



# Local definitions and lexical scope

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# Top-Level Definitions

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We have learned three kinds of definitions thus far:

1. Function definitions, *e.g.*,  
`(define (f x) (+ x 1))`
2. Variable (constant) definitions, *e.g.*,  
`(define two (f 1))`
3. Structure definitions, *e.g.*,  
`(define-struct pair (left right))`

They appear in Dr. Scheme's **Definitions** window and are called *top-level definitions*



# Local Expression

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A **local** *expression* groups together a set of definitions for use in a subcomputation:

**(local** (*def*<sub>1</sub> *def*<sub>2</sub> ... *def*<sub>n</sub>) *exp*)

- *exp* is an arbitrary expression
- *def*<sub>*i*</sub> is a definition in the set
- the variables defined in *def*<sub>1</sub> *def*<sub>2</sub> ... *def*<sub>n</sub> are distinct and only exist (are available for use) within the **local** expression *i.e.*, within *def*<sub>1</sub> *def*<sub>2</sub> ... *def*<sub>n</sub> and *exp*



# Simple Examples

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```
(define x 3)                ;; top-level definition
(local [(define x 3)] (+ x 1)) ;; local expression
(define (f x) (+ x 1))      ;; top-level definition

(local [(define x 2)        ;; local definitions
        (define (f x) (+ x 1))]
  (f x))                    ;; body

(+ (local [(define x 3) ;; embedded local-expression
           (define (f x) (+ x 1))]
    (f x))
  1)
```



# Some Incorrect Examples

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- What's wrong with following expressions?

```
(local [(define x 1)])
```

```
(local [(define x 1)  
        (define x 2)]
```

```
  x)
```

```
(local [(define x 1)  
        (define f (+ x 1))]  
  (f x))
```



# Why local?

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## Reason 1: Avoid namespace pollution

```
;; sort: list-of-numbers -> list-of-number
;; (sort lon) returns the elements of lon in ascending order
(define (sort alon)
  (cond
    [(empty? alon) empty]
    [(cons? alon) (insert (first alon) (sort (rest alon)))]))

;; insert: number list-of-numbers (sorted) -> list-of number
;; (insert n lon) assumes lon is in ascending order and returns a
;; a list containing n and the elements of lon in ascending order

(define (insert an alon)
  (cond [(empty? alon) (list an)]
        [else (if (<= an (first alon))
                    (cons an alon)
                    (cons (first alon) (insert an (rest alon))))]))
```



# Why local?

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- Reason 1: Avoid namespace pollution (cont.)

```
;; sort: list-of-numbers -> list-of-numbers  
(define (sort alon)
```

```
(local  
  [;; insert: number list-of-numbers (sorted) -> list-of numbers  
    (define (insert an alon)  
      (cond [(empty? alon) (list an)]  
            [else (if (<= an (first alon))  
                      (cons an alon)  
                      (cons (first alon)  
                            (insert an (rest alon))))])] ])
```

```
(cond [(empty? alon) empty]  
      [(cons? alon) (insert (first alon) (sort (rest alon)))]))
```

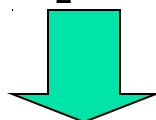


# Why local?

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Reason 1: Avoid namespace pollution

```
(define (mainFun x) exp)
(define (auxFun1 ...) exp1)
(define (auxFun2 ...) exp2)
```



```
(define (mainFun x)
  (local [(define (minFun x) exp)
          (define (auxFun1 ...) exp1)
          (define (auxFun2 ...) exp2)]
    (mainFun x)))
```





# Why local?

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## Reason 2: Avoid repeated computation

```
;; max-num: list-of-number -> number
;; (max=num lon) returns the largest number n in lon;
;;   throws an error if lon is empty
(define (max-num x lon)
  (cond
    [(empty? lon) ...]
    [else ... (first lon)
               ... (max-num x (rest lon)) ...])))
```



# Why local?

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## Reason 2: Avoid repeated computation

```
(define (max-num lon)
  (cond
    [(empty? lon)
     (error "max-num applied to empty list")]
    [else
     (if (>= (first lon) (max-num (rest lon))
         (first lon)
         (max-num (rest lon)))])])
```



repeated work



# Why local?

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Reason 2: Avoid repeated computation

```
(define (max-num lon)
  (cond
    [(empty? Lon)
     (error "max-num applied to empty list")]
    [else
     (local [(define rest-max (max-num (rest lon)))]
       (if (> (first lon) rest-max)
           (first lon)
           rest-max))]))
```



# Why local?

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## Reason 3: Naming complicated expressions

```
;; mult10 : list-of-digits -> list-of-numbers
;; creates a list of numbers by multiplying each digit in alod
;; by (expt 10 p) where p is the number of digits that follow
;; This is bad code used only as an example. Good code
;; requires refactoring techniques we haven't learned yet.
```

```
(define (mult10 alod)
  (cond [(empty? alod) empty]
        [else (cons (* (expt 10 (length (rest alod)))
                        (first alod))
                      (mult10 (rest alod)))]))
```



# Why local?

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- Reason 3: Naming complicated expressions

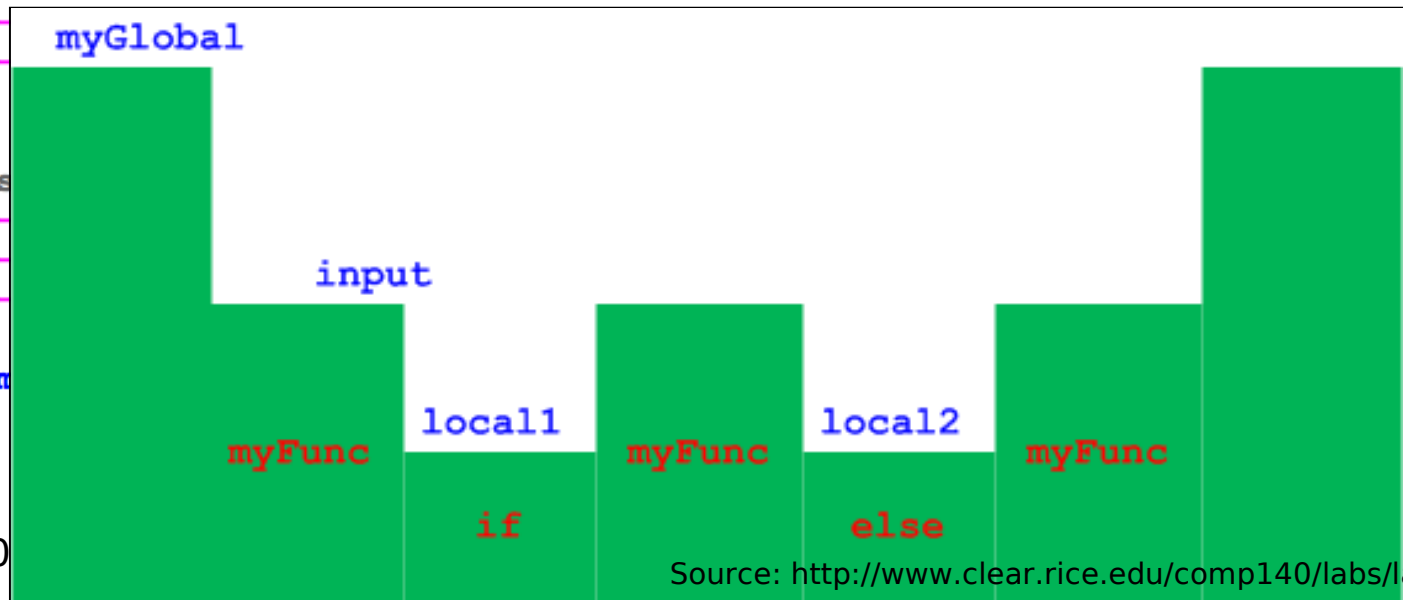
```
;; mult10 : list-of-digits -> list-of-numbers
;; creates a list of numbers by multiplying each digit in alod
;; by (expt 10 p) where p is the number of digits that follow
;; This is bad code used only as an example. Good code
;; requires refactoring techniques we haven't learned yet.
```

```
(define (mult10 alod)
  (cond [(empty? alod) empty]
        [else (local [(define a-digit (first alod))
                        (define the-rest (rest alod))
                        (define p (length the-rest))]
                  (cons (* (expt 10 p) a-digit) (mult10 the-rest)))]))
```

# Recap of Variable Scopes from COMP 140

```
myGlobal = 42
```

```
def myFunc(input):  
    print "myFunc: input = ", input  
    print "myFunc: myGlobal = ", myGlobal # global variable visible here  
    # neither local1 nor local2 are accessible here.  
    if input > 0:  
        local1 = 100  
        # cannot access local2 from here.  
        print "myFunc-if: local1 = ", local1  
        print "myFunc-if: myGlobal = ", myGlobal  
        print "myFunc-if: input = ", input  
    else:  
        local2 = -100  
        # cannot access local1 from here.  
        print "myFunc-else: local2 = ", local2  
        print "myFunc-else: myGlobal = ", myGlobal  
        print "myFunc-else: input = ", input  
    print "myGlobal = ", myGlobal  
myFunc(5)  
myFunc(-5)
```





# Variables and Scope in Scheme

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- Example:

```
(local ((define answer1 42)
        (define (f2 x3) (+ 1 x4)))
  (f5 answer6))
```

- Variable occurrences: 1-6
  - Binding (or defining) occurrences: 1,2,3
  - Use occurrences: 4,5,6
  - Scope = code region where a definition may be used
- Scopes of definitions
  - 1:?
  - 2:?
  - 3:?



# Variables and Scope

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- What will g evaluate to?

```
(define x 0)
```

```
(define f x)
```

```
(define g
```

```
  (local ((define x 1)) f))
```





# Renaming

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Example:

```
(local [(define answer1 42)
        ((define (f2 x3) (+ 1 x4)))]
  (f5 answer6))
```

- Which variable occurrences can be renamed within the local expression?
- Use the same name for “binding occurrence” and all its “use occurrences”.
- Local variables can safely be renamed (no change to the answers produced by a program) without changing anything in the surrounding program.
- What name choices can be used? Any name that does not clash with variable names already visible in same scope. A “fresh” variable name.



# Renaming

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Example:

```
(local [(define answer 42)
        (define (f x) (+ 1 x))])
(f answer)
```

=>

```
(local [(define answer_0 42)
        (define (f_0 x) (+ 1 x))])
(f_0 answer_0)
```

We must rename all occurrences of a variable, both its *binding* occurrence and its *use* occurrences. In the preceding example, both `answer` and `f` have only one *use* occurrence. (Every variable has exactly one *binding* occurrence since each *binding* occurrence *defines* a new variable.) We are using the same underscore number convention for renaming as the DrScheme stepper.



# Renaming

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Recall our example:

```
(local [(define answer 42)
        (define (f x) (+ 1 x))])
(f answer)
```

=>

```
(local [(define answer_0 42)
        (define (f_0 x) (+ 1 x))])
(f_0 answer_0)
```

We could also rename the function parameters within a `local` expression but it is not necessary for our purposes. We simply want to rename all of the variables (including function names) introduced in a `local`.



# Renaming in Evaluating `local`

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Idea: We can promote (move) the block of `defines` introduced in a `local` to the top level (like the other `defines` in our program) *provided* that rename the variables introduced in the `local` so that they cannot clash with variables already `defined` at the top level.

Rule: when the leftmost unevaluated expression is a `local`, rename the variables `defined` in the `local`, lift the block of `defines` in the renamed `local` to the top level, and replace the `local` expression by its renamed body.



# Evaluating `local` Expressions

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Recap: how do we (hand) evaluate Scheme programs with `local`?

- By (i) renaming all of the **defined** variables in the `local` (with *fresh* names to avoid any collisions with variables already defined at the top level), (ii) lifting the renamed local definitions to the top level, and (iii) replacing the `local` expression by its renamed body.

To express this law we need a new format for expressing rules. Why? Because lifting `local` definitions *augments* the set of definitions that constitute the *environment* in which evaluation takes place.



# Hand Evaluation Example

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```
(define x 2)                ;; top-level definition
;; local-expression as part of another expression
(+ (local [(define x 3) (define (f x) (+ x 1))]
    (f x))
  1)
=>
(define x 2)
(define x_0 3)
(define (f_0 x) (+ x 1))    ;; parameters not renamed
(+ (f_0 x_0) 1)
=>
```



# Hand Evaluation Example

---

```
(define x 2)
(define x_0 3)
(define (f_0 x) (+ x 1))
(+ (f_0 3) 1)
```

=>

```
(define x 2)
(define x_0 3)
(define (f_0 x) (+ x 1))
(+ (+ 3 1) 1)
```

=>

```
(define x 2)
(define x_0 3)
(define (f_0 x) (+ x 1))
(+ 4 1)
```



# Hand Evaluation Example

---

=>

```
(define x 2)
(define x_0 3)
(define (f_0 x) (+ x 1))
(+ 4 1)
```

With **local** in the language, each step in the evaluation must carry the environment (the block of **defines** constituting the program) as well as the expression being evaluated.

Confused? Try using the stepper (the menu button shaped like a foot) on examples in DrScheme.





# When naming can cause problems

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Romeo, Romeo! wherefore art thou Romeo?

. . .

What's in a name? That which we call a rose  
by any other name would smell as sweet.

*Romeo and Juliet (II, ii)*