

# Comp 480/580: Assignment #4

Rice University — Due Date: Thursday, 04/19/2019

## 1 Markov Chains: Simple Proofs

Use the materials posted on piazza for reference. The following are also the exercise 2.6, 2.7, 2.8 and 2.9 from the lecture notes

1. For a markov chain, prove that if the graph associated is undirected (i.e.  $P(x,y) = P(y,x)$ ), then aperiodicity is equivalent to graph being non-bipartite.
2. Define the period of  $x$  as  $\gcd \{t : P^t(x,x) > 0\}$ . Prove that for an irreducible Markov chain, the period of every  $x \in \Omega$  is the same. [Hence, if the graph associated is undirected, the period is either 1 or 2.]
3. Suppose a markov chain with stochastic matrix  $P$  is irreducible. Then it is aperiodic if and only if there exists  $t$  such that  $P(x,y)^t > 0$  for all  $x, y \in \Omega$ .
4. Suppose a markov chain with stochastic matrix  $P$  is irreducible and contains at least one self-loop (i.e.,  $P(x,x) > 0$  for some  $x$ ). Then  $P$  is aperiodic.