

## Networks

- Regulatory network: network of control decisions used to turn genes on/off.
- Signaling network: interactions among genes, gene products and small molecules that activate cellular processes.
- Metabolic network: network of proteins that synthesize and breakdown cellular molecules.


From flow cytometry data to signaling networks




## Challenges

- The cell is a complex stochastic domain: signal transduction, metabolic and regulatory pathways all interconnected.
- Pathways are controlled by combination of many mechanisms.
- We only observe mRNA levels and/or phospho-lipid levels.
- Many interactions are not directly observed at the mRNA level
- Measurements are noisy.



## Some initial approaches

- Boolean networks
- Deterministic models of interactions between genes.
- Disadvantage: deterministic. We need stochastic models for representing interactions.


## Why probabilistic models?

Gene regulation occurs at many stages:

- pre-transcriptional (chromatin structure)
- transcription initiation
- RNA editing (splicing) and transport
- Translation initiation
- Post-translation modification
- RNA \& Protein degradation

All these processes are stochastic!
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Bayesian networks: the model

- A Bayesian network $B=(V, E)$ is a directed acyclic graph in which each node in $V$ is annotated with quantitative probability information.
- A set $V$ of random variables are the nodes of the network. They can be continuous or discrete.
- If there is an edge from node $X$ to node $Y$ in $E$, then $X$ is said to be the parent of $Y$.
- Each node $X$ in $V$ has a conditional probability distribution $P(X \mid$ Parents $(X)$ ) associated with it.
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Adapted from
Sachs, 2005
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## Computing joint probability

 distributions- Any entry in the joint probability distribution can be calculated from the Bayesian network.

$$
\begin{aligned}
P(J, M, A, \neg B, \neg E) & =P(J \mid M, A, \neg B, \neg E) P(M, A, \neg B, \neg E) \\
& =P(J \mid A) P(M \mid A, \neg B, \neg E) P(A, \neg B, \neg E) \\
& =P(J \mid A) P(M \mid A) P(A \mid \neg B, \neg E) P(\neg B, \neg E) \\
& =P(J \mid A) P(M \mid A) P(A \mid \neg B, \neg E) P(\neg B) P(\neg E)
\end{aligned}
$$

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## Computing joint probabilities

$P\left(X_{1}=x_{1}, \ldots, X_{n}=x_{n}\right)=\prod_{i=1}^{n} P\left(X_{i}=x_{i} \mid \operatorname{Parents}\left(X_{i}\right)\right)$
$P($ Burglary $\mid$ Alarm $)=0.376$
$P($ Burglary $\mid$ Alarm,Earthquake $)=0.003$


## Conditional probability distributions

- Multinomial model
- Discrete values
- Linear Gaussian model
- $P\left(X \mid u_{1}, u_{2}, \ldots, u_{k}\right)=N\left(a_{0}+\sum, a_{1} u_{i}, \sigma^{2}\right)$


## Advantages of Bayesian networks

- Flexible representation of (in)dependency structure of multivariate distributions and interactions.
- Natural for modeling global processes with local interactions.
- Clear probabilistic semantics.
- Natural for statistical confidence analysis of results and answering of queries.
- Stochastic in nature: models stochastic processes \& deals well with noise in measurements.
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## Need for discretization

- The expression measurements are real numbers.
-We need to discretize them in order to learn general conditional probability distributions. This step entails a loss of information.
- If we don't discretize, we must assume some specific type of conditional probability distribution (like "linear Gaussian"), and this assumption causes loss of modeling fidelity.
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## Learning Bayesian Models

- Using gene expression data $D$, find the Bayesian network $G$ that is most likely given the data, i.e. $G$ that maximizes $P(G \mid D)$.
- Two cases
- Graph structure is known; the conditional probability distributions are unknown.
- Recovering optimal conditional probability distributions when the graph is known is "easy".
- Graph structure and the conditional probability distributions are unknown.
- Recovering optimal graph structure is NP-hard.



| Challenges |
| :--- |
| Ab initio learning of cellular process is |
| difficult - data is extremely limited (few |
| hundred samples). |
| - Data is noisy; measurement and |
| interpretation problems, as well as |
| problems caused by tissue heterogeneity. |
| - Therefore, we need to incorporate |
| available knowledge of biological processes; |
| the role of expression data is to refine |
| known models. |
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## Robustness of EM learning

Leave-one-out Cross validation results for the GSH network

|  | GSH |  |
| :---: | :---: | :---: |
|  | Network |  |
|  | Actual |  |
| Predicted | N | T |
| N | 41 | 8 |
| T | 9 | 44 |

Predictions from GSH network


We can make predictions about metabolite levels from the two learned networks. It is remarkable that we can predict that the level of oxidative stress in tumor cells is much higher in
tumor cells using networks learned from the gene expression data alone! (c) Devika Subramanian, 2006

Bayesian network learning

- Computationally intensive.
- Require lots of data.
- Dynamical Bayesian networks can represent feedback loops and deal with temporal data.
- Dynamical Bayesian networks are generalizations of Hidden Markov Models!


## The Bayesian approach



$$
P(G \mid D) \propto P(D \mid G) P(G)
$$

Prior over Networks

Key idea: Use $P(G / D)$ to evaluate a network given a particular microarray data set.

Learning network structure

- Find the network structure that has maximum likelihood with respect to the data
- Find $G$ that maximizes $P(G \mid D)$.


## Learning network structure

- The structure ( $G$ ) learning problem is NPhard => heuristic search for best model must be applied, generally bring out a locally optimal network.
- It turns out, that richer structures give higher likelihood $P(D \mid G)$ to the data (adding an edge to the graph is always preferable).


## Learning structure



- If we add $B$ to Parents $(C)$, we have more parameters to fit $\rightarrow$ more freedom $\rightarrow$
- But we prefer simpler (more explanatory) networks (Occam's razor!)
- Therefore, practical scores of Bayesian Networks compensate for the likelihood improvement by imposing a penalty on complex networks.


## Local search

We change one edge and evaluate the gains made by this change


## Search algorithm recipe

- Start with a random graph G. Evaluate its likelihood wrt D, P(G|D).
- Until little improvement in likelihood
- Perturb structure $G$ by adding, deleting or reversing edge
- Accept change if likelihood improves.
- End

Randomized restarts


Look for features common to many models
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## Difficulty \#1

- We do not have enough data to uniquely identify a high-scoring network.
- Exponentially many networks with the same P(G|data) score!
- Solution: generate many high-scoring network and extract common features.


## Difficulty \#2

- What space of graph perturbations to consider?
- Solution: sparse candidate algorithm (Friedman 1999)
- Limit potential parents to k most correlated variables.


## Experiment

Data from Spellman et al. (Mol.Bio. of the Cell 1998).

- Contains 76 samples of all the yeast genome:
- Different methods for synchronizing cell-cycle in yeast.
- Time series at few minutes ( $5-20 \mathrm{~min}$ ) intervals.
- Spellman et al. identified 800 cellcycle regulated genes.


The sparse data problem:

## summary

- There are many more genes than experiments Therefore, many different networks suit the data well.
- Shrink the network search space. E.g., in biological systems each gene is regulated directly by only a few regulators.
- Don't believe the learned networks, but use them to find reliable links between genes. (i.e., edges that are present in all learned networks).


## Representing partial models

- Analyze the set of plausible networks and attempt to characterize features that are common to most of these networks.
- Features
- Markov relations: Is $Y$ in the Markov blanket of $X$ ?
- Order relations: Is $X$ an ancestor of $Y$ in all the networks of a given equivalence class?
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## Features contd.

- Question: Given that $X$ and $Y$ are indirectly dependent, who mediates this dependence?
- Separator relation:
- $X$ affects $Z$ who in turn affects $Y$
- $Z$ regulates both $X$ and $Y$





## Estimating statistical confidence in features

- To what extent does the data support a given feature?
- An effective and relatively simple approach for estimating confidence is the bootstrap method.


## The bootstrap method

- For $i=1, \ldots, m$
- Re-sample with replacement $N$ instances from $D$. Denote by $D_{i}$ the resulting dataset.
- Apply the learning procedure on $D_{i}$ to induce a network structure $G$.
- For each feature $f$ of interest calculate

$$
\operatorname{conf}(f)=1 / m \sum_{i=1}^{m} f\left(G_{i}\right)
$$

- where $f(G)$ is 1 if $f$ is a feature in $G$, and 0 otherwise.


## Improving statistical significance

## Sparse Data

- Small number of samples
- "Flat posterior" -- many networks fit the data.
Solution
- estimate confidence in network features
- E.g., two types of features
- Markov neighbors: $X$ directly interacts with $Y$ (have mutual edge or a mutual child)
- Order relations: $X$ is an ancestor of $Y$




## Assembling subnetworks

- Automatic reconstruction
- Goal: Dense sub-network with highly confident pair-wise features
- Score: Statistical significance
- Search: High scoring sub-networks
- Advantages
- Global picture
- Structured context for interactions
- Incorporate mid-confidence features



## Results

- 6 well structured sub-networks representing coherent molecular responses
- Mating
- Iron metabolism
- Low osmolarity cell wall integrity pathway
- Stationary phase and stress response
- Amino acid metabolism, mitochondrial function and sulfate assimilation
- Citrate metabolism
- Uncovered regulatory, signaling and metabolic interactions
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