- Rejection Sampling.

Goal: Sample from $f(x)$

Given: Ability to sample from $g(x)$ s. t. $f(x) \leq M \cdot g(x) \quad \forall x.$ $(M \geq 1)$

<u>Algo</u>: → Sample $X \sim g(x)$

Sample $U \sim U[0,1]$

Accept X if $U \leq \dfrac{f(x)}{M \cdot g(x)}$

$= f(x)$

$P(X \mid \text{Accepted})$

$$= \frac{Pr(X, \text{Accepted})}{P(\text{Accepted})}$$

$g(x)$

$$= \frac{\underline{Pr(\text{Accepted} \mid X)}\ Pr(X)}{Pr(\text{Accepted})}$$

$$Pr\left(U < \frac{f(x)}{M \cdot g(x)} \,\middle|\, x\right)$$

$$\frac{f(x)}{M \cdot g(x)}$$

$$\boxed{Pr(A, B) = Pr(A \mid B)\, Pr(B)}$$

$$= \frac{f(x)}{M\ \boxed{Pr(\text{Accepted})}} \quad \boxed{Pr(\text{Accepted})} = 1/M$$

$= f(x)$

$$\Pr(\text{Accepted}) = \sum_x \Pr(\text{Accepted} \mid X = x)\, \Pr(X = x)$$

$$\Pr\left(U < \frac{f(x)}{M \cdot g(x)} \,\middle|\, X = x\right) \quad g(x)$$

$$\frac{f(x)}{M \cdot g(x)}$$

$$= \sum_x \frac{f(x)}{M} = 1/M \;?$$

1) Inverse Transformation.

Goal: Sample from $f(x)$ (CDF $F(x)$)

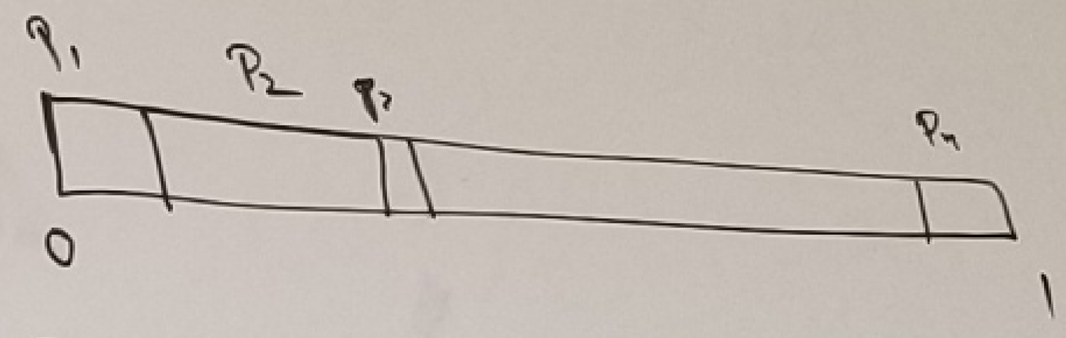
Algo: Sample $U \sim U[0,1]$

return $\underline{\underline{F}}^{-1}(U)$

Ability to compute F^{-1}

$x_1, x_2 \ldots x_n$

$p_1, p_2 \ldots p_n = P(x)$



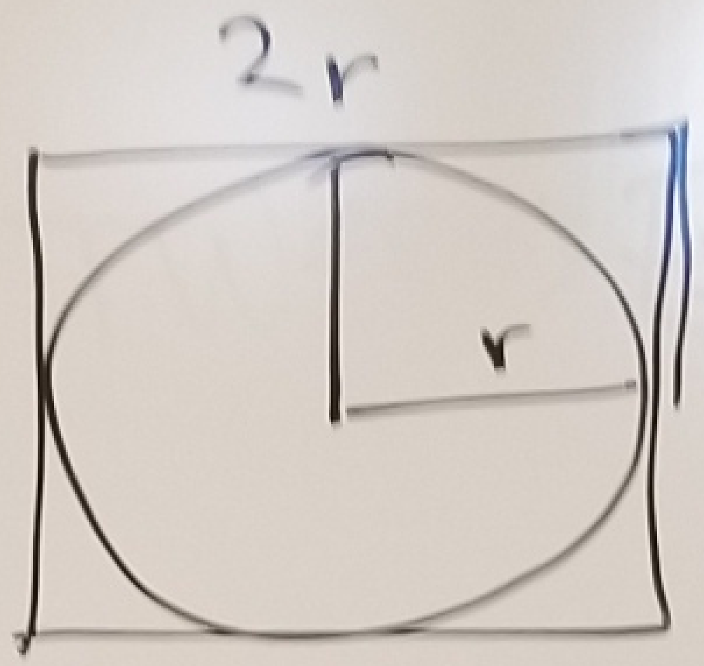
Sample $U \sim U[0,1]$

return x_j if $\sum_{i=1}^{j-1} p_i < U < \sum_{i=1}^{j} p_i$

- Sampling Techniques
  - Transformation Based
    - Inversion Method
    -
    -
    -
  - Rejection Based

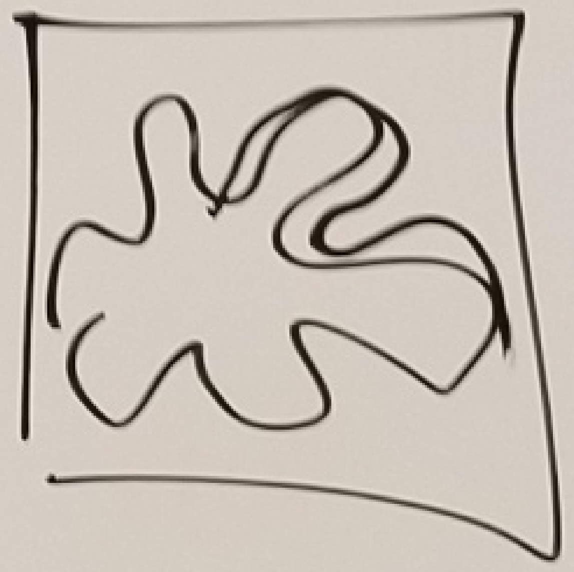
- Importance Sampling *

- Usually beneficial in High Dimentions.



$$\frac{\pi r^2}{4 r^2} = \frac{\pi}{4}$$

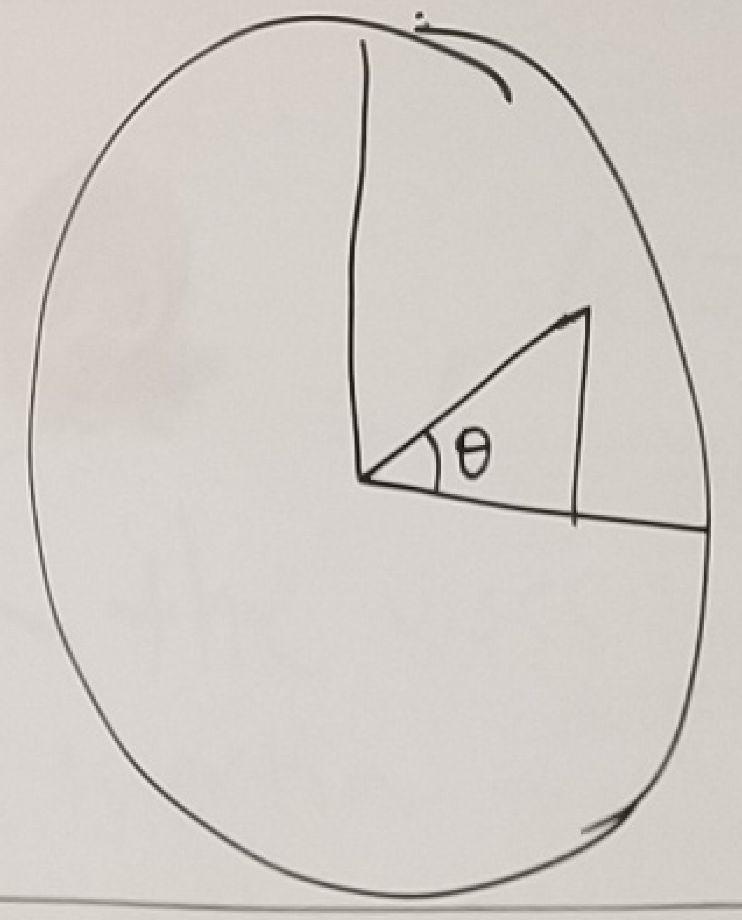
$$\frac{1}{A}$$



Box-Muller Method/Transform

$$X_1 = R\cos\Theta$$
$$X_2 = R\sin\Theta$$

$$\boxed{X_1, X_2 \overset{\text{i.i.d}}{\sim} N(0,1) \iff \begin{array}{l} \Theta \sim U[0, 2\pi] \\ R^2 \sim \text{Expo}(1/2) \end{array}}$$



Algo: Generate $Y_1, Y_2 \overset{\text{i.i.d}}{\sim} U[0,1]$

$$R = \sqrt{-2\log(Y_1)}\,, \quad \Theta = 2\pi \cdot Y_2$$

$$X_1 = R\cos\Theta \qquad X_2 = R\sin\Theta$$

Composition.

$$F(x) = \sum_{i=1}^{\infty} p_i \, F_i(x)$$

$$f(x) = \sum_{i=1}^{N} p_i \, f_i(x)$$

$$\boxed{\begin{gathered} 0 \le p_i \le 1 \\ \sum p_i = 1 \end{gathered}}$$

$$\boxed{f(x) = \sum_{j=1}^{n} \alpha_i \lambda_i e^{-\lambda_i x}}$$

$x_1, x_2 \ldots \quad x_N$

$f_j \quad P_1^j, P_2^j, \ldots \quad P_N^j$

Given.

Sample (f_i) $\forall i$

→ Algorithm.

1) Generate I such that $Pr(I = i) = p_i \quad \forall i$

2) Generate a sample from F_I call it Y

$$Pr(Y = k) = \sum_{i=1}^{N} \underbrace{\boxed{Pr(I=i)}}_{p_i} \times F_i(Y) = F(x)$$