On Uniformly Sampling Traces of a Transition System

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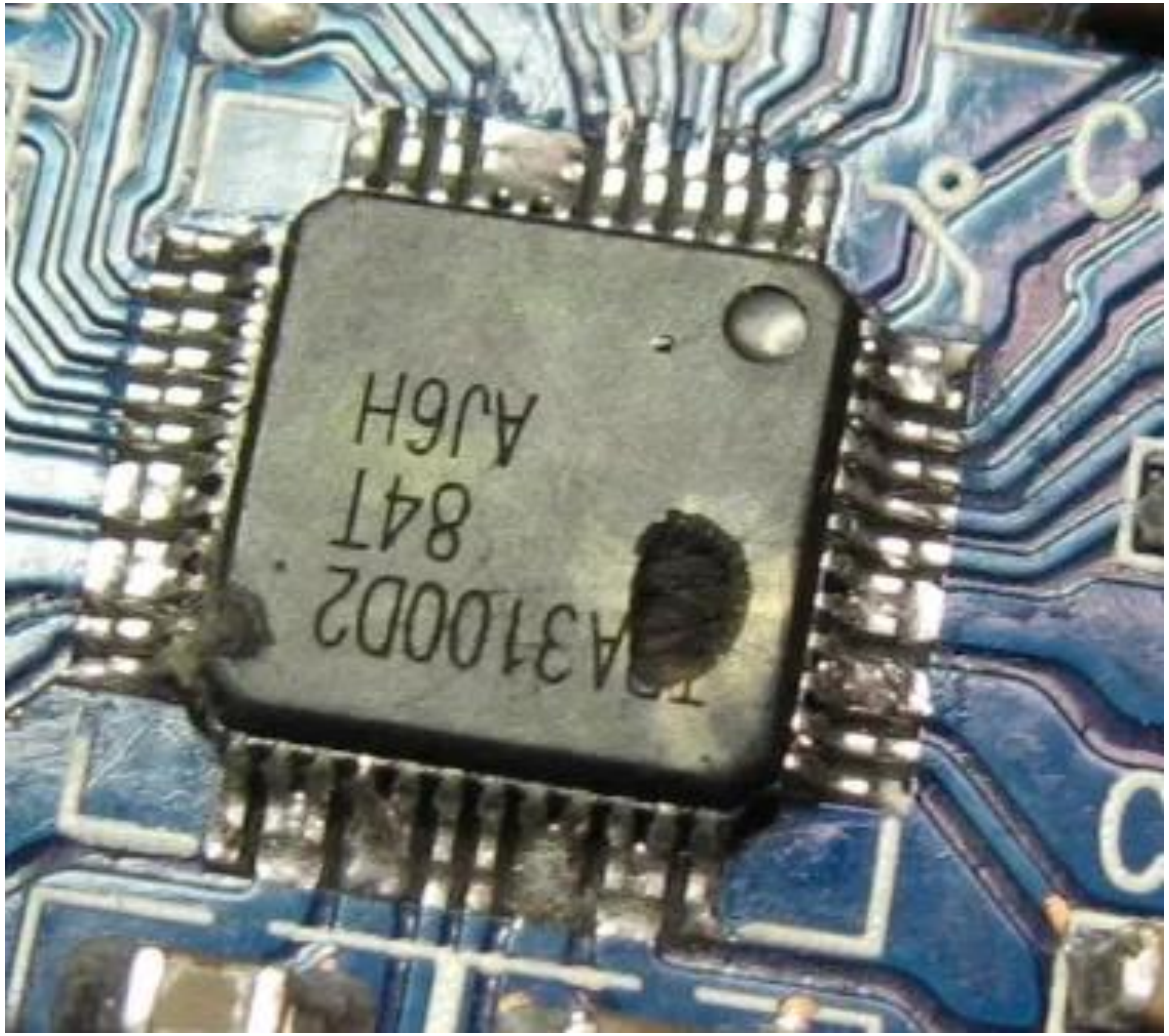
Correctness of large designs

- Enormous size and complexity of modern digital systems
  - Formal verification fails to scale
- Important to catch bugs early
  - Millions of dollars spent on faulty designs
- Constrained Random Verification balances scalability and coverage



Constrained Random Verification

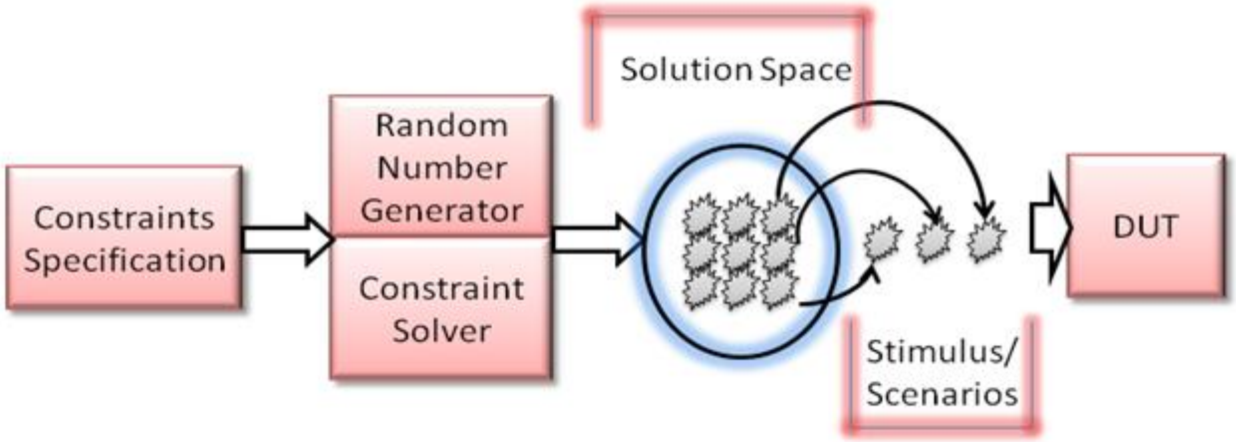


Diagram courtesy www.testbench.in

- Constraints give direction
  - User-defined constraints steer to bug-prone corners
- Randomization enables diversity
  - Inputs sampled at specific simulation steps
- Widely used in industry
  - Ex: SystemVerilog, E, OpenVera etc.

Limitations of Existing CRV Tools

- Provide ‘local’ uniformity over input stimuli
- Insufficient for ‘global’ coverage guarantees
- *Need uniformity of system’s runs or traces*

Our Contributions

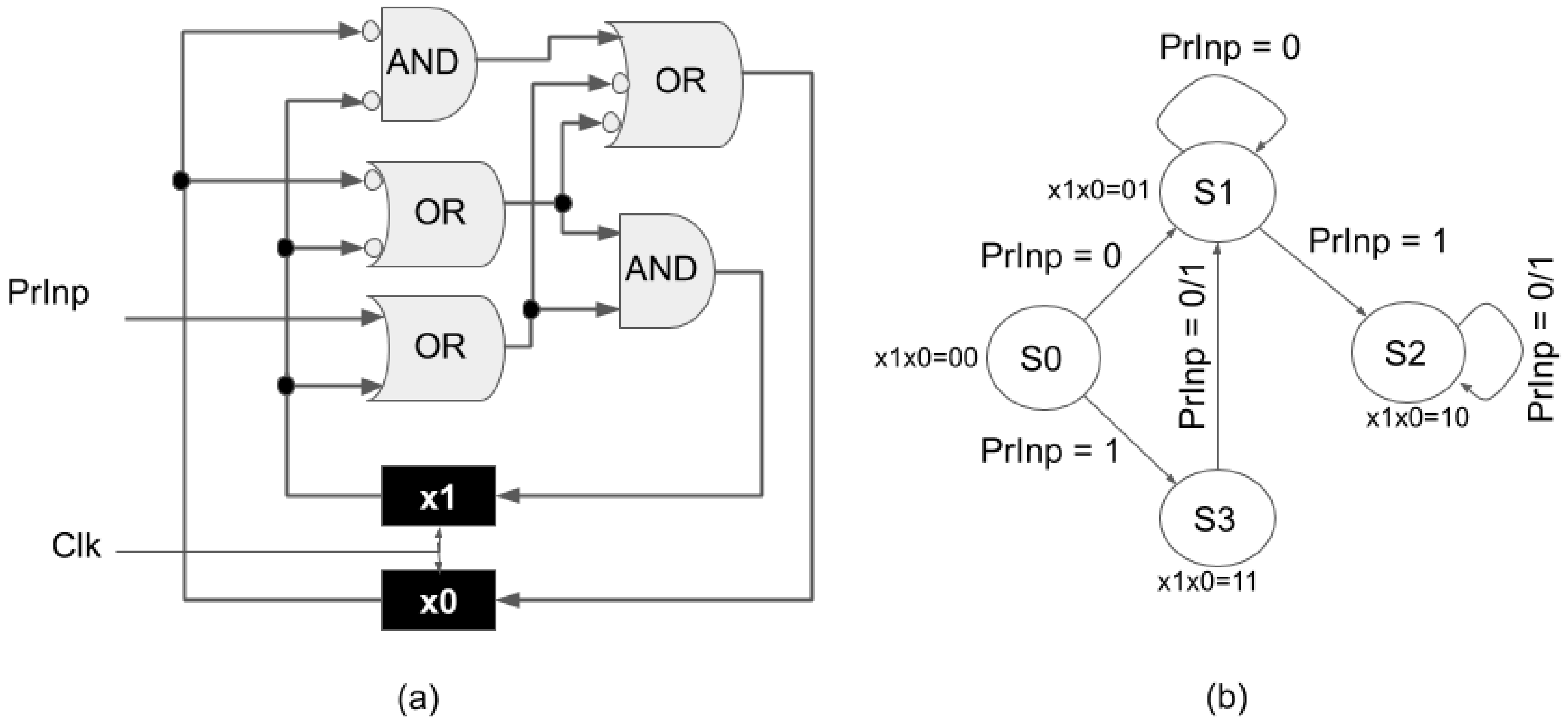
- **TraceSampler:** 1^{st} dedicated algorithm + tool for uniformly sampling traces of a transition system
  - Uses Algebraic Decision Diagrams (ADDs) & enhanced iterative-squaring
  - Easily extensible to weighted sampling

- Empirical comparison to generic samplers based on SAT/CDCL
  - TraceSampler is fastest on ~90% of benchmarks
  - Solves 200 more benchmarks than nearest competitor

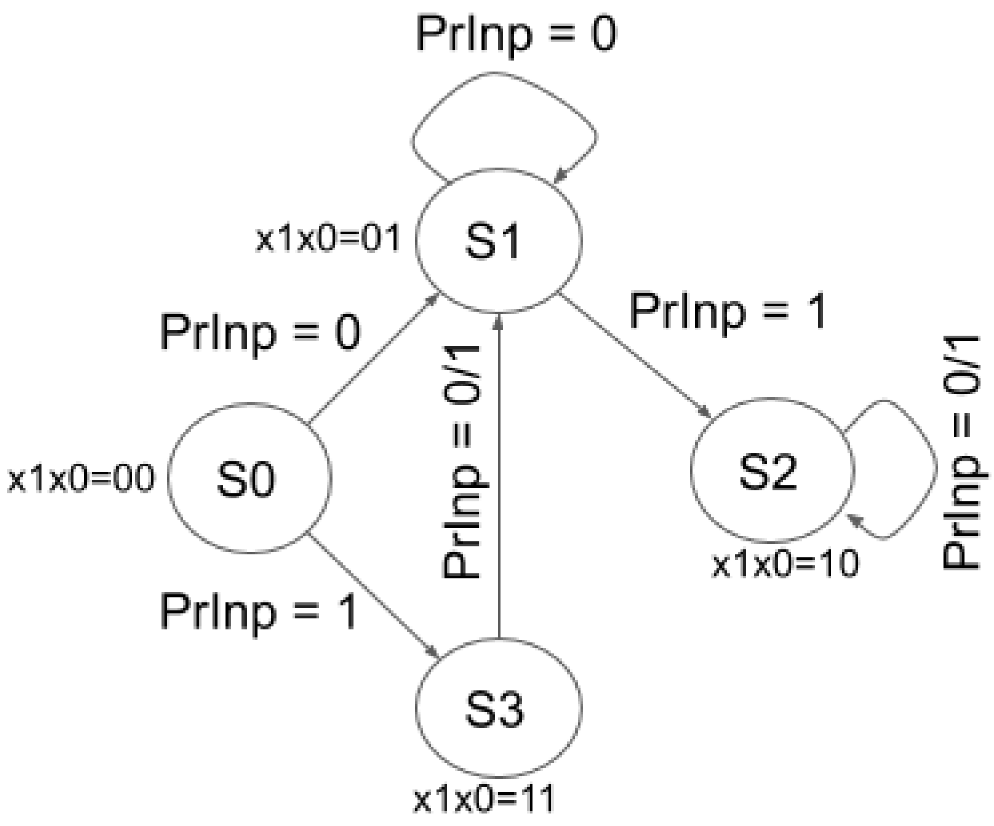
Outline

1. Example + problem definition
2. Inadequacy of Local Uniformity
3. Representing Large Transition Systems Compactly
4. TraceSampler: Two-Phase Algorithm
5. Experimental Results

Example: States, Traces and Uniformity



Example: States, Traces and Uniformity



(b)

Traces with N = 4 transitions (5 states):

1. $s_0 s_1 s_1 s_1 s_1$
2. $s_0 s_1 s_1 s_1 s_2$
3. $s_0 s_1 s_1 s_2 s_2$
4. $s_0 s_1 s_2 s_2 s_2$
5. $s_0 s_3 s_1 s_1 s_1$
6. $s_0 s_3 s_1 s_1 s_2$
7. $s_0 s_3 s_1 s_2 s_2$

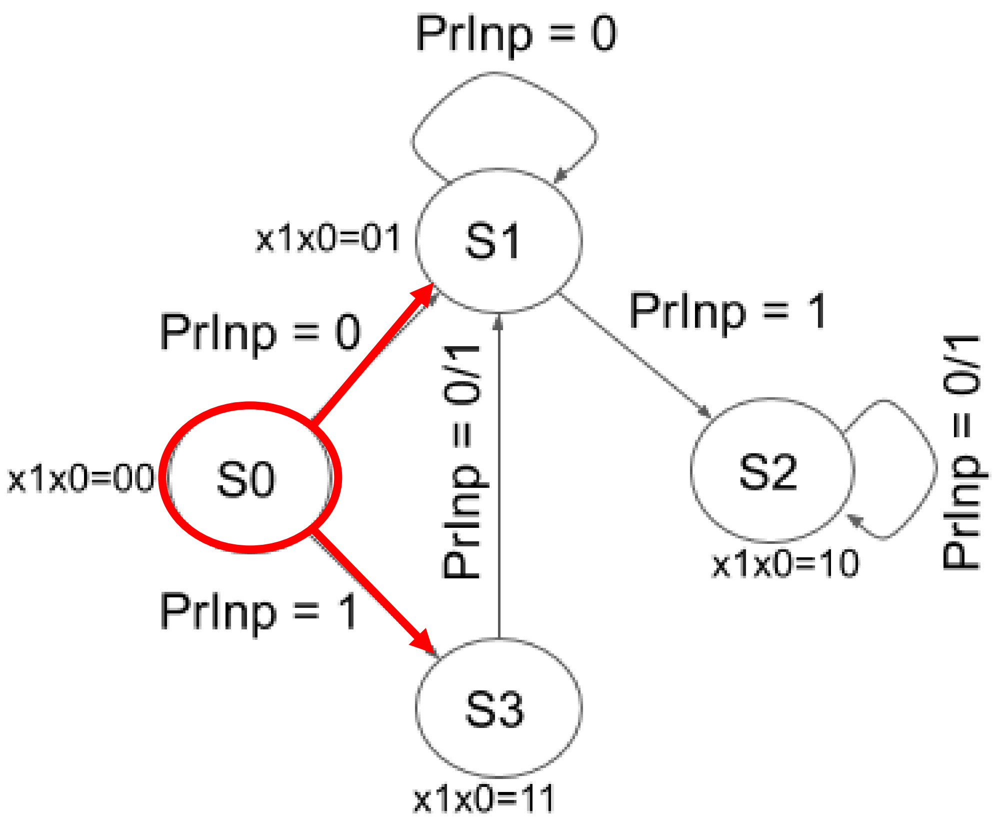
Uniformity: Sample each trace with probability 1/7

Problem Definition

- <u>Given:</u>
  - Transition System
  - Trace-length: N
  - (Optional) Initial States, Final States
- Let **T** be the set of traces of length N, which start in one of the initial states and end in one of the final states
- <u>Goal:</u>
  - Design algorithm that returns a trace T^*, such that

$$\forall T \in \boldsymbol{T} \ \ \Pr[T^* = T] = \frac{1}{|\boldsymbol{T}|}$$

Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

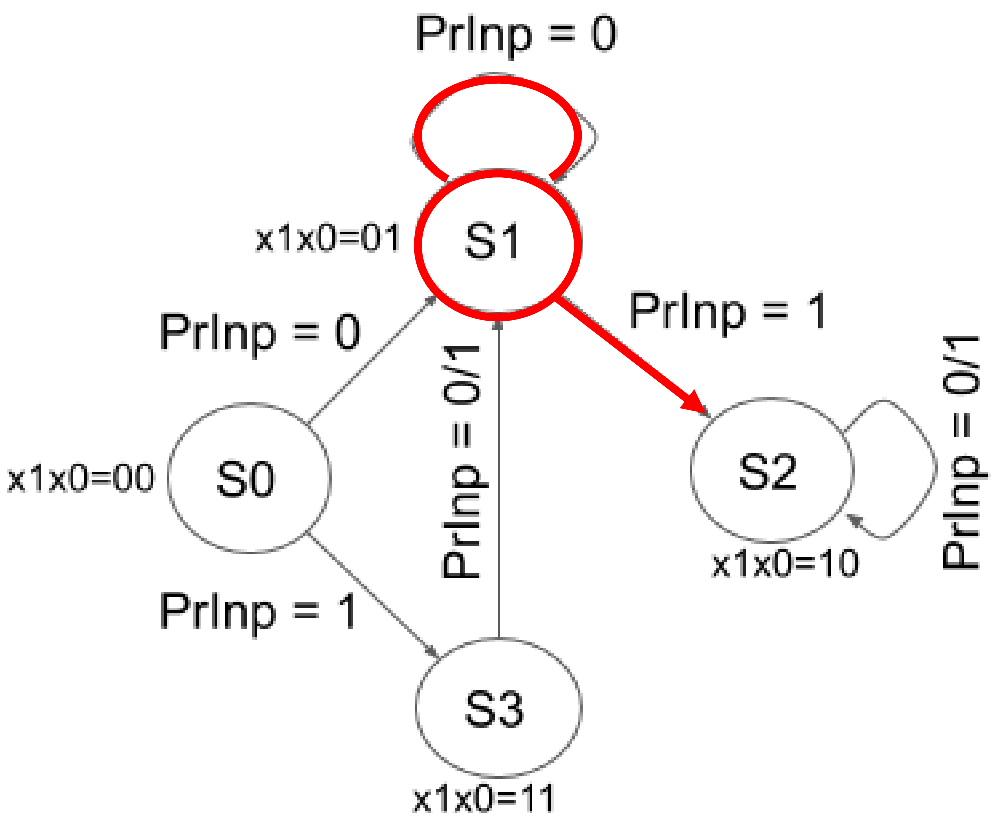
Trace: S_0

Probability: 1

Next State Probabilities:

S_3	0.5
S_1	0.5

Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

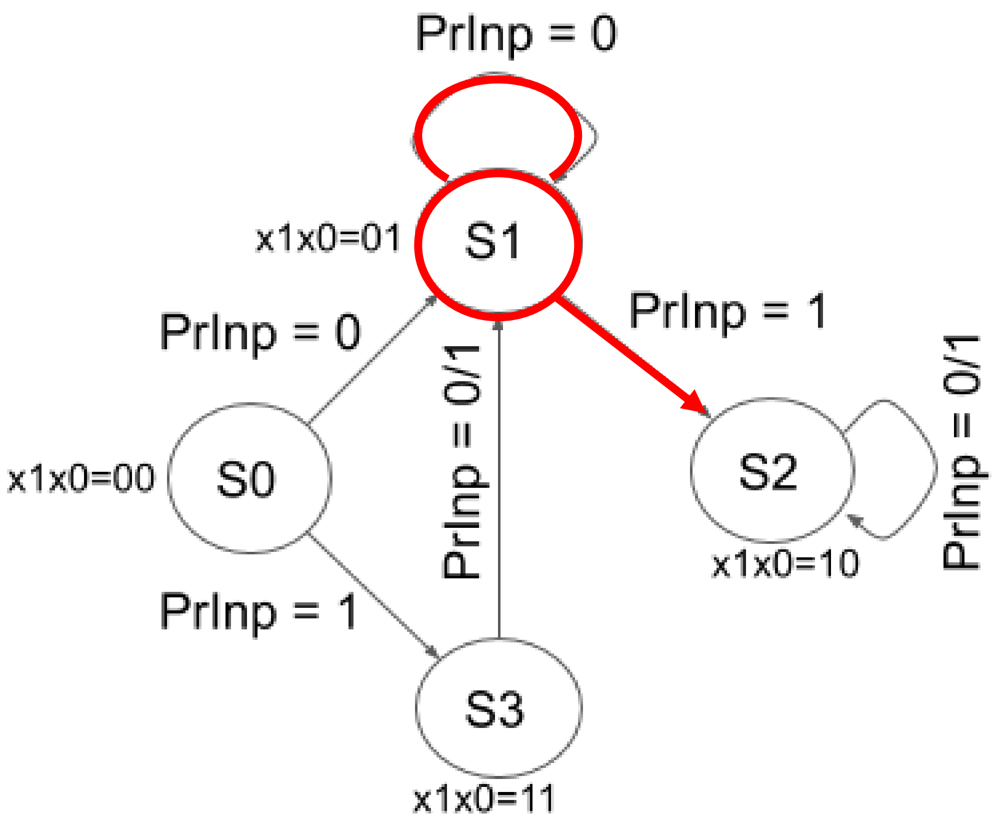
Trace: S_0 S_1

Probability: 1*0.5

Next State Probabilities:

S_2	**0.5**
S_1	**0.5**

Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

Trace: S_0 S_1 S_1

Probability: 1*0.5*0.5

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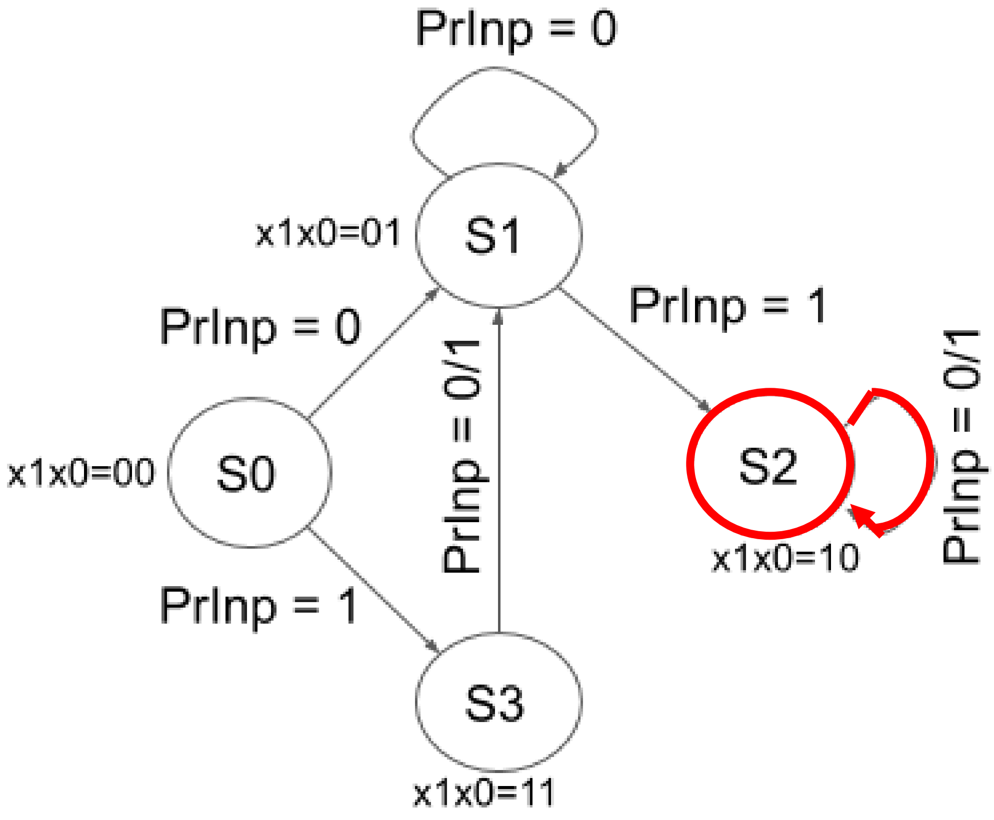
Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

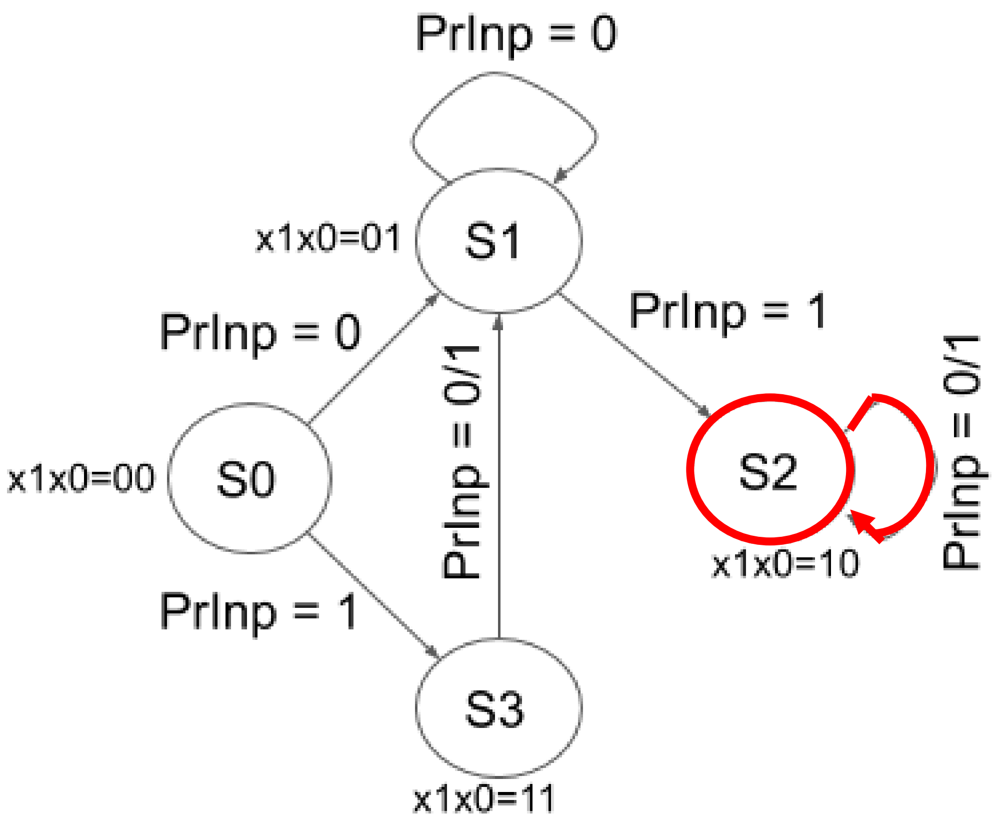
Trace: S_0 S_1 S_1 S_2

Probability: 1*0.5*0.5*0.5

Next State Probabilities:

S_2	1

Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

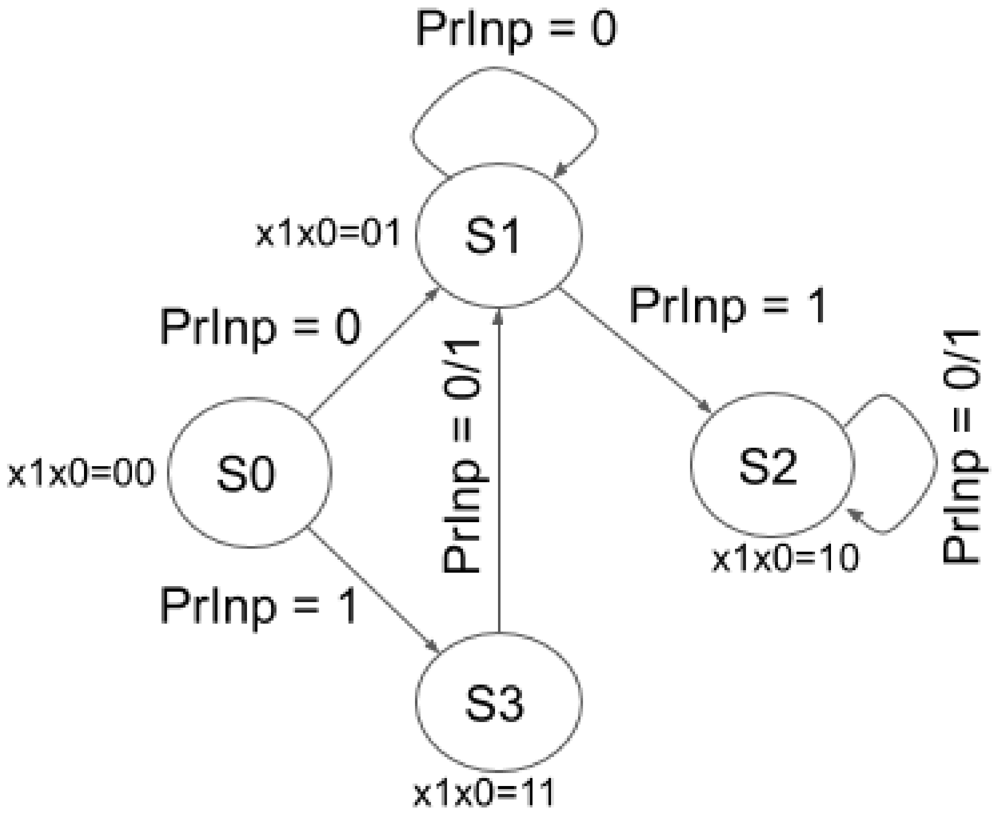
Trace: S_0 S_1 S_1 S_2 S_2

Probability: 1*0.5*0.5*0.5*1 = 0.125

Next State Probabilities:

S_2	**1**

Example: Insufficiency of Local Uniformity



(b)

Current State: S_0

Trace: $S_0\ S_1\ S_1\ S_2\ S_2$

Probability: 1*0.5*0.5*0.5*1 = 0.125

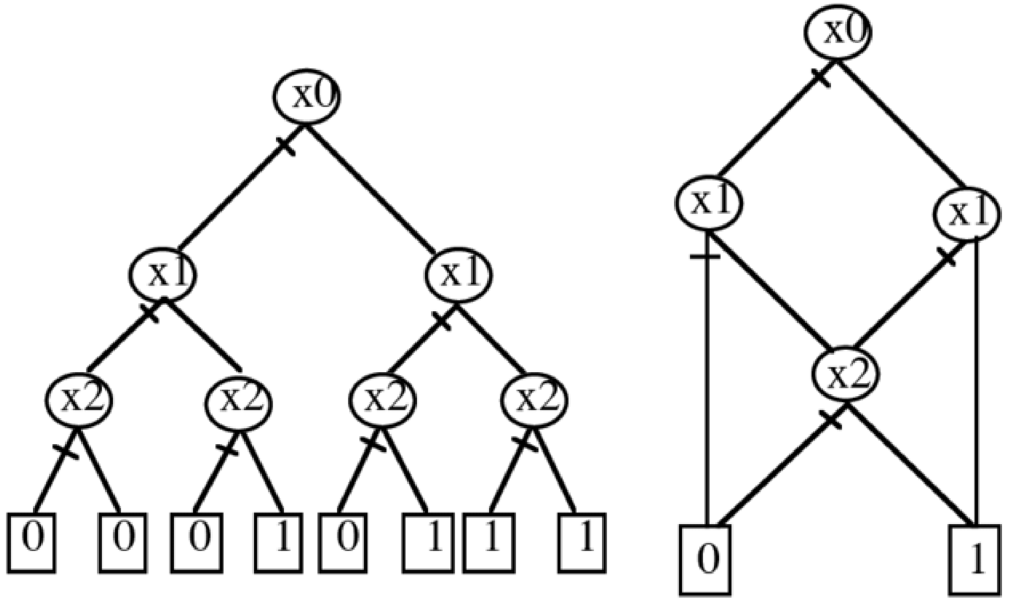
Fact: Pr = 1/7 not possible for **any** assignment of local probabilities

Representing the Transition Function

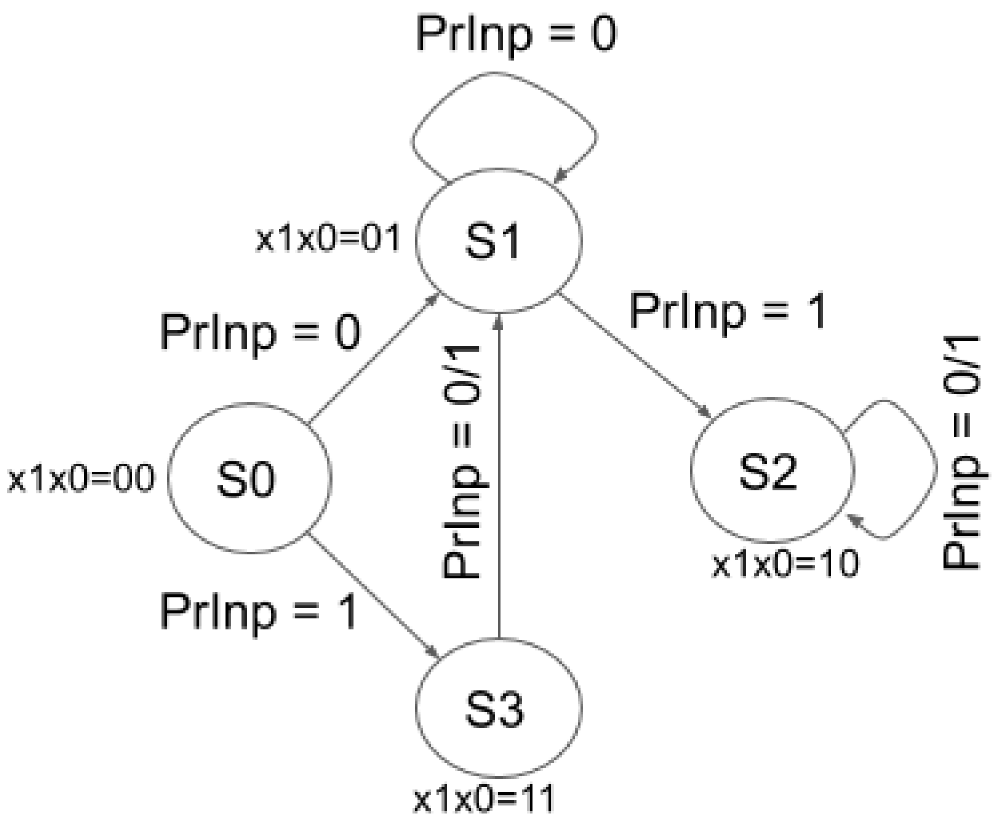
- Transition graph typically very large
  - K latches ➔ 2^k states
  - Cannot represent explicitly
- Binary Decision Diagrams (BDDs) can offer significant compression

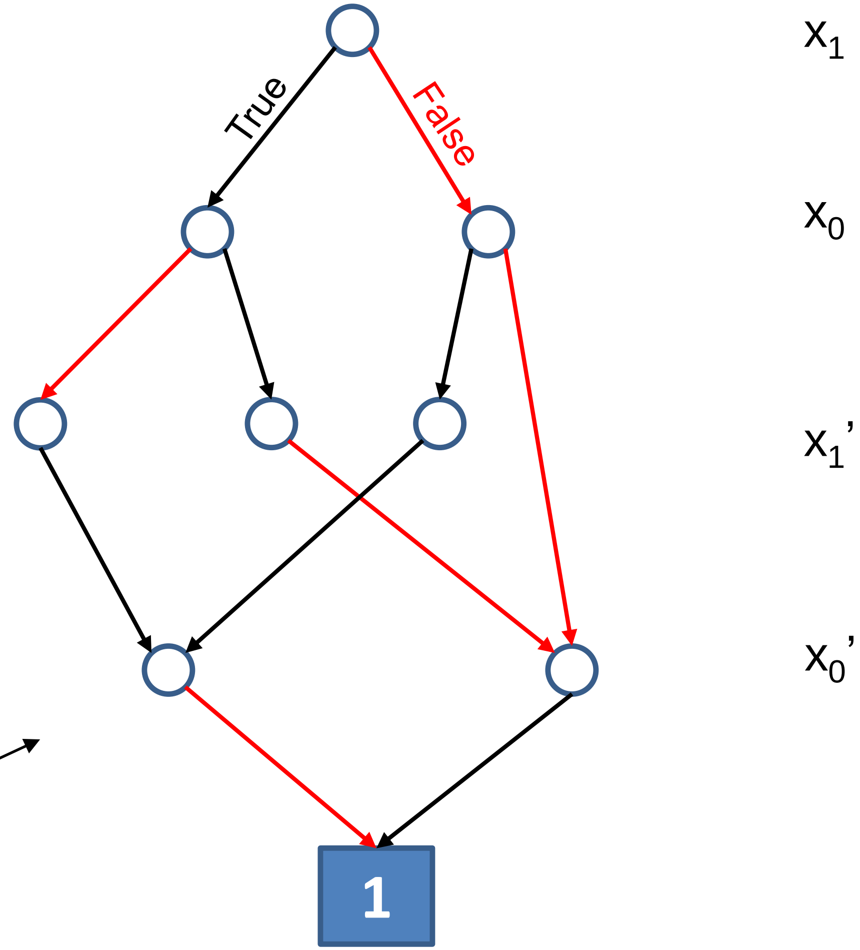
•Represent functions $f: \{0,1\}^n \rightarrow \{0,1\}$

•DAGs with node sharing + fixed variable order



BDD Example

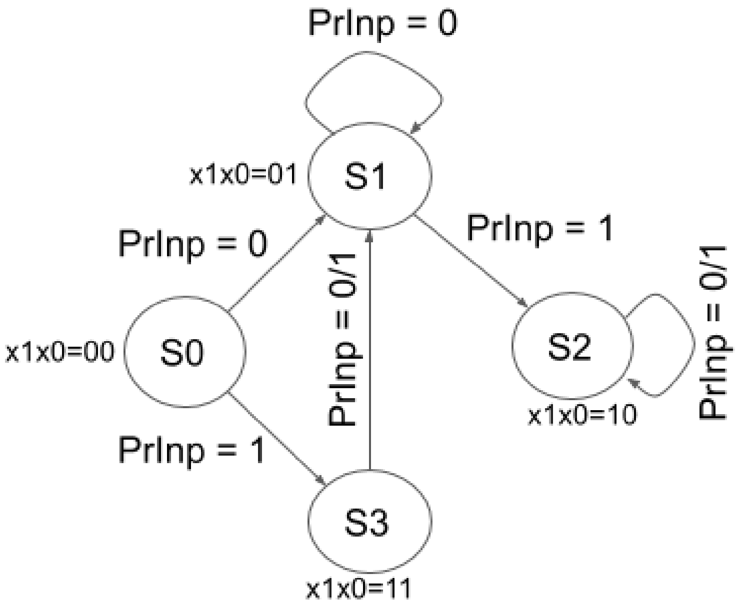




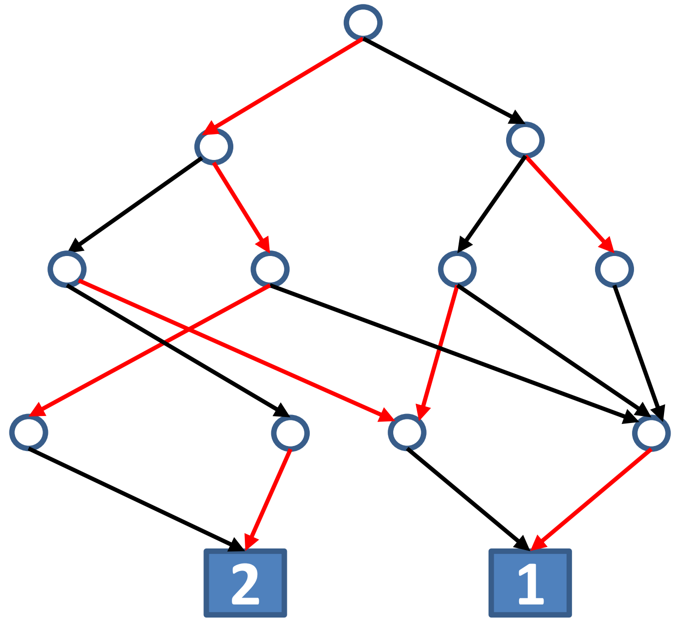
Represents 1-Step Transition Function

Algebraic Decision Diagrams

- Generalize BDDs to real-valued Boolean functions $f: \{0,1\}^n \rightarrow R$
  - DAGs with fixed variable order and node-sharing
  - Operations: Sum, Product, Additive Quantification (∑), ITE



Original Transition Graph



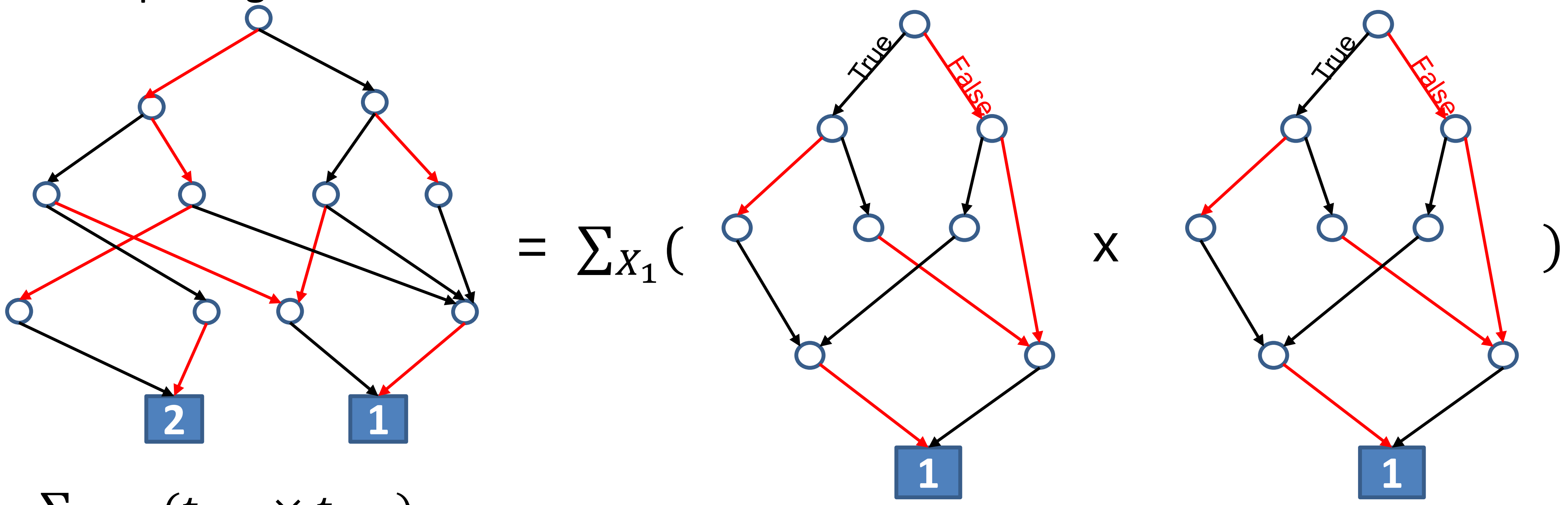
2-Step Transition Relation

TraceSampler: Two-Phase Algorithm

- Compilation Phase:
  - Construct $\log N$ ADDs: $t_1, t_2, t_4, t_8, \ldots, t_N$ by iterative-squaring
    - Aggressively prune ADDs to avoid blowup
- Sampling Phase: Divide & Conquer
  - Recursively split trace while ensuring global uniformity
  - Base case: random walk on ADD from root to leaf

TraceSampler: ADD Compilation Phase

- Iterative-Squaring:



- $t_N = \sum_{X_{N/2}} (t_{N/2} \times t_{N/2})$
- Secret Sauce: Aggressive pruning of ADDs by *novel i-step reachability algorithm*
- Advantages:
  - Only log(N) ADDs necessary: t_1, t_2, t_4, t_8, … , t_N
  - Factored forms offer significant speedup & compression [Dudek et al.’20]

TraceSampler: Sampling Phase

- Recursive Step
  - Sample state at half-way point then sample two halves independently

Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
State											

TraceSampler: Sampling Phase

- Recursive Step
  - Sample state at half-way point then sample two halves independently

Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
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log Nth ADD: t_N

TraceSampler: Sampling Phase

- Recursive Step
  - Sample state at half-way point then sample two halves independently

Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
State	S_0						S_{10}				S_5

log N^{th} ADD: t_N

TraceSampler: Sampling Phase

- Recursive Step
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Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
State	S_0						S_{10}				S_5

log N -1 ADD: $t_{N/2}$

TraceSampler: Sampling Phase

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  - Sample state at half-way point then sample two halves independently

Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
State	S_0				S_{11}		S_{10}		S_8		S_5

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TraceSampler: Sampling Phase

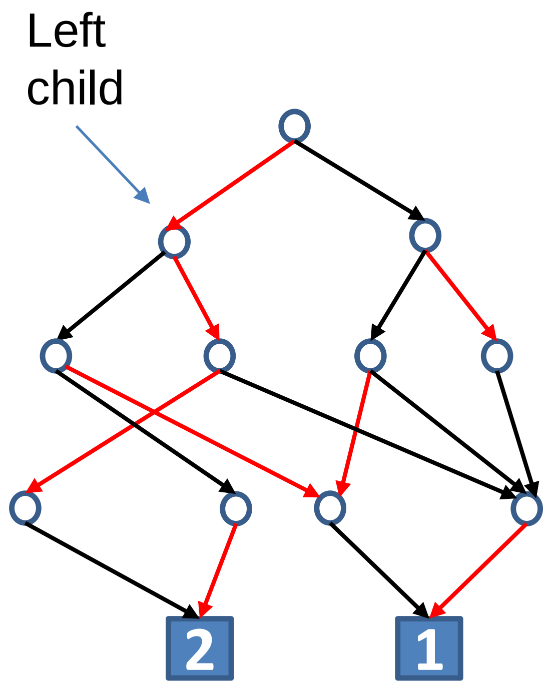
- Recursive Step
  - Sample state at half-way point then sample two halves independently

Trace Position	0	1	2	…	N/4	…	N/2	…	3N/4	…	N
State	S_0				S_{11}		S_{10}		S_8		S_5

log N -2 ADD: $t_{N/4}$

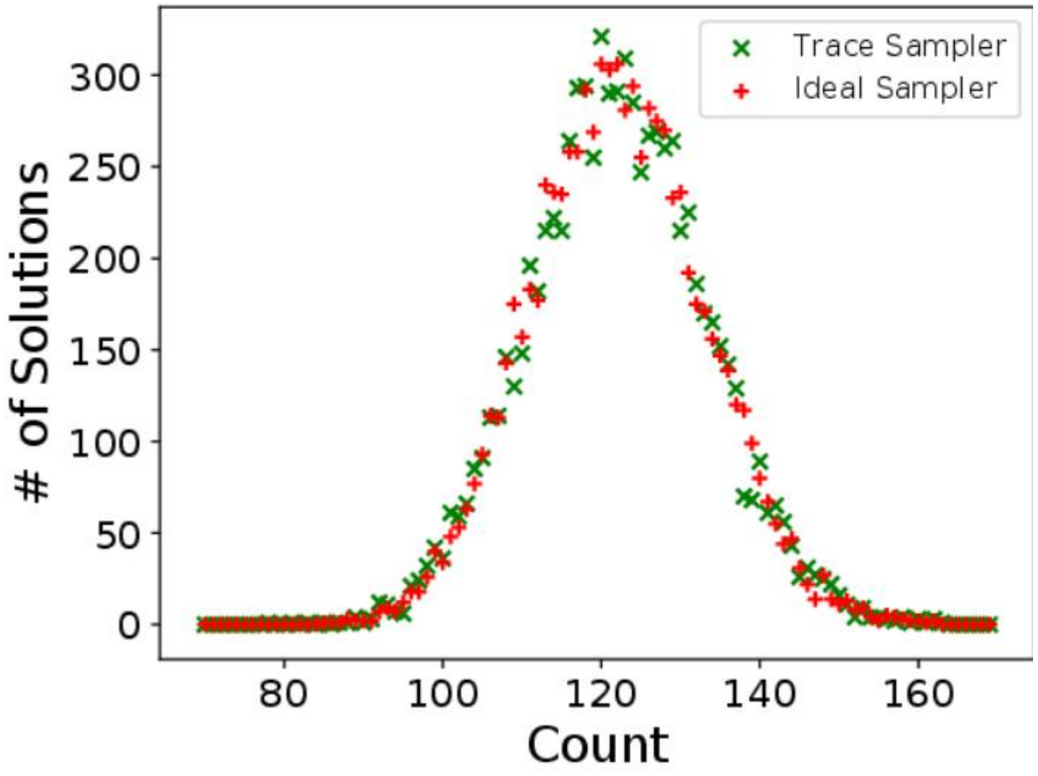
TraceSampler: Sampling Phase

- Base case: sample states from ADD
  - Weighted random walk on ADD
  - Root to leaf traversal
    - Pick child C* with probability $\Pr[C^*] = \frac{wt(C^*)}{\sum_i wt(C_i)}$
    - $wt(C^*)$ = $\sum_{leaves} (num\ paths\ from\ C^*\ to\ leaf) \times val(leaf)$
    - Eg: $wt(left\ child) = 2 \times 2 + 2 \times 1 = 6$



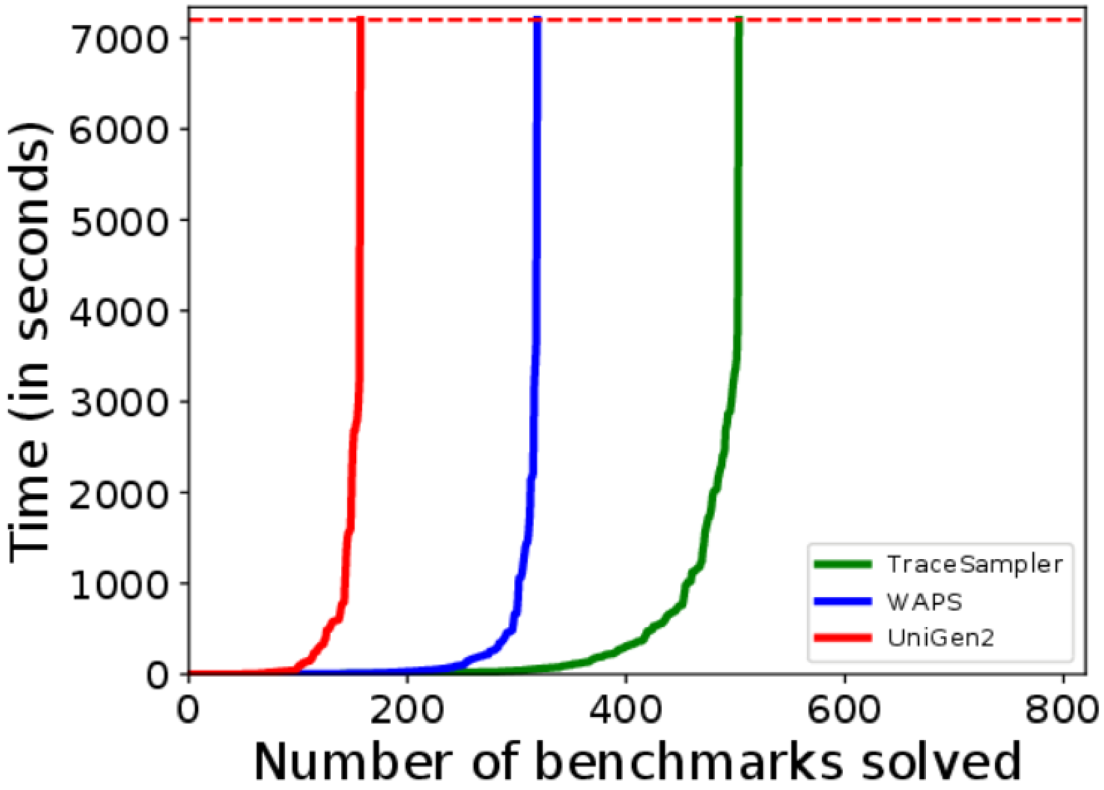
Empirical Evaluation: Uniformity

- Sampled 10^6 traces from small benchmark
  - Using TraceSampler
  - Using Ideal Sampler (WAPS [Gupta et al.])
- **X-axis**
  - Count of how many times a particular trace was sampled
- **Y-axis**
  - Number of traces with specific count
- Distributions are indistinguishable
  - Jensen-Shannon distance: 0.003



Empirical Evaluation: Scalability

- **Benchmarks**: HWMCC'17, ISCAS89
- **Trace Lengths**: 2,4,8,16,…256
- **Comparison**: Encode circuits as CNF and unroll
  - WAPS: Exact uniform sampler [Gupta et al. '19]
  - Unigen2: Approximately uniform sampler
    - [Chakraborty et al. '15]
- **Results**:
  - **TraceSampler** solves 200+ more instances
  - Fastest on ~90% instances
  - Avg. Speedup: 3x to WAPS, 25x to Unigen2
  - Compilation Speedup: 16x to WAPS



Summary and Takeaways

- **TraceSampler**: Novel ADD based algorithm for uniform / weighted sampling of traces
  - Significantly outperforms competing SAT/CDCL-based approaches
  - First prototype; more engineering effort ➔ more scalability
  - Scope for heuristics and time-space tradeoffs

- Use synergistically with traditional CRV solutions?
  - Use CRV to reach bug-prone corner
  - Invoke TraceSampler for strong coverage guarantees

References

- [Dudek et al., ‘20] Jeffrey M Dudek, Vu HN Phan, and Moshe Y Vardi. AAAI 2020. ADDMC: Exact weighted model counting with algebraic decision diagrams

- [Gupta et al., 19] Rahul Gupta, Shubham Sharma, Subhajit Roy, and Kuldeep S Meel. 2019. Waps: Weighted and projected sampling. In International Conference on Tools andAlgorithms for the Construction and Analysis of Systems. Springer, 59–76

- [Chakraborty et al., ‘15] Supratik Chakraborty, Daniel J Fremont, Kuldeep S Meel, Sanjit A Seshia, and Moshe Y Vardi. 2015. On parallel scalable uniform SAT witness generation. In International Conference on Tools and Algorithms for the Construction and Analysis of Systems. Springer, 304–319.