Logic
in
The Computer Science Curriculum

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February 1998

Acknowledgements

This panel resulted from several discussions that took place during the 1995-6 DIMACS Special Year on Logic and Algorithms. DIMACS is an NSF-funded Science and Technology Center for Discrete Mathematics and Theoretical Computer Science and one of the New Jersey Commission on Science and Technology's Advanced Technology Centers.

http://dimacs.rutgers.edu

What is Logic?

Liberal Arts: The Liberal Arts were divided into the Trivium ("the three roads") and the Quadrivium ("the four roads"). The Trivium consisted of: *Grammar*, *Rhetoric*, and *Logic*. The Quadrivium consisted of *Arithmetic* (number in itself), *Geometry* (number in space), *Music* (number in time), and *Astronomy* (number in space and time).

Francis Bacon, 1605

> "Logic differeth from rhetoric in this, that logic handleth reason exact and in truth, and rhetoric handleth it as it is planted in popular opinions and manners."

The Laws of the Mind

George Boole, *Mathematical Analysis of Logic*, 1847:

> "The design of the following treatise is to investigate the fundamental laws of the operations of the mind by which reasoning is performed; to give expressions to them in the symbolic language of a calculus, and upon this foundation to establish the science of logic and construct its methods."

Ernst Schröder, *The Algebra of Logic*, 1890:

> "Getting a handle on the consequences of any premisses, or at least the fastest method for obtaining these consequences, seems to me to be one of the noblest, if not the ultimate goal of mathematics and logic."

Logic as Foundations for Mathematics

K. Gödel, 1931:

> "The development of mathematics towards greater precision has led, as is well known, to the formalization of large tracts of it, so that one can prove any theorems using nothing but mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system of Principia Mathematica on one hand, and the Zermelo-Fraenkel axiom system of set theory on the other. These two systems are so comprehensive that in them all methods of proof used in mathematics are formalized, that is, reduced to a few axioms and rules of inference."

Hilbert's Program

Hilbert's Program:

1. Is mathematics complete in the sense that every statement could be proved or disproved?

2. Is mathematics consistent in the sense that no statement could be proved both true and false?

3. Is mathematics decidable in the sense that there existed a definite method to determine the truth or falsity of any mathematical statement?

Hilbert firmly believed that the answer to all three questions was '*yes*':

> "Every mathematical problem must necessarily be susceptible to an exact statement, either in the form of an actual answer to the question asked, or by the proof of the impossibility of its solution."

Logic in Mathematics Today

The demise of Hilbert's Program:

- Gödel, 1931: Incompleteness of arithmetics
- Church+Turing, 1936-7: Undecidability of FOL

Today (generally):

- Logic has minor impact on mainstream mathematics
- Mathematical logic is a fringe area in mathematics
- Few logicians in mathematics departments
- Most mathematics students do not learn logic

Where Is Logic Today?

AltaVista, February 22, 1998:

- Logic and Mathematics: 54,253 hits
- Logic and Philosophy: 61,259 hits
- Logic and Computer Science: 232,871 hits

Logic in Computer Science

Logic in computer science provides precise specification languages:

- Circuits – Boolean logic
- Problems – SAT
- Data types - abstraction
- Input data – program specification
- Output data – database queries
- Behavior – temporal logic

Impact of Logic in Computer Science

- *Hardware Design*: programmable logic array
- *Relational Databases*: $10B industry
- *Logic Programming*: Prolog
- *Complexity Theory*: NP-completeness
- Computer-Aided Verification: equivalence checking

A New Perspective: logic is a working tool, the "*calculus*" of computer science

M. Jackson: "Description is our business"

Panel Plan

1. D. Leivant: Computer science is applied logic
2. M.Y. Vardi: Algorithmic logic
3. K. Bruce: Logic and programming languages
4. P. Kolaitis: Spread logic around
5. Open Discussion