Mid-term

122/4.
The starting state should be one of the accepting states.

154/15.
a. $L(M) = \varepsilon \cup (b \cup \varepsilon) a(ba)^* a$
b.

161/2.
a. $L(G) = a(b^* a)^*$
b.

182/1(n)
Not regular.
Assume it is regular and $n$ is the constant from the pumping theorem.

$w = a^n b^{n+2}$, so $w \in L$ and $|w| = 2n + 2 > n$.

Let $x = a^\alpha, y = a^\beta, z = a^{n-\alpha-\beta} b^{n+2}$, where $\beta \geq 1, \alpha + \beta \leq n$. 
So \(xyz = w \in L\) and \(xy^i z = a^{n + \beta (i-1)} b^{n+2} \in L\) for \(i \geq 0\).

Let \(i = 0\), then \(xy^0 z = a^{n - \beta} b^{n+2}\).

Since \(\beta \geq 1\), \#_b(xy^0 z) - \#_a(xy^0 z) = (n + 2) - (n - \beta) = 2 + \beta > 2\). So \(xy^0 z \notin L\).

Contradiction.

So \(L\) is not regular.

184/15.

\[
L = (L_1 \cup L_2) - (L_1 \cap L_2).
\]

Since both \(L_1\) and \(L_2\) are regular and regular languages are closed under union, intersection and difference, \(L\) is regular too.

185/19.

a. Yes.

Regular languages are closed under union. And the union of two infinite languages will still be infinite. So IR languages are closed under union.

b. No.

Example: \(a^* b \cap a b^* = ab\).

c. Yes.

Regular languages are closed under Kleene star. And the Kleene star of any infinite language will still be infinite. So IR languages are closed under Kleene star.

185/21.

a. True

Proof:

The number of languages over \(S = \{a, b\}\) is equal to the number of subsets of \(S^*\), which is uncountable. But the number of regular languages is equal to the number of finite state automata which is the countable union of finite sets and hence countable. If
the number of non-regular languages were also countable, then the number of languages over S would be countable, since the union of two countable sets is countable. But the number of languages over S is not countable; therefore the number of non-regular languages cannot be countable.

c. False
Proof:
$L = \{a\}$, which is regular.

Let $L' = L \cup L \cup L \cup \cdots$ (the union of an infinite number of $L$). But $L' = \{a\}$, which is regular.

e. True
Proof:
Since regular languages are closed under difference and union, given that $L_1$ and $L_2$ are both regular, $L_1 \otimes L_2$ is also regular.

g. False
Proof:
$L_1 = a^*b^*$, which is regular. $L_2 = a^*b^*$, which is not regular.

$L = L_1 \cap L_2 = a^*b^*$, which is not regular.

196/1(c).
1. Build a FSM $M_1$ from $\alpha$ using regextofsm($\alpha$). (by theorem 6.1)
2. Build a FSM $M_2$ from $G$ using grammartofsm($G$). (by theorem 7.1)
3. Return equalFSMs($M_1, M_2$). (by theorem 9.5)

By the three theorems, this procedure terminates and gives the right answer.

246/6(d).

$G = \{\{S,a,b\},\{a,b\},R,S\}$, where
$R = \{S \rightarrow aSb \mid aaSb \mid aSba \mid abSa \mid bSaa \mid baSa \mid SS \mid \epsilon\}$
Proof:

Suppose that $b$ is the maximum number of symbols on the right hand side of any rule. Then after $n$ steps we can have at most $b^n$ terminal symbols. But since we have a finite alphabet, the number of strings of length $b^n$ is finite.

277/1\(c\)

\[ a \rightarrow a, \ b \rightarrow b, \ a \rightarrow a, \ b \rightarrow b, \ \varepsilon \rightarrow \varepsilon \]

311/2

a.

\[ G = \{ \{ S, A, B, X, a, b \}, \{ a, b \}, R, S \}, \text{ where } R = \{
\]

\[ S \rightarrow aAa | bBb | a | b | \varepsilon,
A \rightarrow XAX | a,
B \rightarrow XBX | b,
X \rightarrow a | b\} \]

b.
c.

Assume $L$ is regular and $n$ is the constant from pumping theorem.

$w = ab^nab^n a$, so $w \in L$ and $|w| = 2n + 3 > n$.

Case 1:

$x = \varepsilon, y = ab^\beta, z = b^{n-\beta}ab^n a$, where $\beta \geq 0, \beta + 1 \leq n$.

So $xyz = w \in L$ and $xy^iz = (ab^\beta)^ib^{n-\beta}ab^n a \in L$ for $i \geq 0$.

Let $i=0$, $xy^0z = b^{n-\beta}ab^n a \notin L$ since the first and the last character is different.

Case 2:

$x = ab^\alpha, y = b^\beta, z = b^{n-\alpha-\beta}ab^n a$, where $\beta \geq 1, \beta + \alpha \leq n - 1$.

So $xyz = w \in L$ and $xy^iz = ab^\alpha b^\beta b^{n-\alpha-\beta}ab^n a \in L$ for $i \geq 0$.

Let $i=0$, $xy^0z = ab^{n-\beta}ab^n a \notin L$ since the first and the last character are both $a$ but the middle one is $b$.

Contradiction.

So $L$ is not regular.

311/6.

a.

$L_2 = \{a^n b^{2n} c^{4n} : n \geq 0\}, L_3 = \{a^{4n} b^{2n} c^n : n \geq 0\}$

$L_1 = L_2 \cap L_3 = \{\varepsilon\}$

b.

$L_2 = \Sigma^*, L_3 = \{a^n b^n c^n : n \geq 0\}$

$L_1 = L_2 \cap L_3 = \{a^n b^n c^n : n \geq 0\}$

322/1(a).

1. Build FSM $M_1$ for $L(\alpha)$.
2. Convert FSM $M_1$ to DFSM $M_2$. 

3. Build FSM $M_3$ from $M_2$ by flipping accepting and non-accepting states so that $L(M_3) = \overline{L(\alpha)}$.

4. Build PDA $M_4$ so that $L(M_4) = L(M) \cap L(M_3)$ by $\text{intersect}PDAandFSM(M, M_3)$.

5. Construct G for $M_4$ by $\text{PDAtoCFG}(M_4)$.

6. Return $\text{decideCFLempty}(G)$. 