

# CS4501: Introduction to Computer Vision

## Cameras and Image Formation



Various slides from previous courses by:

D.A. Forsyth (Berkeley / UIUC), I. Kokkinos (Ecole Centrale / UCL), S. Lazebnik (UNC / UIUC), S. Seitz (MSR / Facebook), J. Hays (Brown / Georgia Tech), A. Berg (Stony Brook / UNC), D. Samaras (Stony Brook), J. M. Frahm (UNC), V. Ordonez (UVA).

# Make computers understand images and video



What kind of scene?

Where are the cars?

How far is the  
building?

...



# Today's Class

What is a camera?

Who invented cameras?

Image Formation

Brief Introduction to Projective Geometry (Computer Graphics)

# About the Course

## CS4501-001: Introduction to Computer Vision

- Instructor: Vicente Ordóñez
- Email: [vicente@virginia.edu](mailto:vicente@virginia.edu)
- Website: <http://vicenteordonez.com/vision/>
- Class Location: **Olsson Hall 120** (Capacity 148)
- Class Times: **Monday-Wednesday 5pm - 6:15pm**
- Piazza: <https://piazza.com/virginia/fall2019/cs4501001>
- UVA Collab (for submitting assignments / quizzes / etc)

# Pre-requisites

- Python programming skills
- Calculus / Linear Algebra / Probability

# Office Hours



Vicente Ordonez  
(vicente at virginia.edu)

Office Hours:  
Tuesdays from  
4pm to 6pm  
(Rice Hall 310).



Ziyang Yang  
(zy3cx at virginia.edu)

Office Hours:  
Thursdays from  
12:30pm to 2:30pm  
(Rice Hall 442).



Paola Cascante-Bonilla  
(pc9za at virginia.edu)

Office Hours:  
Fridays from  
2pm to 4pm  
(Rice Hall 442)

# Grading

- Assignments: 50% (5 assignments)  
(10% + 10% + 10% + 10% + 10%)
- Quiz: 20% (2 quizzes)  
(10% + 10%)
- Final project: 30% (group project – 3 people max)

# Late Submission Policy

- You shouldn't submit late assignments
  - That being said there is a 3-day grace period, if you submit an assignment late (by one day), you will not be penalized. Any fraction of a day counts as a full day. You can have up to 3 days under the 3-day grace period. This doesn't apply to the final project. It is 3-days for the entire semester not for individual assignments. These exceptions will only be accounted for at the end of the semester. If this policy is confusing please just don't submit late assignments. Also, please don't send emails about submitting late assignments.



# Textbook

- *Computer Vision: Algorithms and Applications*  
by Richard Szeliski. <http://szeliski.org/Book/>

*Computer Vision: A Modern Approach*, Forsyth and Ponce, Prentice Hall 2011.

*Introductory Techniques for 3D Computer Vision*, Trucco and Verri, Prentice Hall 1998.

*Computer Vision: Models, Learning, and Inference* by S. Prince, Cambridge University Press,  
2012: draft at <http://www.computervisionmodels.com/>

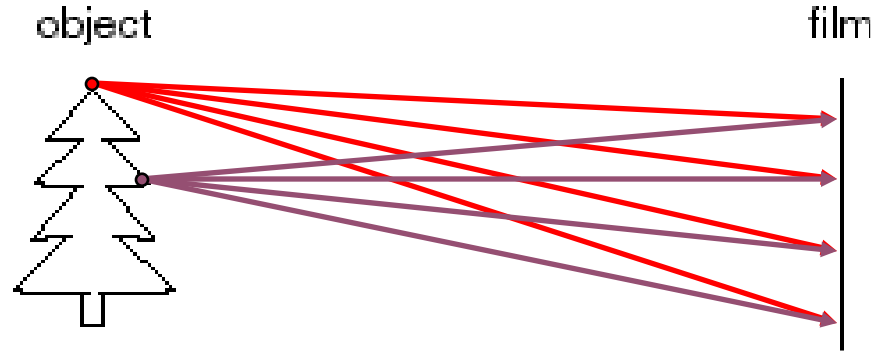
# Cameras



What do you need to make a camera from scratch?



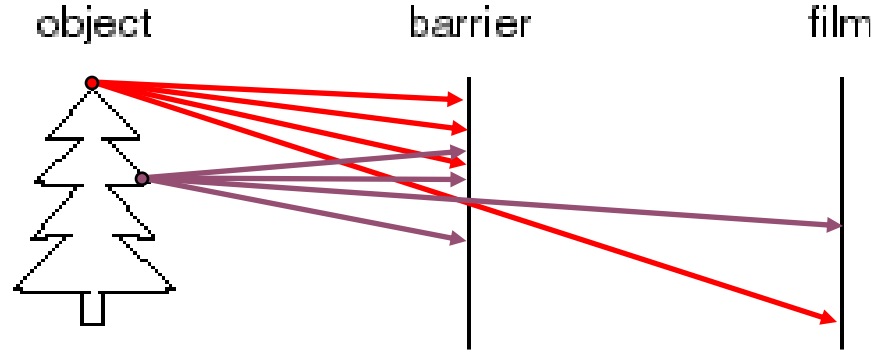
# Image formation



Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

# Pinhole camera



Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

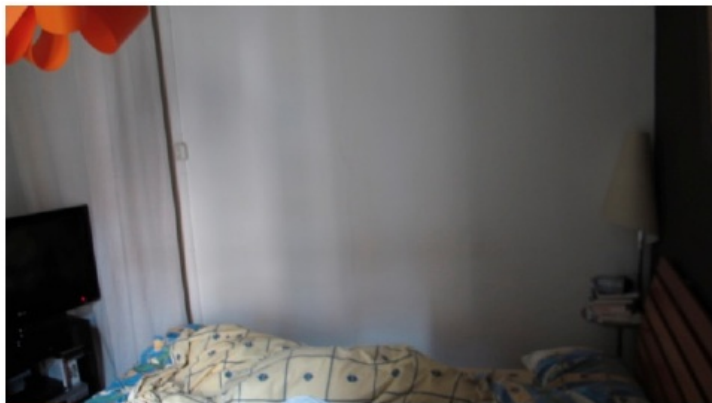
# Accidental Cameras



Accidental Pinhole and Pinspeck Cameras  
Revealing the scene outside the picture.  
Antonio Torralba, William T. Freeman



# Accidental Cameras



a) Input (occluder present)



b) Reference (occluder absent)



c) Difference image (b-a)



d) Crop upside down

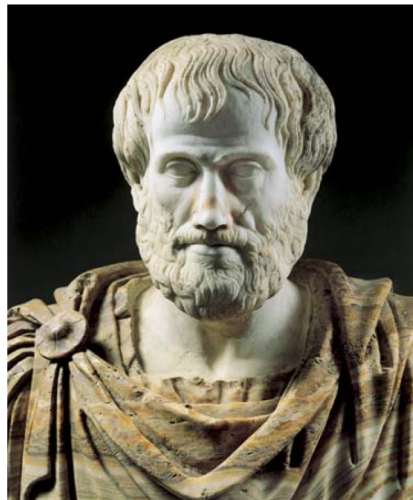


e) True view

## Known by the Greeks and the Chinese 470BC-322BC

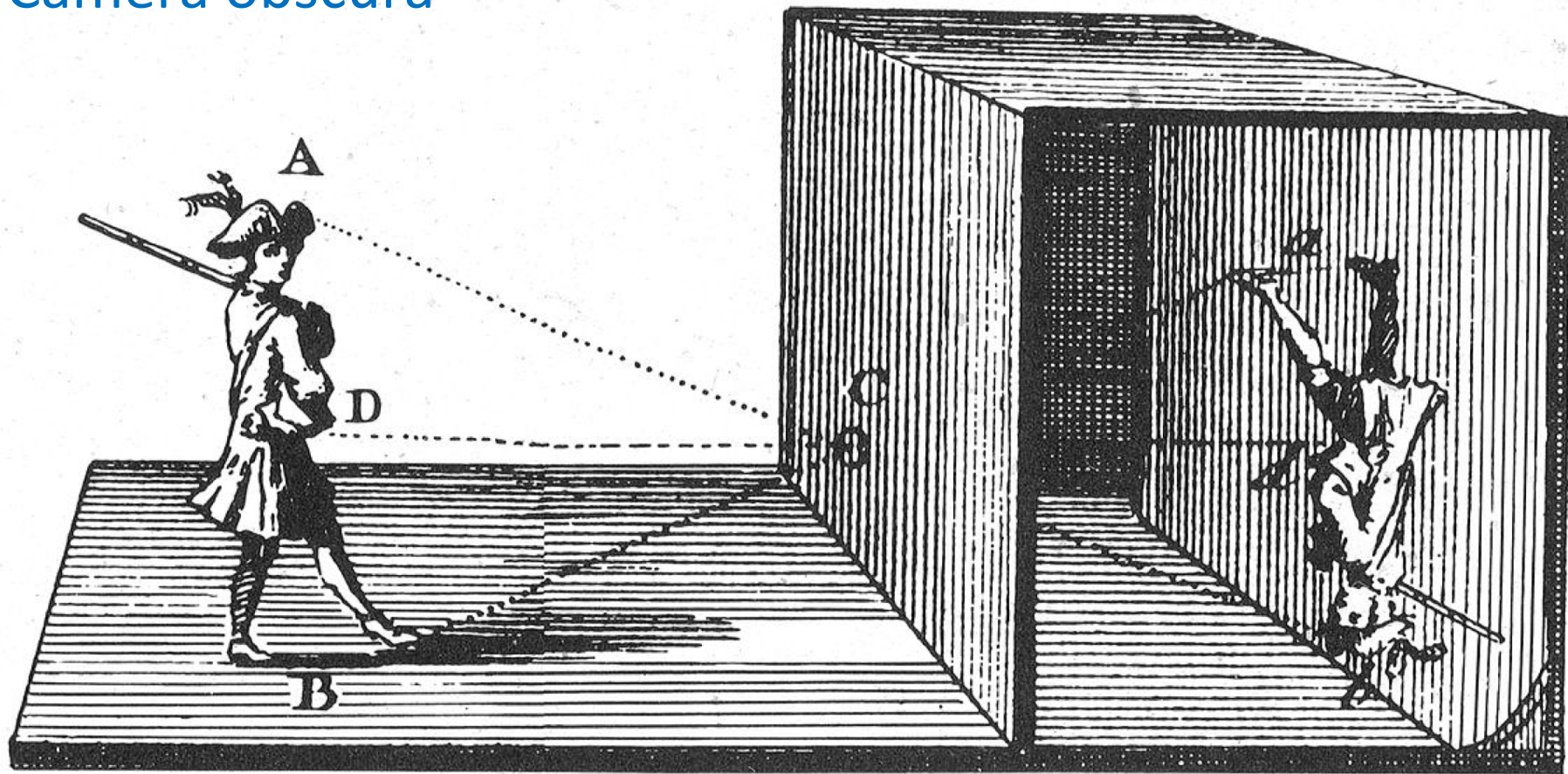


Recorded in writings by Chinese  
Philosopher Mozi ([墨子](#))  
470-390BCE

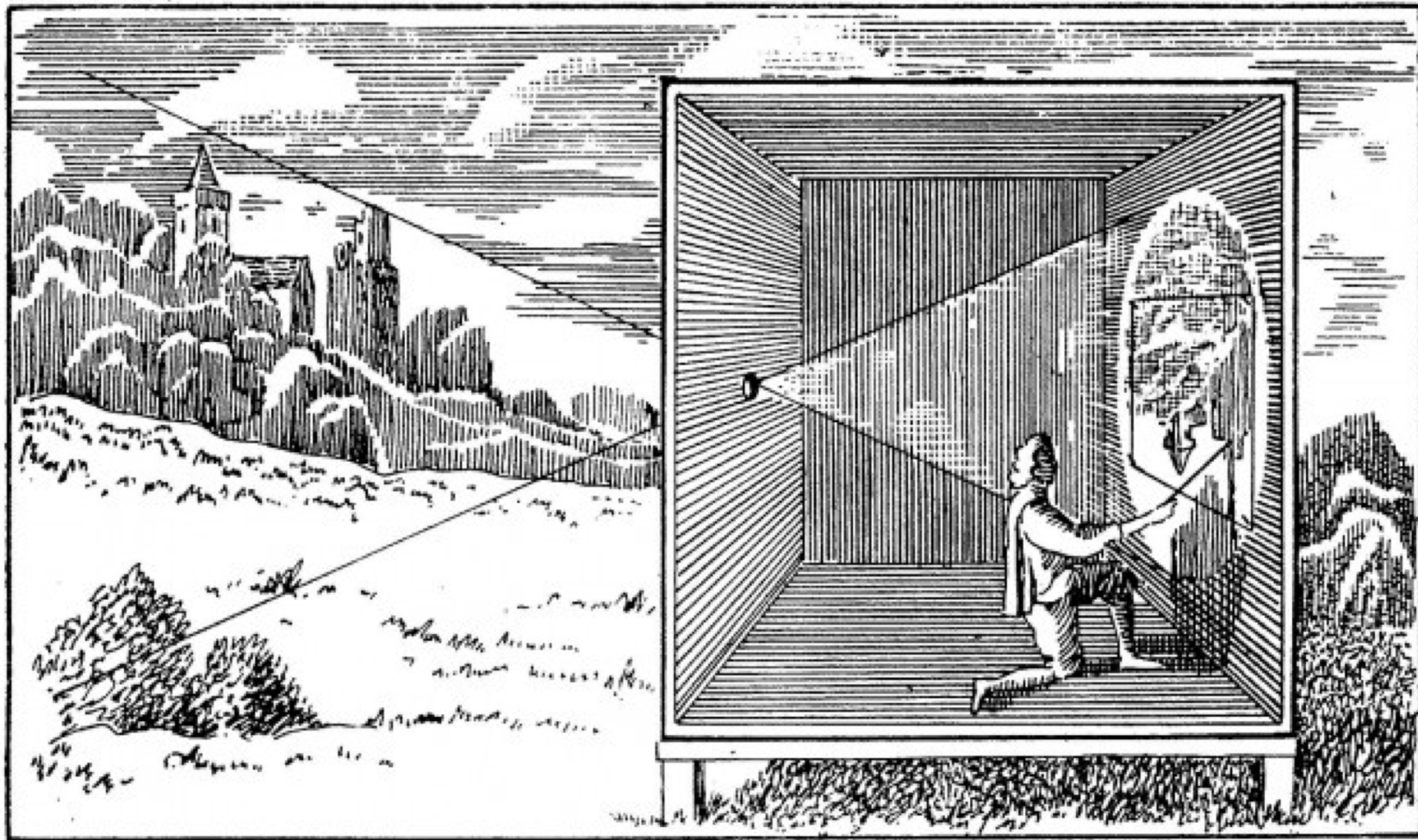


Recorded in writings by Aristotle  
or one of his disciples  
384-322BCE

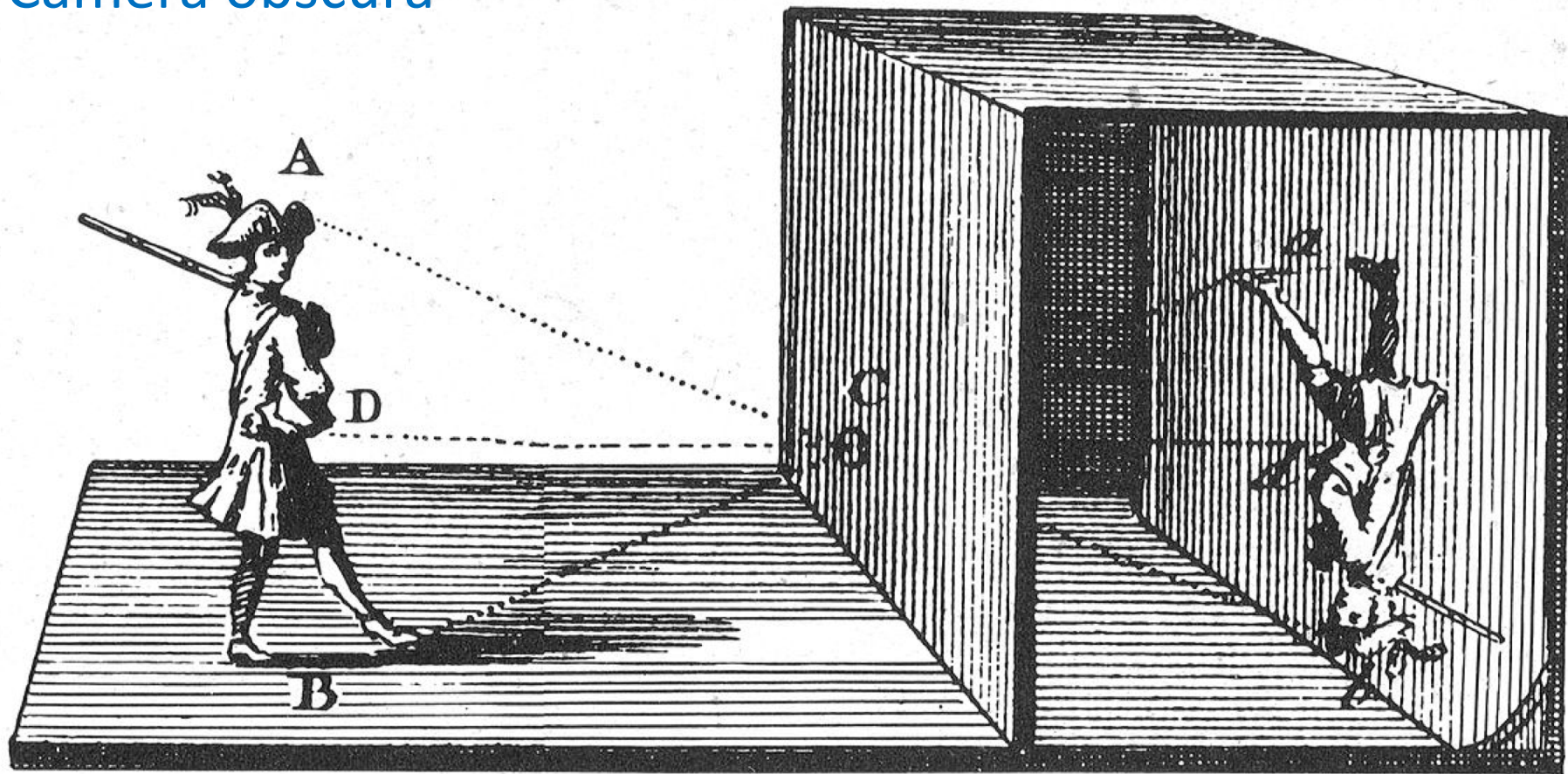
# Camera obscura







# Camera obscura



# Camera obscura: the pre-camera

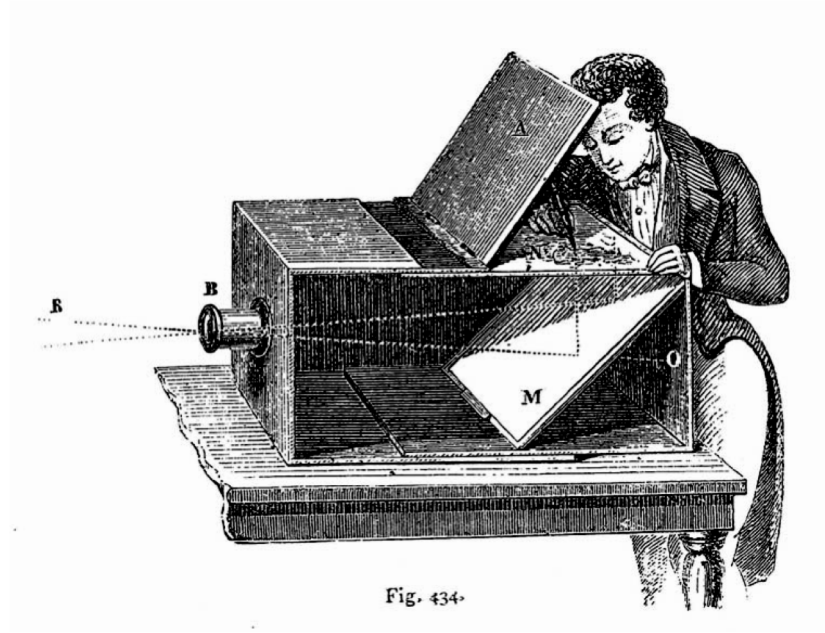


Freestanding camera obscura at UNC Chapel Hill

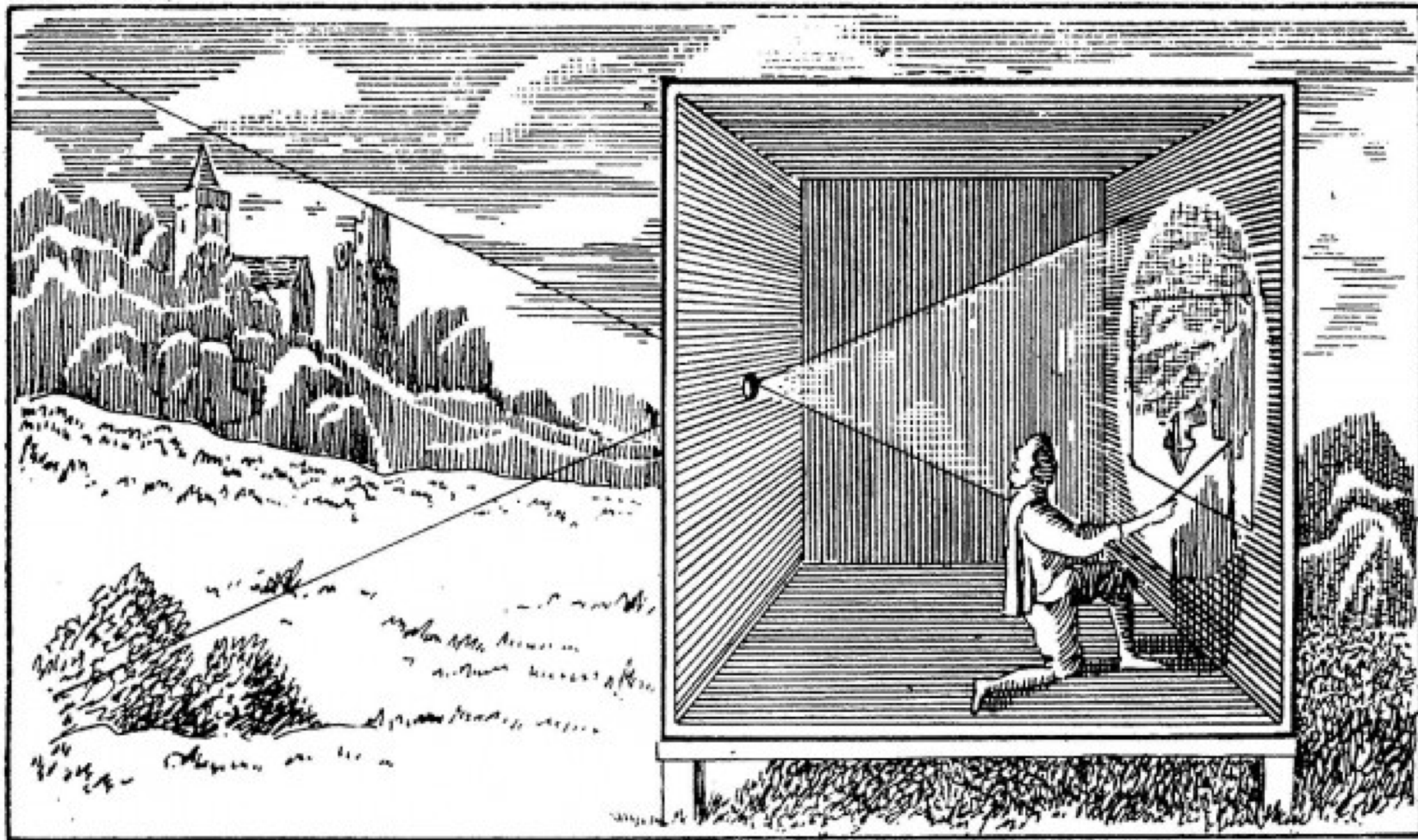
Photo by Seth Ilys



# Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568



# First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate



Joseph Niepce, 1826

Photograph of the first photograph



Stored at UT Austin

Bitumen of Judea: Naturally occurring asphalt that is photo-sensitive

[https://en.wikipedia.org/wiki/Bitumen\\_of\\_Judea](https://en.wikipedia.org/wiki/Bitumen_of_Judea)

# From Joseph Niepce to Louis Daguerre

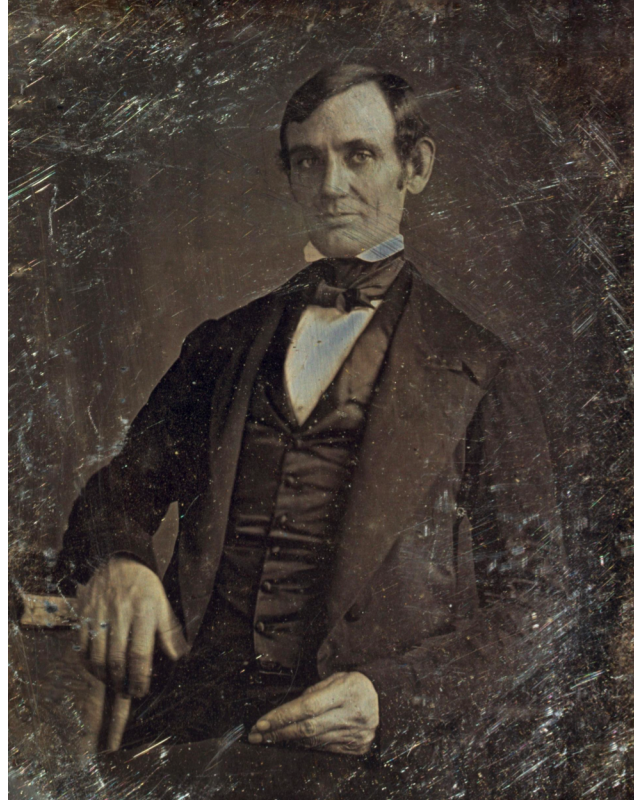


- Act 1: Joseph Niepce tells his nice idea to Louis Daguerre
- Act 2: Good friend Louis Daguerre improves idea and names it Daguerrotypes
- Act 3: Louis Daguerre makes history (and money).

# 1846 Daguerrotype of a young Abraham Lincoln



Daguerreotype camera built by La Maison [Susse Frères](#) in 1839, with a lens by Charles Chevalier



# Hercules Florence's *Photographie*

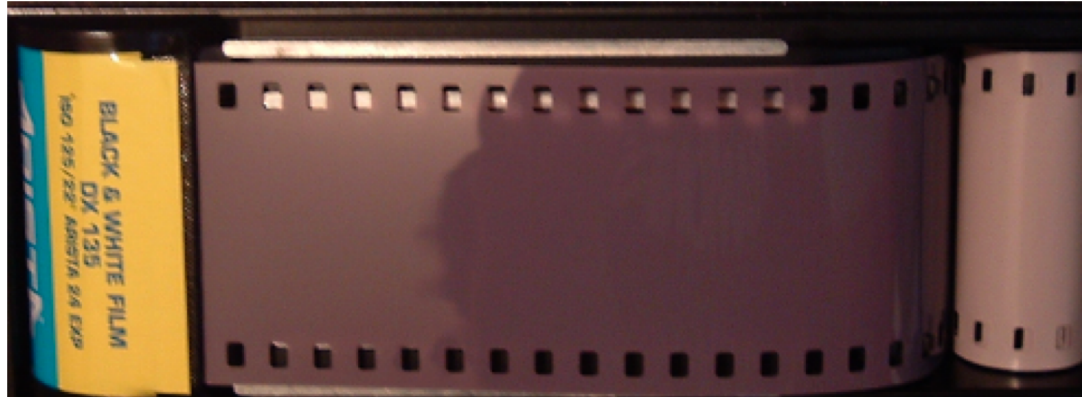
- Brazilian painter and inventor
- Before Daguerre but after Niepce
- Included the idea of negatives





# George Eastman 1885 (Rochester, NY)

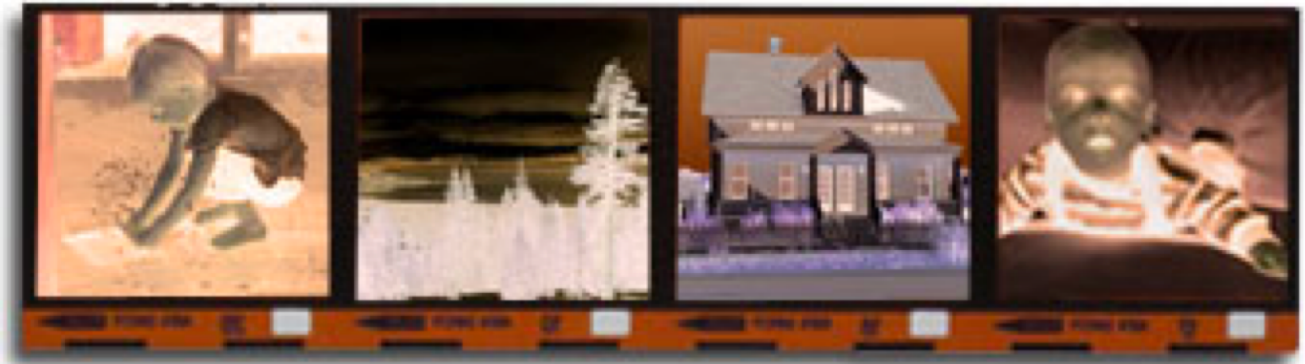
- Founder of pioneering Eastman Kodak Company (Kodak)
- Popularization of film photography (nitrate film)



Should look familiar if you were born in the 80's or earlier or if you are a true modern-day hipster!



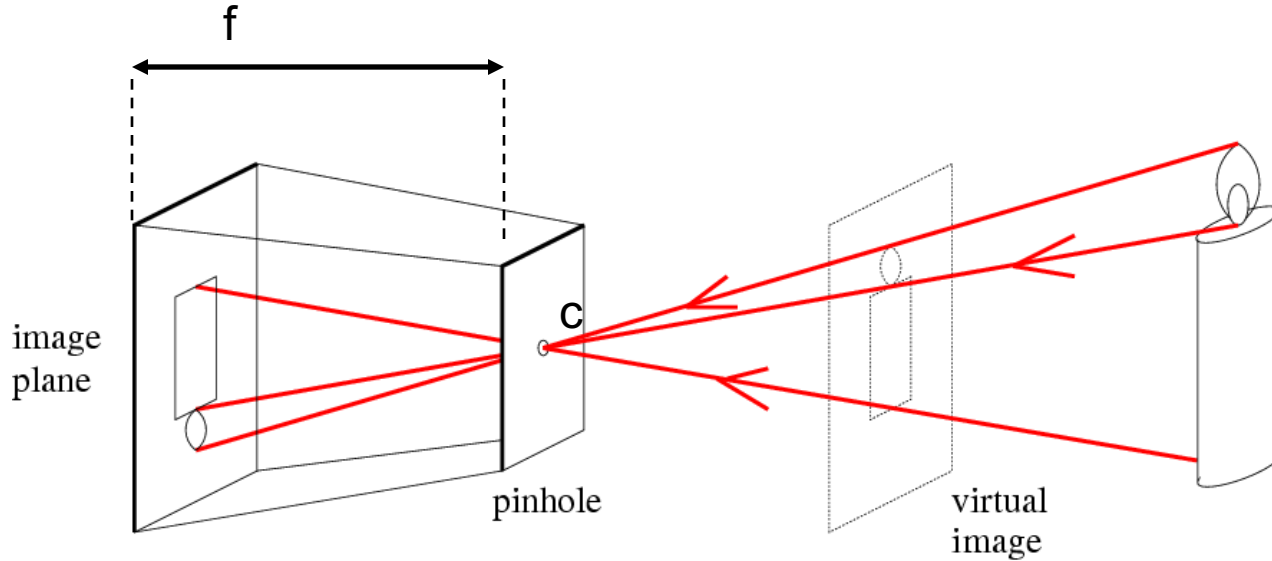
- Photo negatives



So, who invented cameras?

Maybe the wrong question to ask?

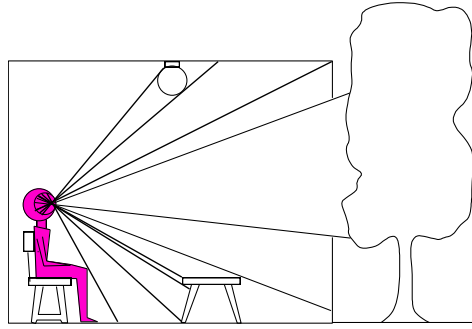
# Back to the Pinhole camera



$f$  = focal length  
 $c$  = center of the camera

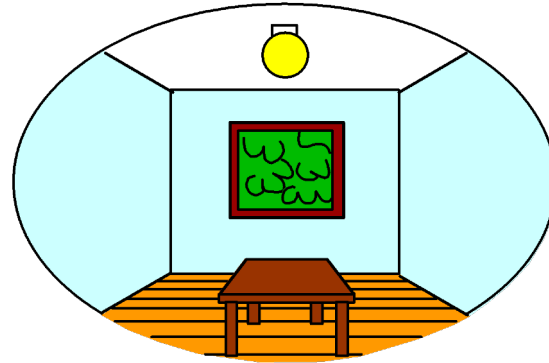
# Dimensionality Reduction Machine (3D to 2D)

*3D world*



Point of observation

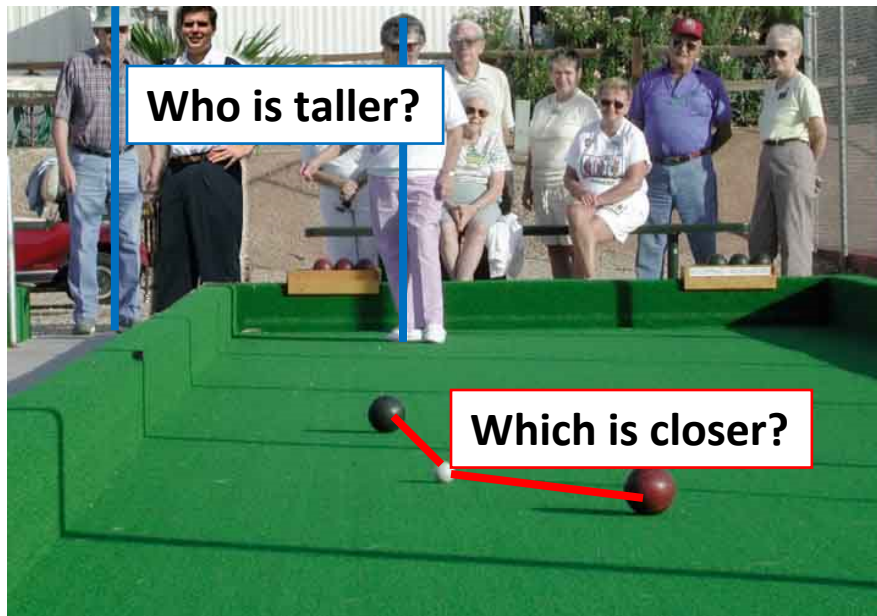
*2D image*



# Projective Geometry

What is lost?

- Length



# Length and area are not preserved

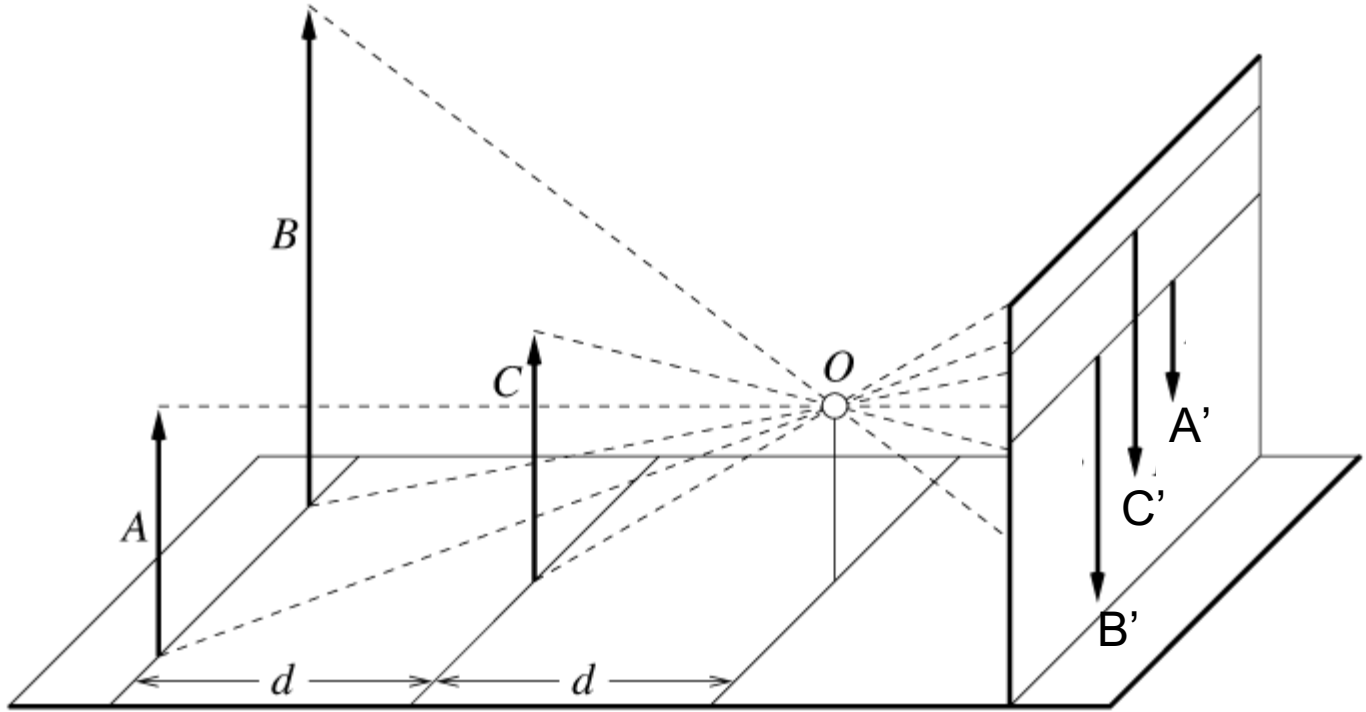


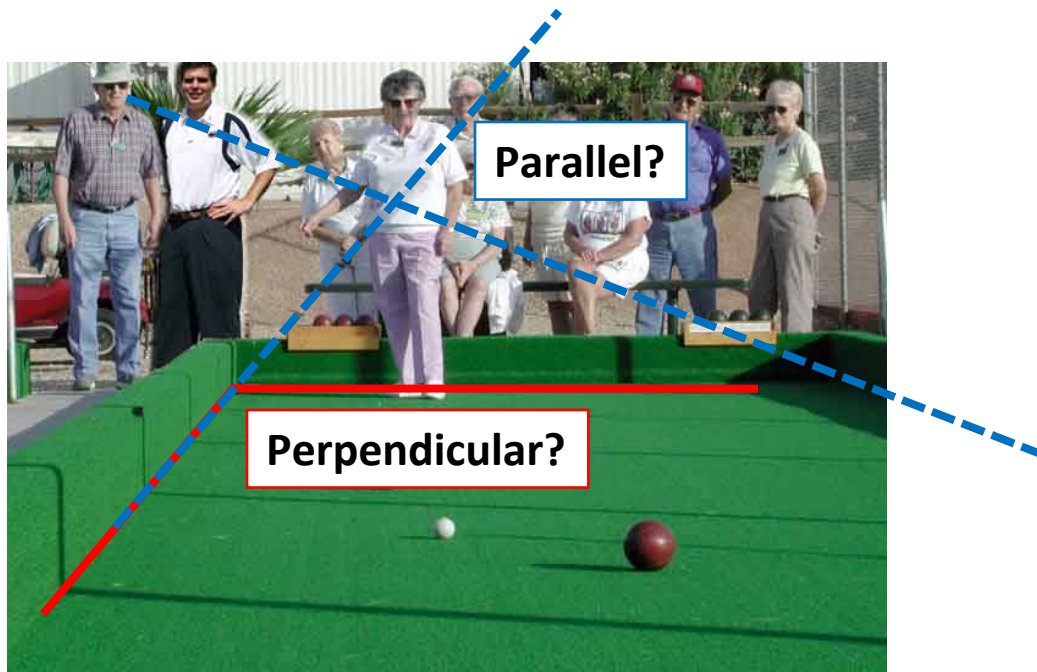
Figure by David Forsyth



# Projective Geometry

What is lost?

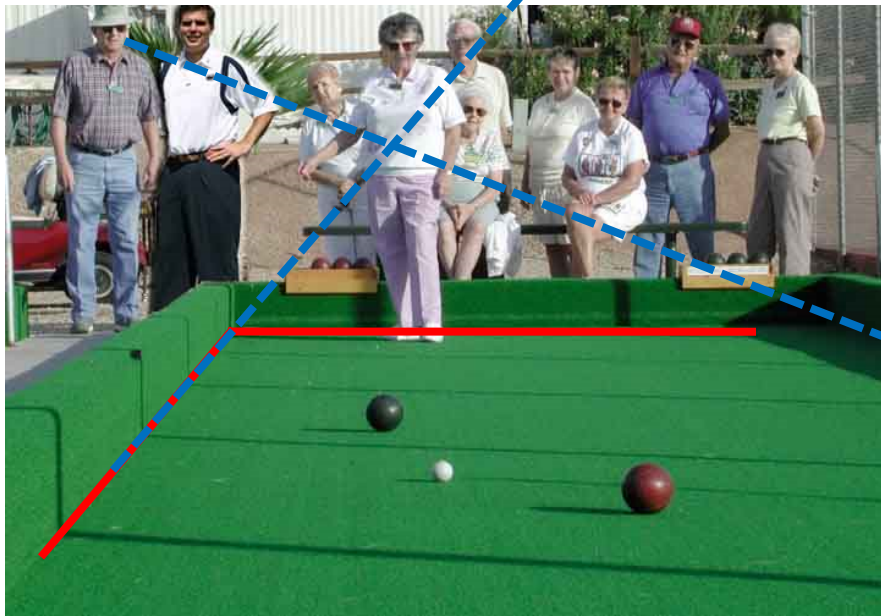
- Length
- Angles



# Projective Geometry

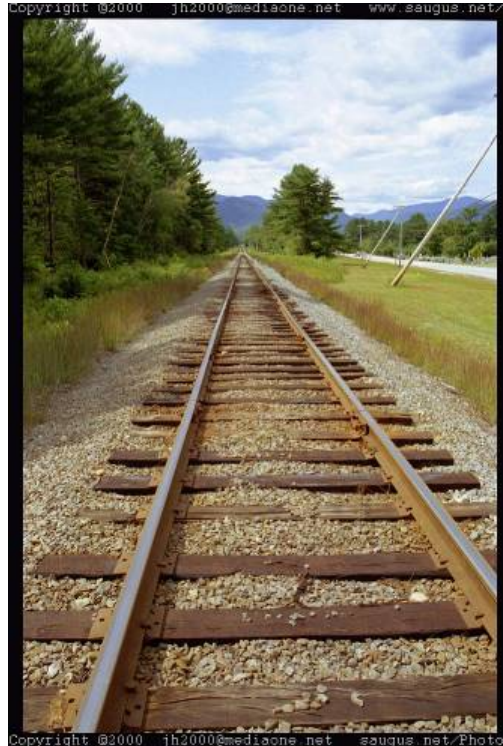
## What is preserved?

- Straight lines are still straight

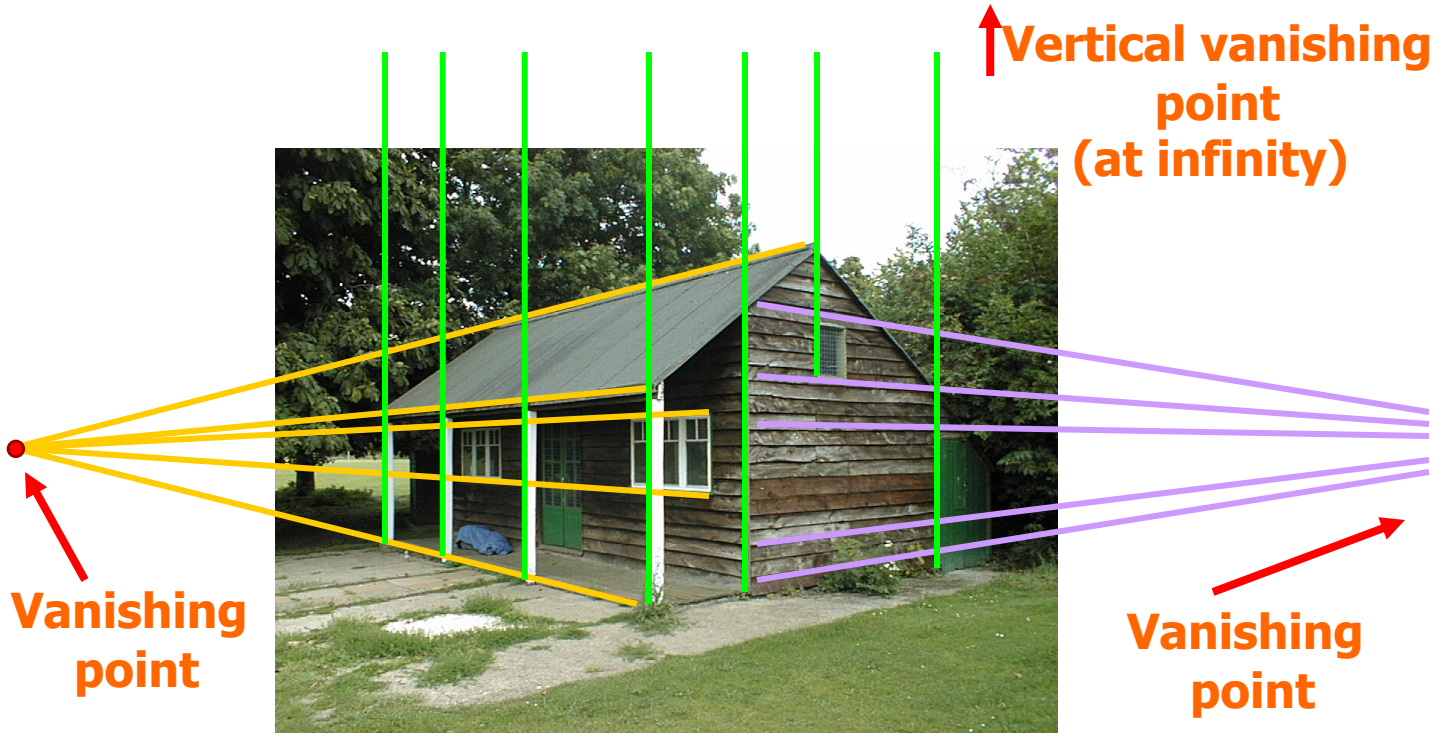


# Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”



# Vanishing points and lines



# Limitations of the Pinhole camera

- Not easy to produce a perfect pinhole in practice
- To photograph some objects you would really need a room sized camera ("camera" means literally room in latin)
- Depending on the type of photo-sensitive material, you would need to keep the light passing through the pinhole a long time (hours? days?).

Solution?

Lenses!

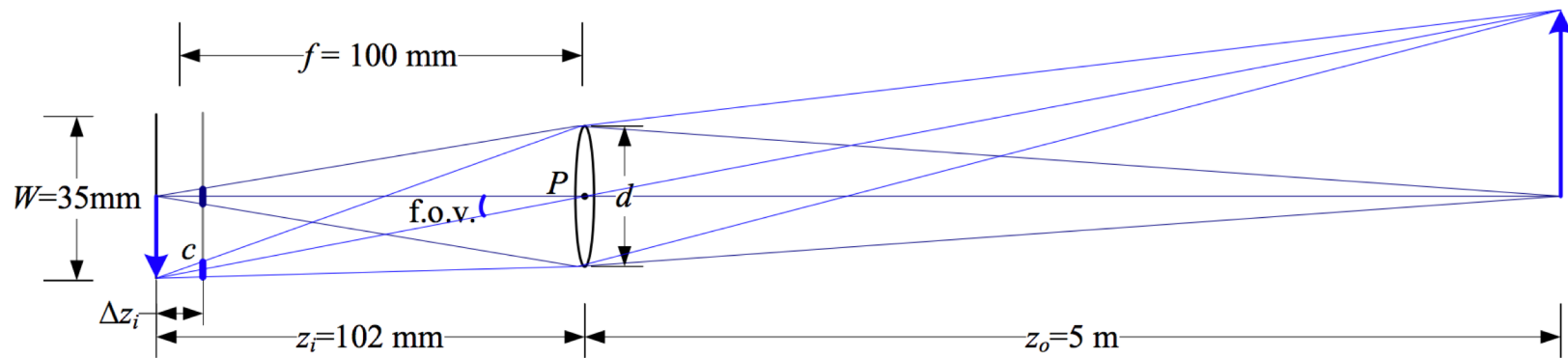


Figure from Szeliski for thin lens



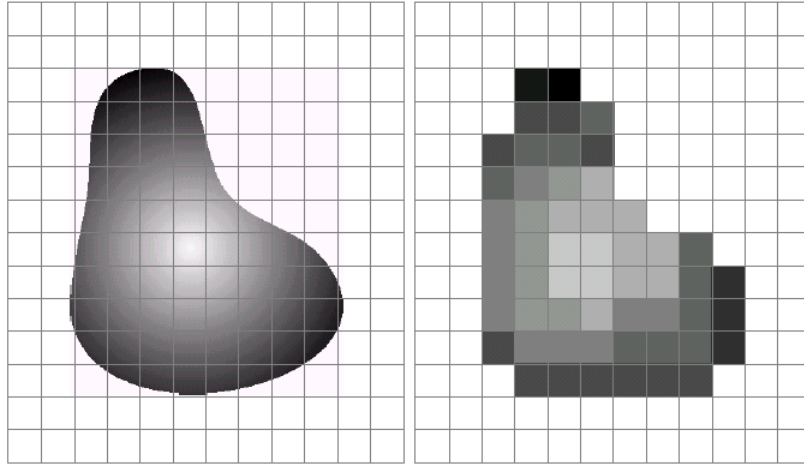
We will mostly use the pinhole camera model  
however as an approximation of actual cameras.

# Digital camera



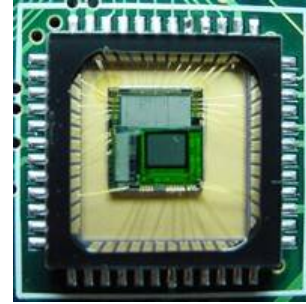
- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode that converts photons to electrons
  - Two common types
    - Charge Coupled Device (CCD)
    - CMOS
  - <http://electronics.howstuffworks.com/digital-camera.htm>

# Sensor Array



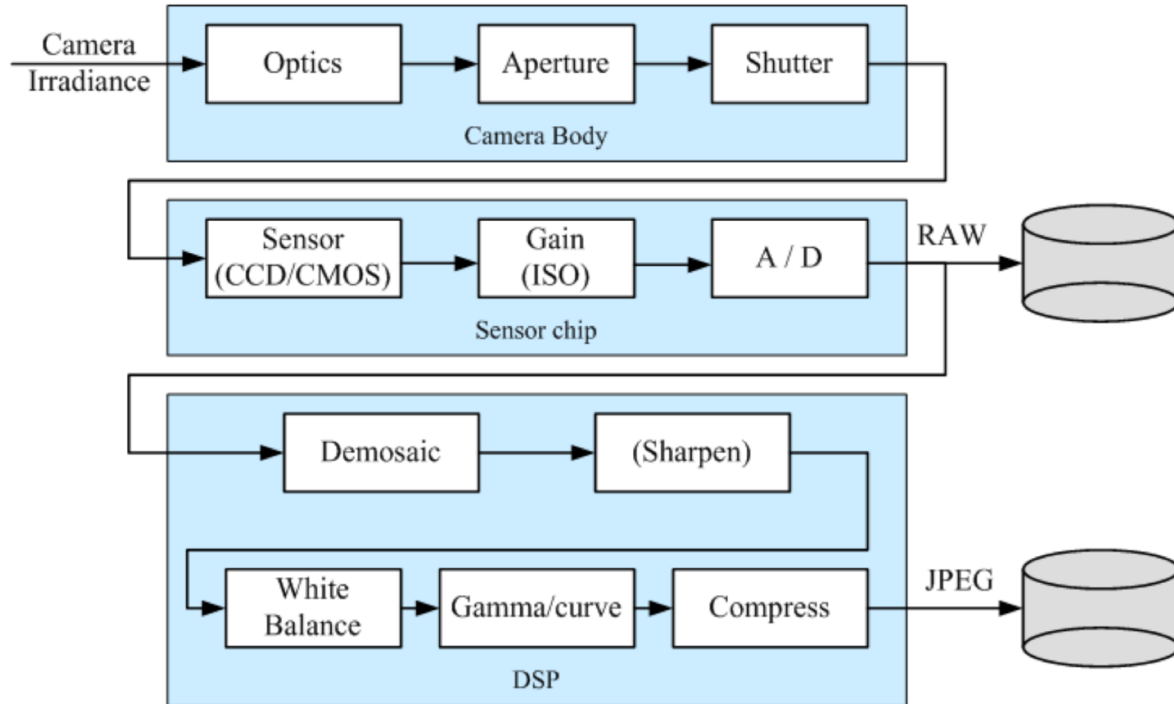
a b

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



CMOS sensor

# Digital Camera Pipeline



# Cameras



Vivitar 20.1MP  
\$30



Polaroid  
\$100



EOS Rebel T6i  
\$900



Canon EOS C300  
\$40,000

Movie Quality

# Netflix Approved Cameras for Content Producers

<https://backlothelp.netflix.com/hc/en-us/articles/217237077-Production-and-Post-Production-Requirements-v2-1>

## Arri Alexa LF w/ 4x 2tb SXR Codex Mags



6+

1 viewed per hour



**\$107,500.00**

+ \$63.90 Shipping

### or Best Offer

Get it by **Thursday, Sep 12** from Culver City, California

- **Used** condition
- No returns, but backed by [eBay Money back guarantee](#)

*"Lightly usedExcellent Shape205 hrs  
1 SxR Adapter for Alexa1 SxS Adapter for Alexa4 SxR Capture  
Drives 2TB w/ Cases 1 SXR Card Reader Codex1 Cable:..."*

[Read full description](#)

[See details](#)



# Cameras



Apple's iPhone X  
\$1000



Google's Pixel 2 XL  
\$800

# Cameras



Nikon D90  
\$1200



Nikon D3300  
\$700

# How to Shoot Photos in Manual?

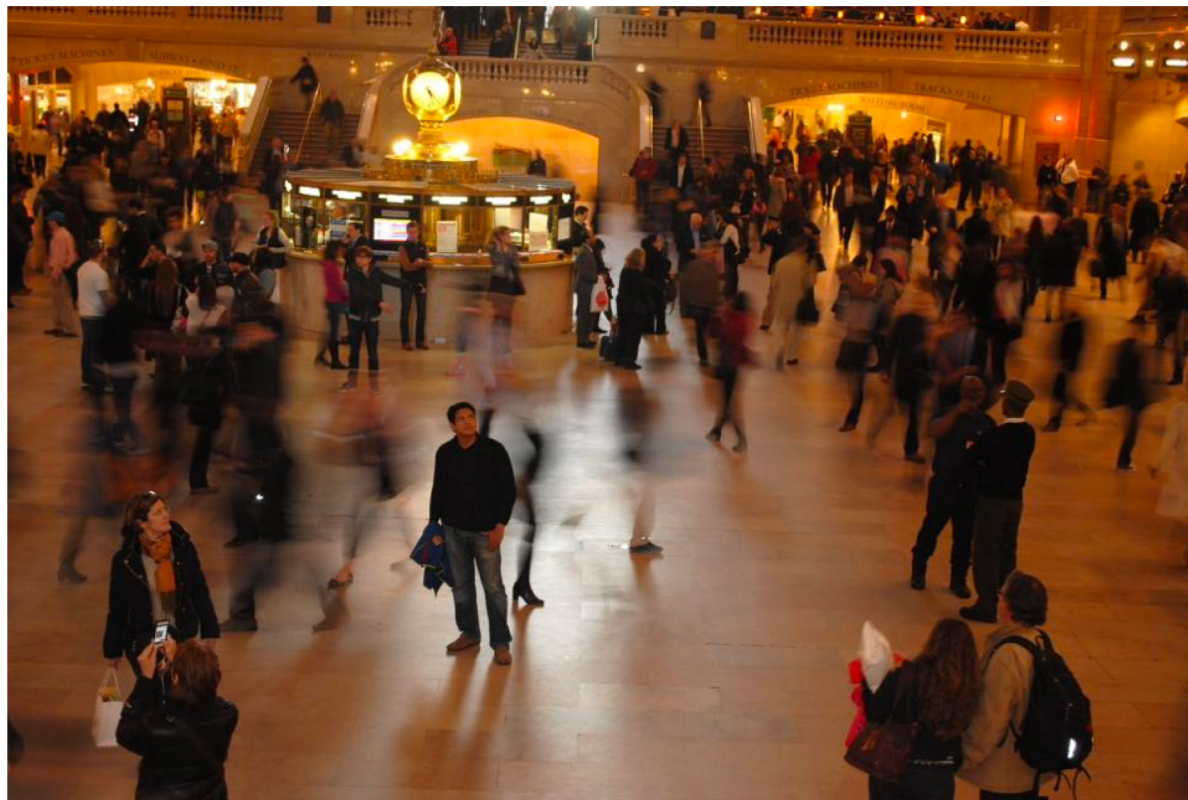
- Shutter time
- Aperture
- ISO
- Focus / Auto-focus (Yes, you can shoot in manual and also probably should focus in manual)

# Small Shutter Time / Speed



<http://www.photographymad.com/pages/view/shutter-speed-a-beginners-guide>

# Long Shutter Time



# Long Shutter Time





# Very Large Shutter Time – 25 seconds



# Long Shutter Time? Think of Buying a Tripod



Aluminum Tripod \$140

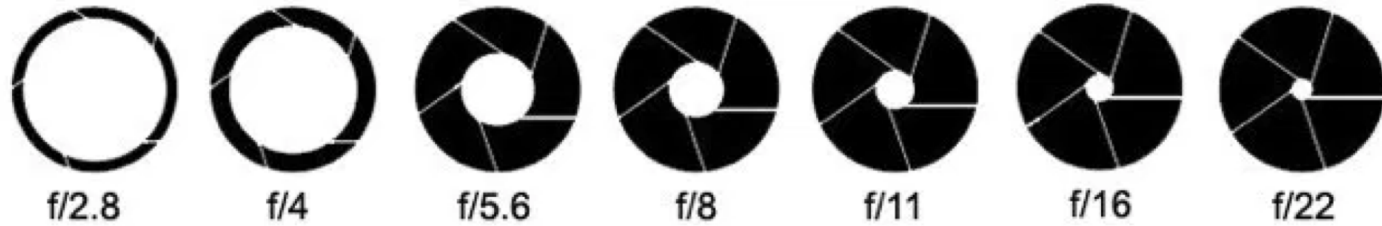


Carbon Fiber Tripod \$200



Manfrotto Mountaineer  
Carbon Fiber Tripod  
\$1300

# Aperture

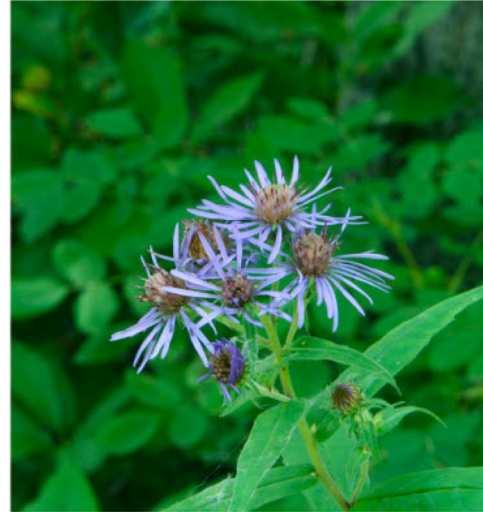


# Large vs Small Aperture + Focus Control

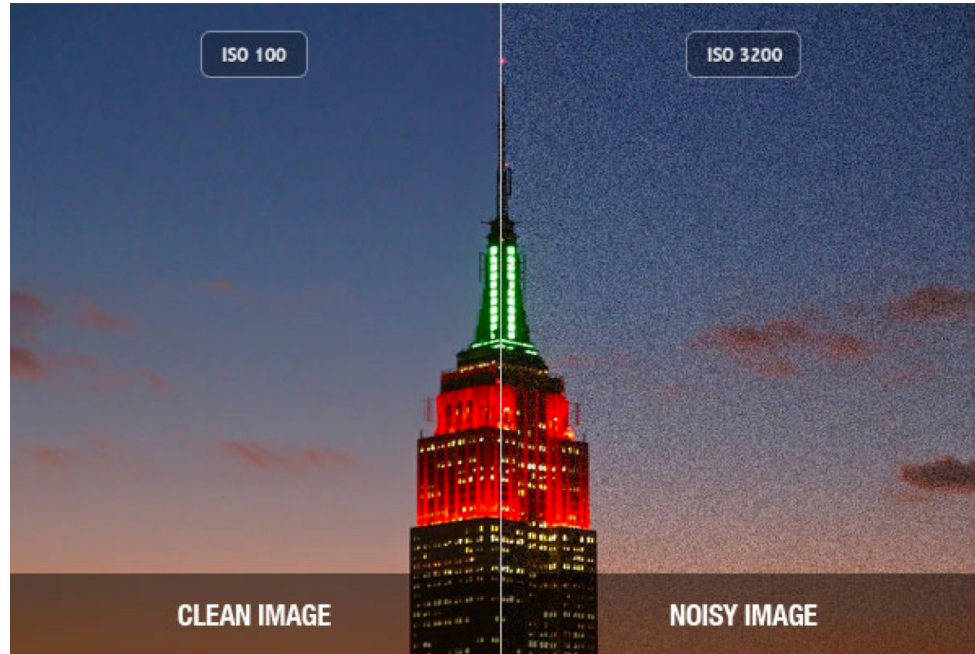
Large Aperture (F4.0)  
Background nicely blurred



Small Aperture (F22)  
Background is distracting



ISO – Should be small ideally



<https://www.exposureguide.com/iso-sensitivity/>

# Trade-offs (We need light to capture a photo)

- Small Aperture leads to less light  
(but allows more focus on objects)
- Small Shutter speed leads to less light  
(but allows capturing fast moving objects)
- Small ISO leads to less light  
(but produces less noisy “grainy” output)

# Most difficult picture to take?

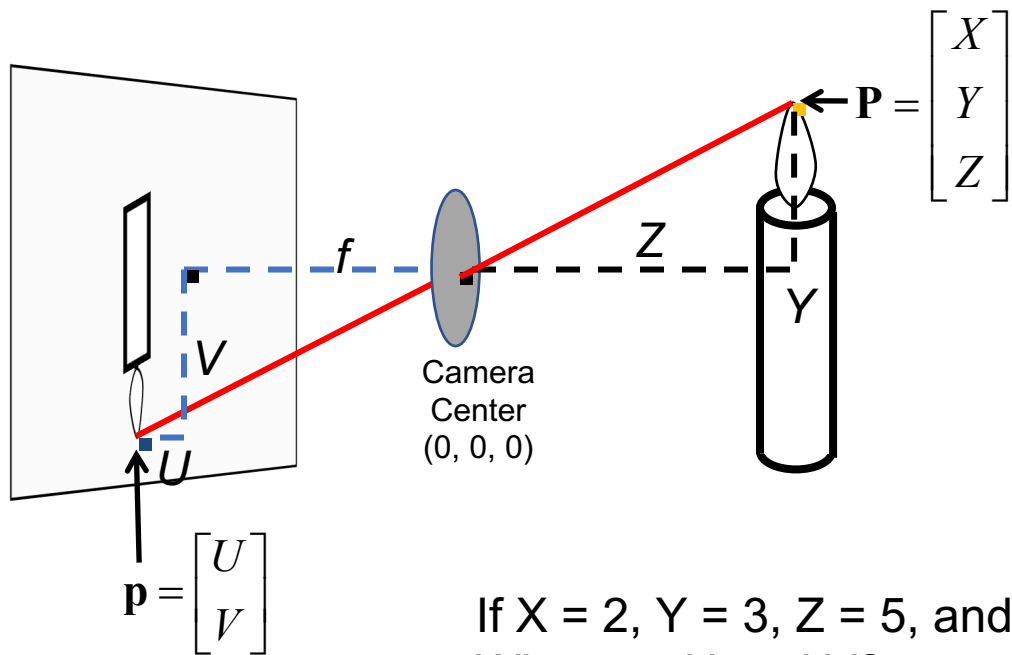
- One where we want focus on a small object in the scene  
(this requires small aperture)
- AND the object is moving fast  
(this requires fast shutter speed)
- AND the object is in a rather poorly illuminated room  
(but if we use a high ISO the picture will be noisy)



## Final Thoughts - Take with grain of salt

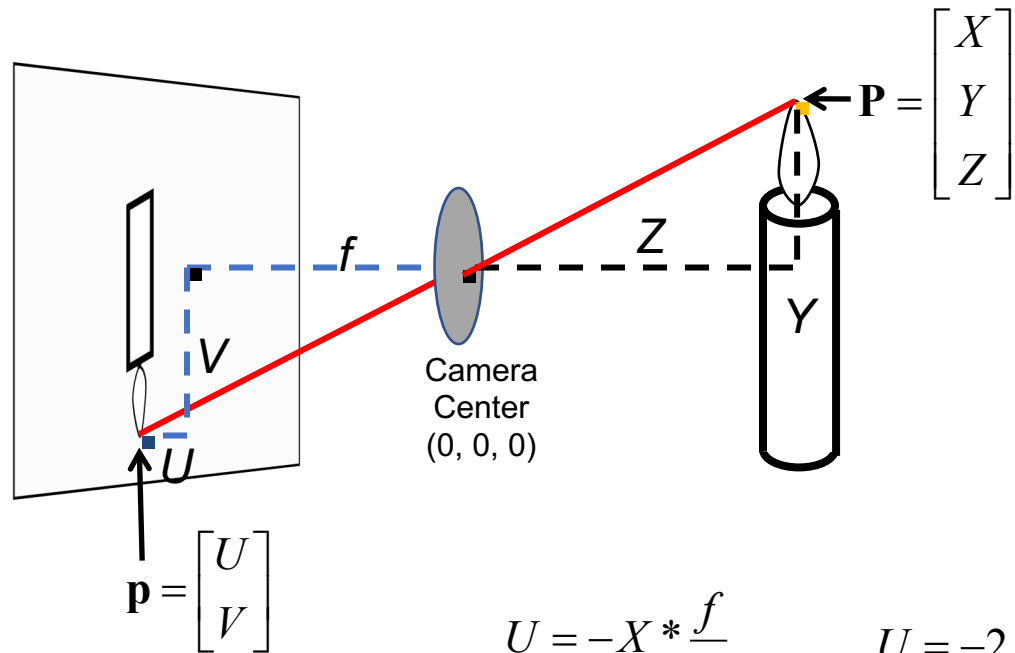
- Shooting in Automatic, especially in low light conditions will often go the easy route of just increasing the ISO all the way up
- Sometimes in low light conditions instead you want to increase the shutter time to compensate the low light, or increase the aperture. (or use Flash)
- No shame in using Automatic in a clear day, unless trying to achieve some effect.

Projection: world coordinates  $\rightarrow$  image coordinates



If  $X = 2$ ,  $Y = 3$ ,  $Z = 5$ , and  $f = 2$   
What are  $U$  and  $V$ ?

Projection: world coordinates  $\rightarrow$  image coordinates



$$U = -X * \frac{f}{Z}$$

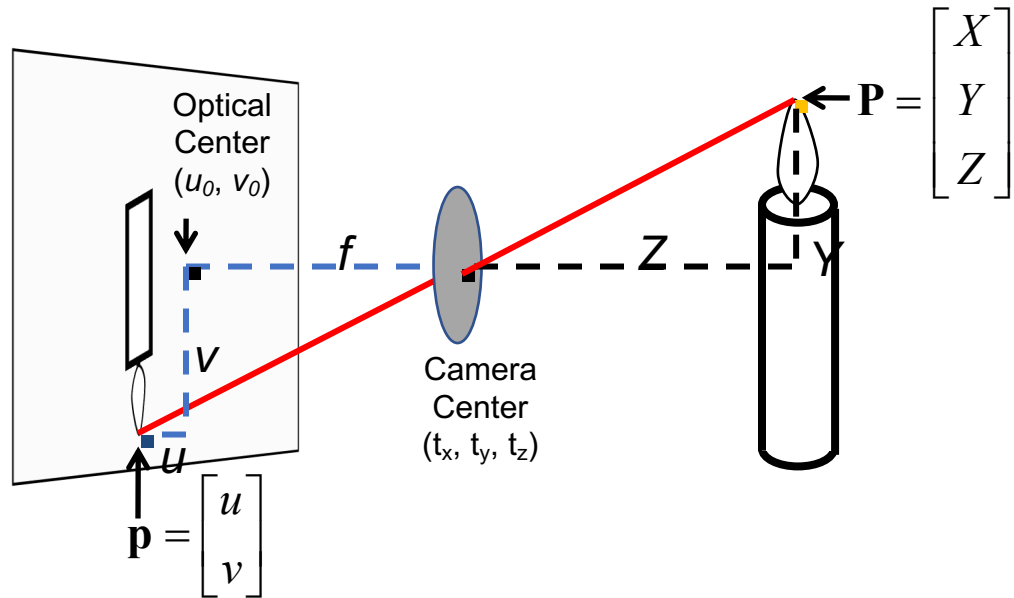
$$U = -2 * \frac{2}{5}$$

$$V = -Y * \frac{f}{Z}$$

$$V = -3 * \frac{2}{5}$$

Sanity check, what if  $f$  and  $Z$  are equal?

Projection: world coordinates  $\rightarrow$  image coordinates



# Homogeneous coordinates

## Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image  
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene  
coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

# Homogeneous coordinates

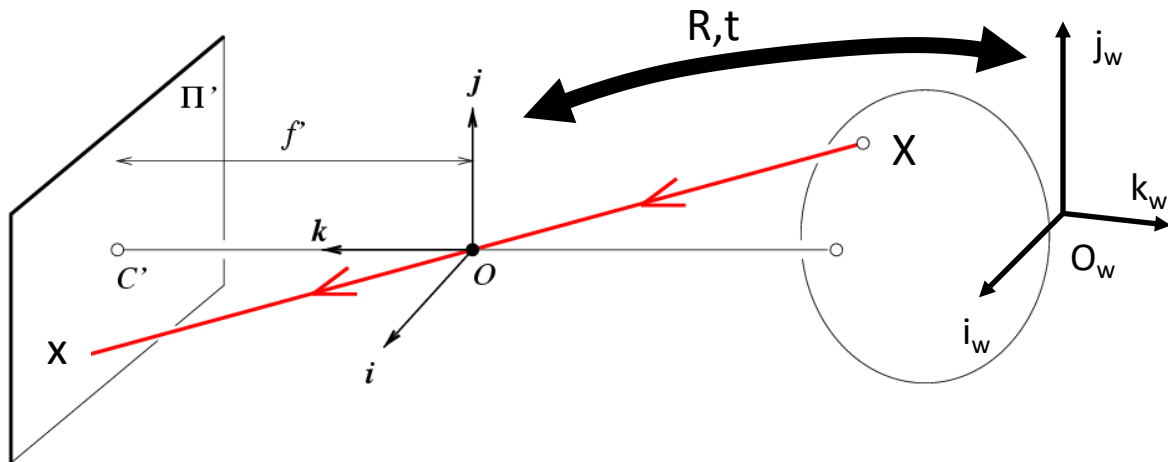
Invariant to scaling

$$k \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \\ \frac{kw}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ 1 \end{bmatrix}$$

Homogeneous Coordinates      Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

# Projection matrix (World Coordinates to Image Coordinates)



$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$

$\mathbf{x}$ : Image Coordinates:  $(u, v, 1)$

$\mathbf{K}$ : Intrinsic Matrix  $(3 \times 3)$

$\mathbf{R}$ : Rotation  $(3 \times 3)$

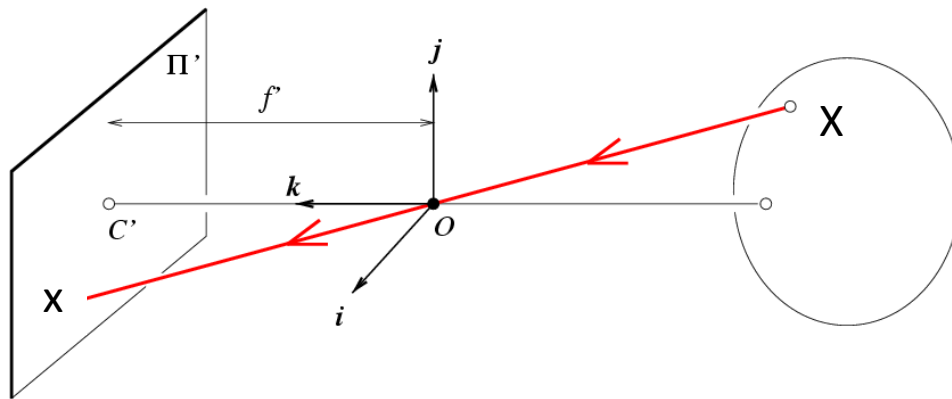
$\mathbf{t}$ : Translation  $(3 \times 1)$

$\mathbf{X}$ : World Coordinates:  $(X, Y, Z, 1)$

Intrinsic Camera Properties:  $\mathbf{K}$

Extrinsic Camera Properties:  $[\mathbf{R} \quad \mathbf{t}]$

# Projection matrix



## Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

## Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow {}^w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

**K**



# Remove assumption: known optical center

## Intrinsic Assumptions

- Unit aspect ratio
- No skew

## Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Remove assumption: square pixels

## Intrinsic Assumptions

- No skew

## Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Remove assumption: non-skewed pixels

Intrinsic Assumptions

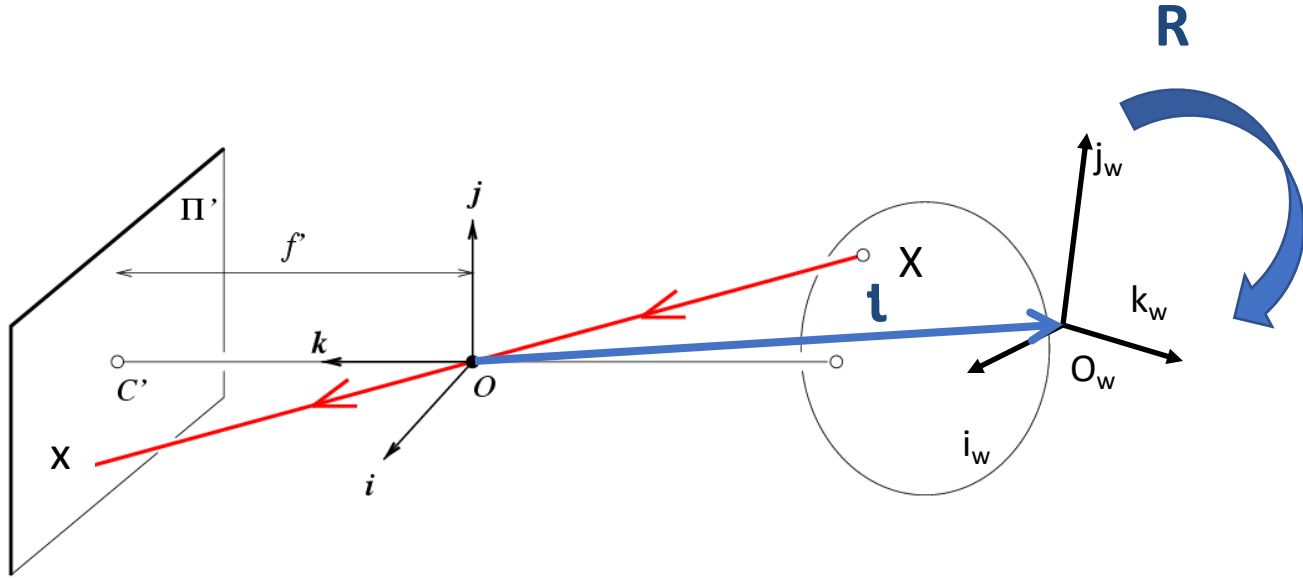
Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

# Oriented and Translated Camera



# Allow camera translation

Intrinsic Assumptions

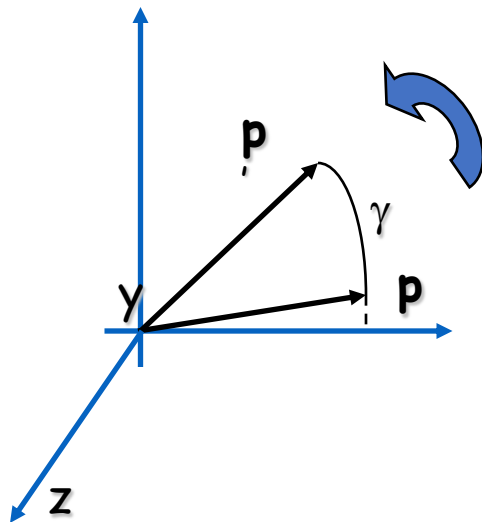
Extrinsic Assumptions

- No rotation

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# 3D Rotation of Points

Rotation around the coordinate axes, **counter-clockwise**:




$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Allow camera rotation

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$


$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



# Degrees of freedom

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$



$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{matrix} & \overset{5}{\begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}} & \overset{6}{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}} \end{matrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Things to Remember for Quiz

- Pinhole camera model
- Focal length in the pinhole camera model
- Shutter Time / Aperture / ISO
- Homogeneous Coordinates
- Extrinsic Camera Properties and Intrinsic Camera Properties
- Describe mathematically (and intuitively) the conversion process from World Coordinates to Image Coordinates

# Next Class: Light

- What determines the color of a pixel?

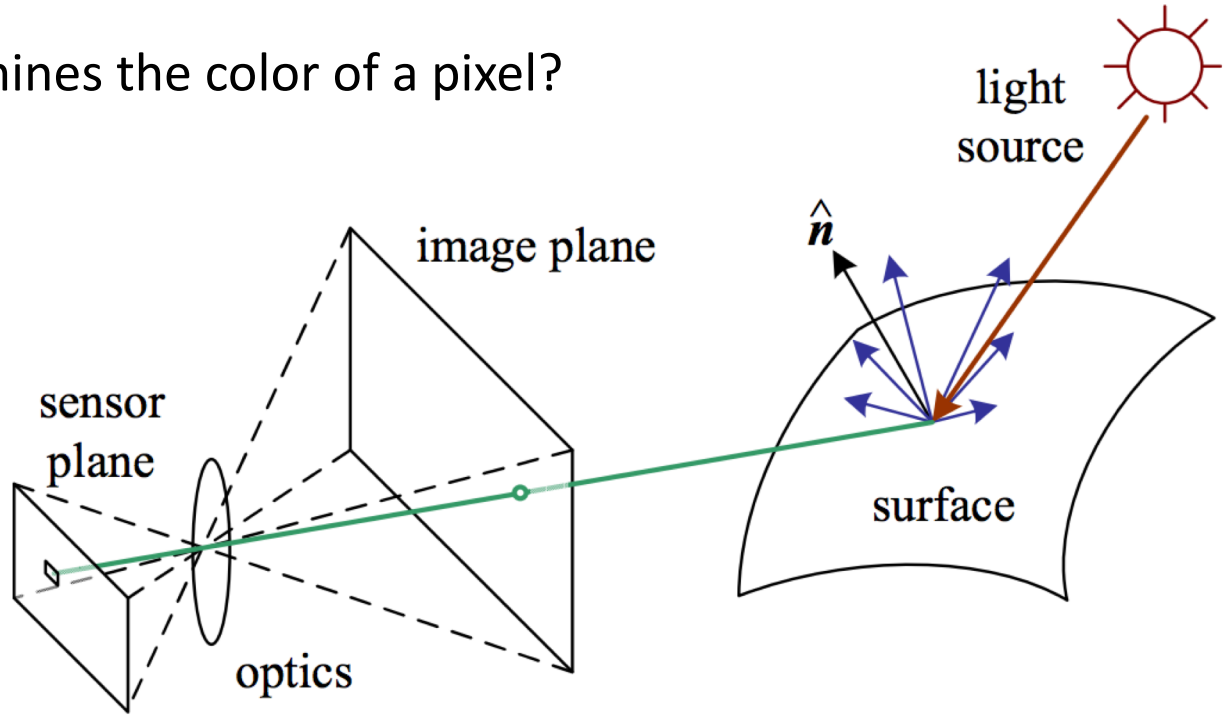
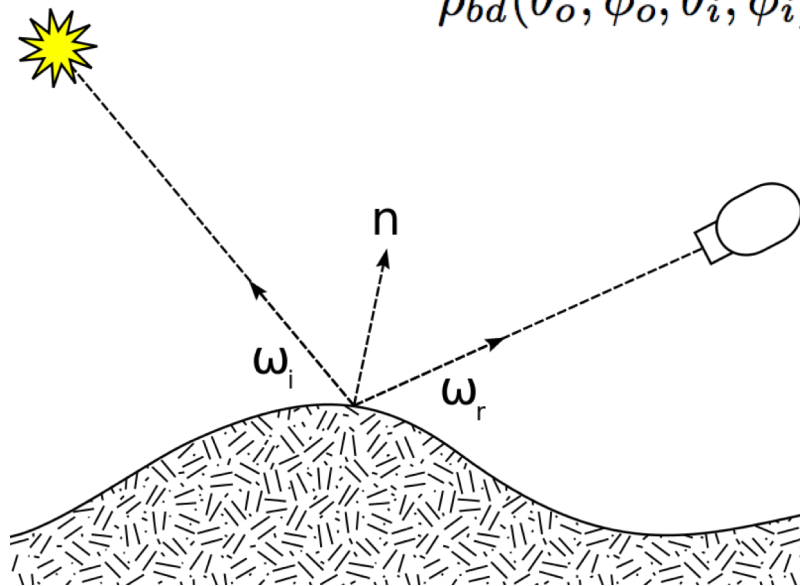


Figure from Szeliski

# BRDF (Bidirectional reflectance distribution function)

$$\rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(\mathbf{x}, \theta_o, \phi_o)}{L_i(\mathbf{x}, \theta_i, \phi_i) \cos \theta_i d\omega}$$



# Questions?