text{hasSubmodule}^-)^* (\text{containsVar} \cup \text{hasSubmodule})
\]

computes the pairs \((m, x)\) such that \( x \) (either a variable, or a module) is visible in module \( m \), according to the usual visibility rules of Algol-like languages.

3. TWO-WAY AUTOMATA

A two-way automaton \([23]\) \( A = (\Gamma, \Sigma, S_0, \rho, F) \) consists of an alphabet \( \Gamma \), a finite set of states \( S \), a set of initial states \( S_0 \subseteq S \), a transition function

\[
\rho : S \times \Sigma \rightarrow 2^{\{0, 1, \ldots, \infty\}}
\]

and a set of accepting states \( F \subseteq S \).

Intuitively, a transition indicates both the new state of the automaton, and whether the head reading the input should move left (-1), right (1), or stay in place (0). If for all \( s \in S \) and \( a \in \Gamma \) we have that \( \rho(s, a) \subseteq S \times \{1\} \), then the automaton is a traditional nondeterministic finite-state automaton (also called one-way automaton).

A configuration of \( A \) is a pair consisting of a state and a position represented as a natural number. A run is a sequence of configurations. The sequence \((s_0, a_0, \ldots, (s_m, j_m)) \) is a run of \( A \) on a word \( w = a_0, \ldots, a_{n-1} \) in \( \Gamma^* \) if \( s_0 = i \), \( j_0 = 0 \), \( j_m \leq n \), and for all \( i \in \{0, \ldots, m - 1\} \), we have that \( 0 \leq j_i < n \), and there is some \((t, k) \in \delta(s_i, a_i)\) such that \( s_{i+1} = t \) and \( j_{i+1} = j_i + k \). This run is accepting if \( j_m = n \) and \( s_m \in F \). \( A \) accepts \( w \) if it has an accepting run on \( w \). The set of words accepted by \( A \) is denoted \( L(A) \).

It is well known that two-way automata define regular languages, and that, given a two-way automaton \( A \) with \( n \) states, one can construct a one-way automaton \( B_1 \) with \( O(2^n \times \log n) \) states such that \( L(B_1) = L(A) \), and a one-way automaton \( B_2 \) with \( O(2^n) \) states such that \( L(B_2) = \Gamma^* - L(A) \) \([23; 35]\).

The ability of two-way automata to scan words in both directions allows us to use them to evaluate RPQIs over DBs. To gain some intuition on how two-way automata capture computations of RPQIs over DBs, we show how to construct, with linear blow-up, from an RPQI \( E \) a two-way automaton \( A_E \) that accepts all words \( w \in \Sigma^* \cdot \delta \) such that \( w \) satisfies \( E \). To construct \( A_E \), we assume that \( E \) is represented as a finite-state (one-way) automaton \( E = (\Sigma, S, I, \delta, F) \) over the alphabet \( \Sigma \). Then \( A_E = (\Sigma_A, S_A, I, \delta_A, \{s_f\}) \), where

- \( \Sigma_A = \Sigma \cup \{\$\} \),
- \( S_A = S \cup \{s_f\} \cup \{s^-^w | s \in S\} \), and
- \( \delta_A \) is defined as follows:
  1. \( (s^-^w, -1) \in \delta_A(s, \ell) \), for each \( s \in S \) and \( \ell \in \Sigma_A \). Such transition makes the automaton ready to scan one step backward by placing it in “backward mode”.
  2. \( (s_2, 1) \in \delta_A(s_1, r) \) and \( (s_2, 0) \in \delta_A(s_1^-, r^-) \), for each transition \( s_2 \in \delta(s_1, r) \) of \( E \). These transitions correspond to the transitions of \( E \) that are performed forward or backward according to the current “scanning mode”.
  3. \( (s_f, 1) \in \delta_A(s, \$) \), for each \( s \in F \).

Notice that the \( \$ \) symbol, which acts as a terminator in the words accepted by \( A_E \), is necessary to allow the automaton
THEOREM 2. Let $E$ be an RPQI, and $A_E$ the two-way automaton constructed from $E$ as specified above. Then $w$ satisfies $E$ iff $w \in L(A_E)$.

4. VIEW-BASED QUERY REWRITING

Let $E_0$ be an RPQI, and $\mathcal{E} = \{E_1, \ldots, E_k\}$ a finite set of RPQIs over the alphabet $\Sigma$. We assume that associated with $\mathcal{E}$ we always have an alphabet $\Sigma_\mathcal{E}$ containing exactly one symbol for each query in $\mathcal{E}$, and we denote the query associated to the symbol $p \in \Sigma_\mathcal{E}$ with $def(p)$.

Intuitively, we want to rewrite $E_0$ in terms of the views $\mathcal{E} = \{E_1, \ldots, E_k\}$, where $p_i$ is the name of the $i$-th view, and $def(p_i) = E_i$ is its definition. The only constraint we impose on the rewriting is that it is an expression denoting a set of words in the alphabet $\Sigma_\mathcal{E} = \Sigma_\mathcal{E}_c \cup \{p^* \mid p \in \Sigma_\mathcal{E}_c\}$, i.e., the target query language for the rewriting is a formalism for specifying languages over $\Sigma_\mathcal{E}$. This means that in the rewriting, the inverse operator can be applied to symbols in $\Sigma_\mathcal{E}$, and therefore we have to specify the semantics of such applications.

For this purpose, we extend the function $def$ to all symbols in $\Sigma_\mathcal{E}$ as follows: if $p \in \Sigma_\mathcal{E}$, then $def(p^-)$ is the regular expression $inv(def(p))$ over $\Sigma$, where $inv$ is defined as follows:

- $inv(a) = a$.
- $inv(a^-) = a$.
- $inv(e_1 \cdot e_2) = inv(e_2) \cdot inv(e_1)$.
- $inv(e_1 \cup e_2) = inv(e_1) \cup inv(e_2)$.
- $inv(e^-) = inv(e)^*$.

Given a language $\ell$ over $\Sigma_\mathcal{E}$, we denote by $expand_\mathcal{E}(\ell)$ the language over $\Sigma$ defined as follows:

$$expand_\mathcal{E}(\ell) = \bigcup_{e_1, \ldots, e_n \in \ell} \{w_1 \cdots w_n \mid w_i \in L(def(e_i))\}$$

DEFINITION 3. Let $R$ be any language over $\Sigma_\mathcal{E}$. We say that $R$ is a rewriting of $E_0$ wrt $\mathcal{E}$ if $\text{ans}(\text{expand}_\mathcal{E}(R), B) \subseteq \text{ans}(E_0, B)$, for every DB $B$.

The following theorem is a useful characterization of “bad” rewritings, based on the notion of satisfaction of a query by a word, as defined in Section 2.

THEOREM 4. Let $R$ be a language over $\Sigma_\mathcal{E}$. Then $R$ is not a rewriting of $E_0$ wrt $\mathcal{E}$ iff there exists a word in $\text{expand}_\mathcal{E}(R)$ that does not satisfy $E_0$.

PROOF. If $R$ is not a rewriting of $E_0$ wrt $\mathcal{E}$, then there exists a DB $B$ with nodes $x, y$ such that $(x, y) \in \text{ans}(\text{expand}_\mathcal{E}(R), B)$ and $(x, y) \notin \text{ans}(E_0, B)$. This means that there is a semipath from $x$ to $y$ in $B$ labeled by a word in $\text{expand}_\mathcal{E}(R)$ that does not satisfy $E_0$.

If there exists a word $w_1 \cdots w_n$ in $\text{expand}_\mathcal{E}(R)$ that does not satisfy $E_0$, then the DB $B$ is constructed by the single semipath $(x, w_1, \ldots, w_n, y)$ such that $(x, y) \in \text{ans}(\text{expand}_\mathcal{E}(R), B)$ and $(x, y) \notin \text{ans}(E_0, B)$. □

We are interested both in maximal rewritings, i.e., rewritings that capture in the best possible way the query $E_0$, and in exact rewritings, which capture exactly the query $E_0$.

DEFINITION 5. Let $R$ be a rewriting of $E_0$ wrt $\mathcal{E}$. $R$ is said to be maximal if for each rewriting $R'$ of $E_0$ wrt $\mathcal{E}$ we have $\text{ans}(\text{expand}_\mathcal{E}(R'), B) \subseteq \text{ans}(\text{expand}_\mathcal{E}(R), B)$, for every DB $B$. $R$ is said to be exact if for every DB $B$, $\text{ans}(\text{expand}_\mathcal{E}(R), B) = \text{ans}(E_0, B)$.

Our technique is based on characterizing the words in $\Sigma_\mathcal{E}$ that do not belong to a rewriting of $E_0$ wrt $\mathcal{E}$. By Theorem 4, these are the words $w \in \Sigma_\mathcal{E}$ such that some word in $\text{expand}_\mathcal{E}((\ell))$ does not satisfy $E_0$. Based on the use of two-way automata, we define an automaton that accepts exactly such words, and then complement such automaton to get the rewriting.

Let $A_0$ be the one-way automaton over $\Sigma$ corresponding to the query $E_0$, and let $\Sigma_\mathcal{E} = \{e_1, \ldots, e_n\}$. We consider words over $\Sigma \cup \Sigma_\mathcal{E} \cup \{\&, \cdot\}$ of the form

$$\$e_{i_1}w_{i_1}\$ \cdots \$e_{i_m}w_{i_m}\$$$

with $e_{i_j} \in \Sigma_\mathcal{E}$, and $w_{i_j} \in \Sigma^*$.

Following the idea described in Section 3, we construct a two-way automaton $A_1$ that accepts words of the above form such that $w_{i_1} \cdots w_{i_m}$ satisfies $E_0$. Let $A_2$ be a one-way automaton that complements $A_1$. Let $A_3$ be a one-way automaton that accepts a word of the form

$$\$e_{i_1}w_{i_1}\$ \cdots \$e_{i_m}w_{i_m}\$$$

iff for every $i_j$, the word $w_{i_j}$ is in $L(def(e_{i_j}))$, i.e., iff the word $w_{i_1} \cdots w_{i_m}$ is in $\text{expand}_\mathcal{E}((\ell))$ and does not satisfy $E_0$. Let $A_4$ accept all words $e_{i_1} \cdots e_{i_m}$ that are projections on the $e_{i_j}$’s of the words

$$\$e_{i_1}w_{i_1}\$ \cdots \$e_{i_m}w_{i_m}\$$$

that are accepted by $A_3 \cap A_2$. By construction, $A_4$ accepts all words $e_{i_1} \cdots e_{i_m}$ such that there is a word in $\text{expand}_\mathcal{E}((\ell))$ that does not satisfy $E_0$. Finally, let $R_{\mathcal{E}, E_0}$ be the complement of $A_4$. By virtue of the above construction, $R_{\mathcal{E}, E_0}$ accepts all words $e_{i_1} \cdots e_{i_m}$ such that every word in $\text{expand}_\mathcal{E}((\ell))$ satisfies $E_0$.

Theorem 6. \( R_{\mathcal{E}},\mathcal{E}_0 \) is a maximal rewriting of \( E_0 \) wrt \( \mathcal{E} \).

Proof. By exploiting Theorem 4, it is easy to show that \( R_{\mathcal{E},\mathcal{E}_0} \) is a rewriting of \( E_0 \) wrt \( \mathcal{E} \). To see that \( R_{\mathcal{E},\mathcal{E}_0} \) is maximal, consider a word \( e_1 \cdots e_m \in \Sigma_\mathcal{E} \) that is not accepted by \( R_{\mathcal{E},\mathcal{E}_0} \). Obviously, \( e_1 \cdots e_m \) is accepted by \( A_4 \), and therefore there is a word \( \$e_1w_1\$ \cdots \$e_mw_m\$ \$ \) that is accepted by \( A_3 \cap A_2 \). It follows that \( \text{expand}_\mathcal{E}(\{e_1 \cdots e_m\}) \) contains a word that does not satisfy \( E_0 \). By Theorem 4, this in turn implies that \( e_1 \cdots e_m \) does not belong to a rewriting of \( E_0 \) wrt \( \mathcal{E} \). □

Observe that the only constraint we put on the rewriting is that it defines a language over \( \Sigma_\mathcal{E} \). Theorem 6 shows that the language over \( \Sigma_\mathcal{E} \) (and therefore also the language over \( \Sigma \)) defined by the maximal rewriting is regular (indeed, \( R_{\mathcal{E},\mathcal{E}_0} \) is a finite-state automaton).

Theorem 7. The problem of generating the maximal rewriting of an RPQI wrt a set of RPQIs is in \( 2\text{EXPTIME} \).

Proof. The size of \( A_1 \) is polynomial in the size of \( E_0 \), the size of \( A_2 \) is exponential in the size of \( A_1 \), and the size of \( A_3 \) is polynomial in the size of \( \mathcal{E} \). Finally, the size of \( A_4 \) is polynomial in the size of \( A_3 \) and \( A_2 \). The claim follows from the fact the size of \( R_{\mathcal{E},\mathcal{E}_0} \) is exponential in the size of \( A_4 \), and therefore double exponential in the size of \( E_0 \), and exponential in the size of \( \mathcal{E} \). □

By exploiting the techniques and the results in [10], it is easy to prove the following theorem, which shows that our method for computing the maximal rewriting of an RPQI is essentially optimal.

Theorem 8. The problem of verifying the existence of a nonempty rewriting of an RPQI \( E_0 \) wrt a set \( \mathcal{E} \) of RPQIs is \( \text{EXPSPACE-complete} \).

Next we address the problem of verifying whether the rewriting \( R_{\mathcal{E},\mathcal{E}_0} \) of \( E_0 \) wrt \( \mathcal{E} \) is exact. We proceed as in [10], and construct a one-way automaton \( B \) that accepts \( \text{expand}_\mathcal{E}(R_{\mathcal{E},\mathcal{E}_0}) \), by replacing each edge labeled by \( e_i \) in \( R_{\mathcal{E},\mathcal{E}_0} \) by a one-way automaton \( A_i \) such that \( L(A_i) = L(\text{def}(e_i)) \). We then check whether \( \text{ans}(L(A_0) \cup \cdots \cup L(A_k)) \subseteq \text{ans}(L(B)) \), for every \( \mathcal{E} \). Using the techniques described in [12], the following result can be proven.

Theorem 9. The problem of verifying the existence of an exact rewriting of an RPQI \( E_0 \) wrt a set \( \mathcal{E} \) of RPQIs is \( 2\text{EXPSPACE-complete} \).

5. VIEW-BASED QUERY ANSWERING

As pointed out in [2; 21; 24; 11], the problem of view-based query answering comes in different forms, depending on various assumptions about how accurate is the knowledge on both the objects of the DB, and the pairs satisfying the views.

Consider a DB that is accessible only through a set of views \( V_1, \ldots, V_k \), and suppose we want to answer an RPQI only on the basis of our knowledge on the views. Specifically, associated to each view \( V_i \) we have

- its definition \( \text{def}(V_i) \) in terms of an RPQI;
- information about its extension in terms of a set \( \text{ext}(V_i) \) of pairs of objects;
- a specification \( \text{as}(V_i) \) of which assumption to adopt for the view \( V_i \) in interpreting \( \text{ext}(V_i) \) with respect to the answer set of \( \text{def}(V_i) \).

The possible assumptions on views are:

- Sound View Assumption (SVA). We say that a view \( V_i \) is sound (satisfies SVA) with respect to a DB \( \mathcal{B} \), if \( \text{ext}(V_i) \subseteq \text{ans}(\text{def}(V_i), \mathcal{B}) \).
- Complete View Assumption (CVA). We say that a view \( V_i \) is complete (satisfies CVA) with respect to a DB \( \mathcal{B} \), if \( \text{ext}(V_i) \supseteq \text{ans}(\text{def}(V_i), \mathcal{B}) \).
- Exact View Assumption (EVA). We say that a view \( V_i \) is exact (satisfies EVA) with respect to a DB \( \mathcal{B} \), if \( \text{ext}(V_i) = \text{ans}(\text{def}(V_i), \mathcal{B}) \).

We say that a DB \( \mathcal{B} \) is consistent with a view \( V_i \) if \( V_i \) satisfies the assumption \( \text{as}(V_i) \) with respect to \( \mathcal{B} \).

We observe that, complete views can be reformulated in terms of exact views, by exploiting union in RPQIs [11]. Hence, in the following, we will focus on sound and exact views only.

With respect to the information available on the objects in the DB, one can further distinguish between: Closed Domain Assumption (CDA) and Open Domain Assumption (ODA). Under CDA, the exact set of objects in the DB coincides with the set of objects that appear in the view extensions, while under ODA the DB may contain additional objects. Formally, we call \( \mathcal{D}_V \) the set of objects appearing in \( \text{ext}(V_1) \cup \cdots \cup \text{ext}(V_k) \), and we say that a DB \( \langle \mathcal{D}, \mathcal{E} \rangle \) is consistent with \( \mathcal{D}_V \) under CDA (resp. ODA) if \( \mathcal{D}_V \) is equal to (resp. is a subset of) \( \mathcal{D} \).

Definition 10. Given (i) \( \text{def}(V_i) \), \( \text{ext}(V_i) \), and \( \text{as}(V_i) \), for \( 1 \leq i \leq k \); (ii) a query \( Q \); (iii) a pair of objects \( c, d \in \mathcal{D} \), view-based query answering under CDA (resp. ODA) consists in deciding whether \( \langle c, d \rangle \in \text{ans}(Q, \mathcal{B}) \), for every \( \mathcal{B} \) that is consistent with \( \mathcal{D}_V \) under CDA (resp. ODA), and consistent with \( V_1, \ldots, V_k \).

We assume that objects are represented by constants and we adopt the unique name assumption [29], i.e., different constants denote different objects, and therefore different nodes.
We observe that view-based query answering can be interpreted as checking whether \((c, d)\) is a certain answer to \(Q\) [2]. On the other hand, we may be interested in checking whether \((c, d)\) is a possible answer to \(Q\), i.e., whether \((c, d) \in \text{ans}(Q, B)\), for some \(B\) that is consistent with the views. Following the method in [11], it is possible to show that computing possible answers can be reduced to computing certain answers. Therefore, in the following we concentrate on certain answers.

The complexity of the problem can be measured in three different ways [34]:

- **data complexity**, as a function of the size of \(\text{ext}(V_1) \cup \cdots \cup \text{ext}(V_k)\);
- **expression complexity**, as a function of the size of \(Q\) and of the expressions \(\text{def}(V_1), \ldots, \text{def}(V_k)\);
- **combined complexity**, as a function of the size of both \(\text{ext}(V_1) \cup \cdots \cup \text{ext}(V_k)\) and the expressions \(Q, \text{def}(V_1), \ldots, \text{def}(V_k)\).

### 5.1 Closed domain assumption

Under the CDA, the techniques for view-based query answering for RFQs in [11] extend directly to RPQIs. Indeed, such techniques are based on guessing a DB \(B\) and then checking that \(B\) is consistent with the views and does not satisfy the query. Such check basically requires to evaluate each view and the query on \(B\). It is easy to see that the presence of the inverse operator does not influence the complexity of the evaluation.

**Theorem 11.** View-based query answering under CDA for RPQIs is co-NP-complete w.r.t data complexity, expression complexity, and combined complexity.

### 5.2 Open domain assumption

Under ODA, the presence of the inverse operator requires introducing novel techniques. As we said in the introduction, the techniques we present are based on searching for a “counterexample DB” represented in a linearized form as a special word, and using two-way automata to check that candidate counterexample DBs satisfy all required conditions.

First of all, it can be shown that it is sufficient to restrict the search for counterexamples on canonical DBs.

**Definition 12.** Let \(D_V\) be the set of objects appearing in the extensions of the views. A DB is called canonical if it is composed of a set of simple semipaths a \(a_1 \rightarrow a_2 \cdots a_m \rightarrow b\), where \(a, b \in D_V\) and \(a_1, \ldots, a_{m-1}\) are not in \(D_V\) and do not occur in any other semipath in the set.

**Theorem 13.** Given an RPQI \(E\), a set of views, and a DB \(B\) that is consistent with the views and such that \((c, d) \notin \text{ans}(E, B)\), there exists a canonical DB \(B'\) that is consistent with the views and such that \((c, d) \notin \text{ans}(E, B')\).

The importance of canonical DBs lies in the fact that they are suitable for being linearized. Each canonical DB \(B = (D, E)\) can be represented by a word \(w_B\) over the alphabet \(\Sigma_A = \Sigma \cup D_V \cup \{8\}\), which has the form

\[
\$d_1 w_1 d_2 \$w_3 \$d_4 \$ \cdots \$w_{2m-1} d_m \$w_m \$d_m 8
\]

where \(d_1, \ldots, d_{2m}\) range over \(D_V\), \(w_i \in \Sigma^*\), and the \(\$\) acts as a separator. Specifically, \(w_B\) consists of one subword \(d_2 \ldots d_{2m-1} w_m d_m\), for each simple semipath in \(B\) from \(d_{2m-1}\) to \(d_m\) conforming to \(w_m\). Observe that the same DB can be represented by several words that differ in the direction considered for the semipaths and in the order in which the subwords corresponding to the semipaths appear.

We show now how a two-way automaton \(A_{(E, a, b)}\) can be used for the fundamental task of verifying whether a pair \((a, b)\), with \(a, b \in D_V\), is in the answer set of an RPQI \(E\) over a linearized DB. The automaton \(A_{(E, a, b)}\) accepts a word \(w_B\) over the alphabet \(\Sigma_A\) representing a DB \(B\) iff \((a, b) \in \text{ans}(E, B)\). To do so we exploit not only the ability of two-way automata to move on the word both forward and backward (see Section 3), but also the ability to “jump” from one position in the word representing a node to any other position (either preceding or succeeding) representing the same node. These two capabilities ensure that the automaton evaluating the RPQI on the word simulates exactly the evaluation of the query on the DB.

To construct \(A_{(E, a, b)}\) we assume that \(E\) is represented as a one-way automaton \(E = (\Sigma, S, I, \delta, F)\) over the alphabet \(\Sigma\). Then \(A_{E} = (\Sigma_A, S_A, \{s_0, s_f\}, \delta_A, \{s_f\})\), where

- \(S_A = S \cup \{s_0, s_f\} \cup \{s^-| s \in S \} \times S \times D\), and
- \(\delta_A\) contains the following transitions\(^2\):

\(^2\)The transitions defined by items 1 and 2 are the same as those defined by items 1 and 2 in Section 3, considering the current definition of \(\delta_A\).
1. \((s^+, -1) \in \delta_A(s, t)\), for each \(s \in S\) and \(t \in \Sigma \cup \mathcal{D}_V\). At any point such transition makes the automaton ready to scan one step backward by placing it in “backward mode”.

2. \((s_2, 1) \in \delta_A(s_1, r)\) and \((s_2, 0) \in \delta_A(s_1, r\text{-}_2)\), for each transition \(s_2 \in \delta(s_1, r)\) of \(E\). These transitions correspond to the transitions of \(E\) which are performed forward or backward according to the current “scanning mode”.

3. \((s_0, 1) \in \delta_A(s_0, \ell)\), for each \(\ell \in \Sigma_A\), and also \((s, 0) \in \delta_A(s_0, a)\) for each \(s \in I\). These transitions place the head of the automaton in some randomly chosen occurrence of \(a\) in the input string and set the state of the automaton to some randomly chosen initial state of \(E\).

4. For each \(s \in S\) and each \(d \in \mathcal{D}_V\)
   \[
   \begin{align*}
   (s, d, 0) &\in \delta_A(s, d) \\
   (s, d, 0) &\in \delta_A(s^+, d) \\
   (s, d, 1) &\in \delta_A(s, d, \ell), \quad \text{for each } \ell \in \Sigma_A \\
   (s, -1) &\in \delta_A(s, d, -1), \quad \text{for each } \ell \in \Sigma_A \\
   (s, 0) &\in \delta_A(s, d, d) \\
   (s, 1) &\in \delta_A(s, d, d)
   \end{align*}
   \]
   Whenever the automaton reaches a symbol representing a node \(d\) (first and second clause), it may enter into “search (for \(d\) mode)” and move to any other occurrence of \(d\) in the word. Then the automaton exits search mode (second last clause) and continues its computation either forward (last clause) or backward (see item 2).

5. \((s_f, 0) \in \delta_A(s, b)\), for each \(s \in F\) and \((s_f, 1) \in \delta_A(s, \ell)\), for each \(\ell \in \Sigma_A\). These transitions place the automaton in the final state when it reads \(b\) in a final state of \(E\). Then the head moves to the end of the input string to accept.

The following theorem characterizes the relationship between an RPQI and the corresponding two-way automaton.

**Theorem 14.** Let \(Q\) be an RPQI, \(B\) a canonical DB, \(w_B\) the word representing \(B\), and \(a, b\) two objects in \(\mathcal{D}_V\). Then \(A_{(Q, a, b)}\) accepts \(w_B\) iff \((a, b) \in \text{ans}(Q, B)\).

Given a set of views \(V_1, \ldots, V_n\), an RPQI \(Q\), and objects \(c, d \in \mathcal{D}_V\), we can exploit the construction above to build a two-way automaton that checks the existence of a canonical DB \(B\) that is consistent with the views and such that \((c, d) \notin \text{ans}(Q, B)\). In particular:

1. We construct the one-way automaton \(A_0\) that accepts the language \((\$ \cdot \mathcal{D}_V \cdot \Sigma^* \cdot \mathcal{D}_V)^* \cdot \$\), hence enforcing the general structure of words representing canonical databases.

2. For each (sound or exact) view \(V_i\) and for each pair \((a, b) \in \text{ext}(V_i)\), we construct the automaton \(A_{(\text{def}(V_i), a, b)}\) as specified above.

3. For each exact view \(V_i\) we build a two-way automaton \(A_{V_i}\) that checks whether a pair of objects other than those in \(\text{ext}(V_i)\) is in \(\text{ans}(\text{def}(V_i), B)\). More precisely, \(A_{V_i}\) is the union of the following automata, which are all obtained by adapting the construction above:
   - an automaton \(A_{(V_i, a)}\) for each object \(a\) appearing as the first component in a pair in \(\text{ext}(V_i)\), which starts by placing the head on some randomly chosen occurrence of \(a\), evaluates \(\text{def}(V_i)\), and after that makes a transition to the final state only if it does not read a symbol \(b\) such that \((a, b) \in \text{ext}(V_i)\).
   - an additional automaton \(A_{(V_i, b, \text{exact})}\), which starts by placing the head on some randomly chosen symbol in the word different from any \(a\) that appears as the first component in a pair in \(\text{ext}(V_i)\), and accepts if it can evaluate the query \(\text{def}(V_i)\) from there.

4. Finally, we construct the automaton \(A_{(Q, c, d)}\) as specified above.

To check whether \((c, d) \notin \text{ans}(Q, B)\) for some DB \(B\) consistent with the views, we check for nonemptiness of the one-way automaton \(A_{O \text{DA}}\) obtained as the intersection of the one-way automata corresponding to each \(A_{(\text{def}(V_i), a, b)}\) constructed at point 1. of the one-way automata corresponding to the complement of each \(A_{V_i}\) constructed at point 2. and of the one-way automaton corresponding to the complement of \(A_{(Q, c, d)}\) constructed at point 3.

By exploiting Theorems 13 and 14 it is possible to prove the following correctness result.

**Theorem 15.** Let \(V_1, \ldots, V_n\) be a set of views, \(Q\) an RPQI, and \(c, d\) objects in \(\mathcal{D}_V\). Then \((c, d) \notin \text{ans}(Q, B)\) for some DB \(B\) consistent with \(V_1, \ldots, V_n\) iff the one-way automaton \(A_{O \text{DA}}\) constructed as specified above is nonempty.

Observe that all two-way automata constructed above are of linear size in the size of \(Q\). \(\text{def}(V_1), \ldots, \text{def}(V_n)\), and \(\text{ext}(V_1), \ldots, \text{ext}(V_n)\). Hence, the corresponding one-way automata would be exponential. However, we do not need to construct \(A_{O \text{DA}}\) explicitly. Instead, we can construct it “on the fly” while checking for nonemptiness, by exploiting the fact that the transition function of the one-way automaton corresponding to a two-way automaton or its complement can be computed verifying local conditions only [35], and that the same observation holds for intersection.

Considering the lower bounds for view-based query answering under ODA for RPQs in [11], we obtain the following characterization of computational complexity.

**Theorem 16.** View-based query answering under ODA for RPQIs is PSPACE-complete w.r.t expression complexity and combined complexity.

As for data complexity, it is known that the lower bound is co-NP [11]. To get a matching upper bound, the construction above cannot be directly applied. To isolate the impact
of the extensions of the views on complexity, we have to look into the transformation from two-way to complementary one-way automata. In particular, such transformation is based on searching for the existence of a sequence \(T_0 \cdots T_n\) of sets of states of the two-way automaton labeling an input word \(w\) of length \(n\), such that the sequence satisfies certain conditions on adjacent sets [35].

For simplicity, we consider only the case where the two-way automaton checks whether there is some word \(w\) representing a canonical DB \(B\) such that \((a, b) \not\in ans(Q, B)\). In our case, the number of states of the two-way automaton depends on the objects in the extensions of the views only due to those states used to implement “search mode”. Hence, we need to avoid search mode in the two-way automaton. Therefore, we consider the two-way automaton that computes \(Q\) over words representing databases as specified in Section 3. and simulate search mode by ensuring that for each occurrence \(d_i\) of an object \(d \in D_V\) in the input word, the set of states in which the two-way automaton may be when the head scans \(d_i\) is the same. Then, to check whether the two-way automaton accepts some word \(w_b\), we guess a labeling of each object \(d\) with a set of states of the two-way automaton, and then check for the existence of the sequence of sets of states that completes such labeling. Such check can be done in polynomial time with respect to the number of objects in the extensions of the views.

**Theorem 17.** View-based query answering under ODA for RPQIs is co-NP-complete wrt data complexity.

The summary of our results on the complexity of view-based query answering for regular-path queries with inverse is reported in Table 1. Entries with “all sound” (resp. “all exact”) in the column named “Assumption on views” refer to the case where all views are assumed to be sound (resp., exact), whereas “arbitrary” means that for each view \(V\), as(V) can be either SVA, CVA, or EVA. Each entry of the table referring to a complexity class \(C\) means that the corresponding problem is complete with respect to \(C\). The table shows that view-based query answering for regular-path queries with inverse has the same computational complexity as for standard regular-path queries.

6. CONCLUSIONS

We have presented a novel technique for view-based query processing in the context of semistructured data, and in particular for regular-path queries extended with the inverse operator. The technique is based on the idea of encoding candidate counterexamples as words, and then checking candidates with two-way automata.

The ability of two-way automata to move back and forth on words is crucial for checking candidate counterexamples, and allows us to show that the inverse operator does not increase the complexity of view-based query rewriting and query answering.

We believe that, besides the specific results, our method has the merit of showing the power of two-way automata in reasoning on complex queries. Indeed, the techniques described in this paper can be adapted to reasoning about queries of more general forms. First results in this direction are reported in [12] for the problem of checking containment of conjunctive regular-path queries with inverse.

7. ACKNOWLEDGMENTS

This work was supported in part by the NSF grant CCR-9700061, by MURST, by ESPRIT LTR Project No. 22469 DWQ (Foundations of Data Warehouse Quality), and by the Italian Space Agency (ASI) under project “Integrazione ed Accesso a Basi di Dati Eterogenee”.

8. REFERENCES


S. Chaudhuri, S. Krishnamurthy, S. Potarjianos, and K. Shim. Optimizing queries with materialized views. In Proc. of the 11th IEEE Int. Conf. on Data Engineering (ICDE'95), Taipei (Taiwan), 1995.


A. Y. Levy. Obtaining complete answers from incomplete databases. In Proc. of the 22nd Int. Conf. on Very Large Data Bases (VLDB'96), pages 402–412, 1996.


