Making Situation Calculus Indexical*

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Abstract

In the spirit of Levesque and Brachman[1985], we study a new class of representations made popular in the world of reactive planning [Firby, 1987], called indexical-functionals [Agre and Chapman, 1987]. We present a knowledge level analysis [Newell, 1982] of some indexical terms that clarifies their logical content and helps us identify the source of their computational power. In particular, we demonstrate that indexical terms can be formalized in a restriction of the situation calculus. Indexical theories gain descriptional as well as computational efficiency by using terms whose referents are determined in context by the perceptual machinery of an agent, and by selecting those and only those terms that are essential for determining a course of action for the agent. We show how indexical theories can be synthesized from a situation calculus description of a planning problem together with knowledge of the goals of the agent. This perspective on indexical terms provides us not only with a way of understanding what their ontological underpinnings are, but also helps us analyze the conditions under which they are useful.

1 Introduction

The intractability of classical planning and the need to actively monitor plans in a complex, dynamic world has led to the development of reactive planners that build plans at execution time based solely on the situation existing then [Firby, 1987]. Several designs for reactive planners have been proposed in the recent literature: the theory of situated automata and their combinational circuit compilation by Rosenschein and Kaelbling [1986], the theory of indexical-functional aspects in the reactive planner Pengi proposed by Chapman and Agre, and Rod Brooks' [1987] subsumption architecture. For the purposes of this paper we focus on the indexical-functionals described by Agre and Chapman in [1987] with a view to understanding the assumptions about the world and the planning process they make in order to contain the complexity of planning in their framework. The results we seek are a declarative specification of the worlds in which Pengi style planning works and an analysis of their computational and descriptional efficacy. Our results apply equally well to the other architectures.

The impetus to perform this analysis comes from two sources. One, the existence of the fundamental tradeoff between expressiveness and efficiency in knowledge representation [Levesque and Brachman, 1985] indicates that the computational advantage in indexical-functional representations ought to come from expressiveness limitations. If we can identify them, we can determine the worlds for which they are an epistemologically adequate formalism. Two, we would like to design methods for compiling out indexical-functional aspects from more expressive situation calculus descriptions of the world. Presently the designers of Pengi synthesize these aspects by hand. Should the rules of the world change, they will have to manually reconstruct these indexical-functionals and the situation-action rules that use them. Specifying modifications at the level of individual situation-action rules (or worse circuits) is clumsy and for moderately complex worlds almost impossible. An analysis of this representation in terms of situation calculus allows us to build a compiler for indexical-functional aspects and thus to specify modifications at a reasonable level of description.

The designers of Pengi state that "registering and acting on indexical-function aspects is an alternative to representing and reasoning about complex domains, and avoids combinatorial explosions". We demonstrate that indexical-functionals can be constructed out of the objective ontology of the world assumed by a situation calculus representation using the processes of indexicalization and propositionalization. Theo-

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ries that contain indexical-functionals simply make different expressiveness-computational tradeoffs as compared to the full situation calculus.

There are many indexical terms in English, "Me", "You", "This", "That", "Here", "There", "Now", "The Car Behind Me" and so on. We begin by introducing time indexicals into situation calculus. A simple example of a time indexical is *Now*. An agent that has its present theory of the world expressed in terms of the logical constant `Now` alone, is computationally simpler than one that requires expression of statements about time points in terms of some fixed time point other than *Now*. This idea is made rigorous in Section 2. Section 2.1 introduces the axioms needed for meaningful introduction of *Now* in the situation calculus. In Section 3, we use these axioms to reformulate a standard first order predicate logic sentence that assumes an objective ontology into an indexical one that uses *Now* and that is tuned for one-step planning. The final step is propositionalization that leads to the construction of propositional theories with their well-known efficiency properties.

The introduction of *Now*, sets the stage for introducing further indexicals pertaining to space (*Here* and *There*), and to objects (*The Block-that-is-behind-me*). However, introducing additional indexical terms, necessitates making assumptions about the process of determining their referents. This topic is pursued further in the longer version of this paper [Subramanian and Woodfill, 1989]. In Section 4 we discuss the expressive limitations of indexicals and present an analysis of the time and space costs associated with using them for monitoring and one-step planning. We conclude with a summary of the main points of the paper in Section 5.

2 A Conceptualization with a View

To use logic one must choose a point of view. Traditional situation calculus starts from a sort of god's eye or objective point of view. This objective stance allows one to frame eternal sentences, e.g. "Block A is on block B at 3:13, May 3rd, 1989". "block A" and "3:13, May 3rd, 1989" are considered as rigid designators that denote the same things for all time. Eternal sentences have the nice property that any conjunction of eternal sentences is also an eternal sentence. If an agent acquires true beliefs in the form of eternal sentences, it can always conjoin these new beliefs to its old true beliefs without fear of contradiction.

It is also possible to develop a situation calculus from an agent's subjective point of view. Consider a subjective ontology. What things and relations are central to an agent's point of view? First, the current time seems interesting, *Now* must be a distinguished thing for an agent. *Tomorrow* and *Yesterday* are important, but less so than *Now*. The place, *Here*, must be something special. If the agent is sitting at its desk typing, the typewriter in front of it, the chair under it, the paperback at its elbow, are all central objects. The agent could have a theory of here-and-now couched in symbols denoting these various elements of the ontology of here-and-now.

As a first example of an indexical theory, let us consider a rather narrow minded agent that makes one distinction, and hence deals with one predicate, *Rich*. *Rich* is true when the agent is rich, and false when the agent is not. Suppose further that the agent owns AI stock that was worth millions yesterday. Yesterday, the agent could have a theory containing the one sentence `Rich`. Today, the stock market fell and the stock is worth nothing; the agent can add $\neg$`Rich` to its supply of facts. The agent seems committed to the sentence: `Rich` $\wedge$ $\neg$`Rich`.

Rich must be made situation dependent. From an agent's point of view, the current moment (not S_0, the first situation) is the most interesting moment, so we introduce a term `Now` which now denotes the current situation. Our agent can maintain $\neg$`Rich(Now)`. The agent's state of richness yesterday can be stated today if we introduce a function *Before* which maps one state to its predecessor: `Rich(Before(Now))` Notice that `Rich-Now` is a quite useful propositional symbol where `Rich-S0` is not. The question of whether I am rich now is always fairly relevant, while the question of being rich at some fixed time loses significance. In what follows, we will describe how we can convert a theory that contains a situation-dependent predicate like *Rich* expressed in the full situation calculus that allows access to arbitrary situations, into an indexical one that allows reference only to the current situation and its predecessors via the *Before* function.

Two obstacles prevent the use of *Now* and *Before* and their corresponding symbols `Now` and `Before` in standard situation calculus: first, *Before* is not definable in standard situation calculus as there is no unique predecessor of a situation; second, it seems that an agent's theory becomes highly non-monotonic across time. `Rich(Now)` was in the theory yesterday, and it is not today. The first obstacle is overcome by adding appropriate axioms for situations and *Before*. The second obstacle is stepped around by accepting the idea of having a different, current, theory for each new situation in time, and considering the issue of transforming one theory of here-and-now at t_1 to another theory of here-and-now at t_2.

2.1 The Reformulated Situation Calculus

The need to provide more axioms for the situation calculus arises because *Before* is not explicitly definable in the standard situation calculus. To see that *Before* is not definable consider a world containing a compass C and four directions in which the compass' needle can point: N, E, S and W. The two actions in this world, *Clockwise*, and *Anti-Clockwise*, turn the compass. One model of this world has four situations, one for each direction the needle of the compass can point to.

$$S1 = C(N), S2 = C(E), S3 = C(S), S4 = C(W)$$

If *Before* were definable, $Before(S2)$ would have to be either $S1$ or $S3$. But $Do(Clockwise, S2) = S1$ and $Do(Anti\text{-}Clockwise, S3) = S1$. Clearly there can be no function defined on these four situations which picks out the previous situation.

The following axioms restrict the models of situation calculus to those in which there is a unique previous situation. The variables a1 and a2 range over actions, and s1 and s2 range over situations.

Intuitively, we would like to define a function $Before$ which maps a situation into its predecessor, just as the function "the day before" maps today into yesterday. This intuition is captured by the following definition for a relation:

(A). $\{\langle \mathtt{x}, \mathtt{y}\rangle \mid \exists \mathtt{a}\ \mathtt{Do}(\mathtt{a}, \mathtt{x}) = \mathtt{y}\}$

However for many models of situation calculus, this formula is not functional. The definition becomes functional if *Do* satisfies the following sentence.

(B). $\forall \mathtt{y}\ \exists \mathtt{a}\ \mathtt{x}\ \mathtt{Do}(\mathtt{a}, \mathtt{x}) = \mathtt{y}$
$\Rightarrow (\forall \mathtt{z}\ \mathtt{Do}(\mathtt{a}, \mathtt{z}) = \mathtt{y} \Rightarrow \mathtt{x} = \mathtt{z})$

After restricting *Do* in this way, $Before$ is still not defined by (A) for situations which are not in the range of *Do*. We choose to introduce an axiom relating $Before$ and *Do* which entails (B) and forces $Before$ to agree with (A):

$(ISC1)$. $\forall \mathtt{a}\ \mathtt{s}\ \mathtt{Before}(\mathtt{Do}(\mathtt{a}, \mathtt{s})) = \mathtt{s}$

As in our everyday world, given this axiom, each situation has a unique predecessor. However, there may be several distinct actions which take one situation to the same successor. Although not necessary for the construction, in accord with common intuition, we provide an axiom which forces situations with distinct histories to be distinct:

$(ISC2)$. $\forall \mathtt{a1}\ \mathtt{a2}\ \mathtt{s}\ \mathtt{a1} \neq \mathtt{a2}$
$\Rightarrow \mathtt{Do}(\mathtt{a1}, \mathtt{s}) \neq \mathtt{Do}(\mathtt{a2}, \mathtt{s})$ *optional*

Later when we discuss transforming one theory of here-and-now to the next theory of here-and-now, we will require that every situation be *before* some situation, (that $Before$ be onto) for a soundness condition to hold:

$(ISC3)$. $\forall \mathtt{s1}\ \exists \mathtt{s2}\ \mathtt{Before}(\mathtt{s2}) = \mathtt{s1}$

We introduce $(ISC4)$ in order to pick out an initial situation for a set of standard points, just as 0 picks out an initial element in the standard points of Peano arithmetic.

$(ISC4)$. $\neg\exists \mathtt{a}\ \mathtt{s}\ \mathtt{Do}(\mathtt{a}, \mathtt{s}) = \mathtt{S0}$ *optional*

Given the notion that a situation has a predecessor, the identity of the action resulting in a situation becomes an object of interest[1]. $(ISC5)$ is a definition of an additional relation *Action-that-generated* in terms of *Do*.

[1] If $(ISC2)$ is omitted, the *identities* of the actions which could have resulted in a situation are objects of interest.

$(ISC5)$. $\forall \mathtt{a}\ \mathtt{s1}\ \mathtt{Action\text{-}that\text{-}generated}(\mathtt{s1}, \mathtt{a})$
$\iff \exists \mathtt{s2}\ \mathtt{Do}(\mathtt{a}, \mathtt{s2}) = \mathtt{s1}$ *definition*

Action-that-generated(S, A) is true if performing A in some situation would result in S.

A useful consequence of these axioms and definitions is:

$(ISC6)$.
$\forall \mathtt{a}\ \mathtt{s}\ \mathtt{Action\text{-}that\text{-}generated}(\mathtt{s}, \mathtt{a}) \iff$
$\mathtt{Do}(\mathtt{a}, \mathtt{Before}(\mathtt{s})) = \mathtt{s}$ [*ISC1*, *ISC5*]

2.2 Transforming Theories

At each situation an agent is best served by having the best theory for that situation. In some environments the best theory for a situation is a theory couched in the subjective terms denoting the objects present to the agent in that situation. Each new situation will have a different theory. The theory of one situation is not, in general, a subset of the theory of the succeeding situation. We need a method for generating the current theory in terms of the current subjective ontology that describes the preceding situation. For now, only the term Now changes its reference from situation to situation We define a translation which, given a theory of the previous situation, produces a theory that is true in the current situation. The translation maps the term Now in the first theory to a term which designates the same thing in the second theory. Since Before(Now) now designates the situation which Now designated in the previous situation, the translation π maps Now to Before(Now), and all other parameters to themselves. To generate the theory of a new situation from the theory of the previous situation one applies the syntactic translation corresponding to π, replacing all occurrences of Now by Before(Now). This transformation has the desirable property that it preserves the consequences of a theory modulo the shift in designation for Now.

Lemma 1 *Given a set of sentences $\mathcal{T}$, and letting ψ denote the sentence* $\forall \mathtt{s1}\ \exists \mathtt{s2}\ \mathtt{s1} = \mathtt{Before}(\mathtt{s2})$ *(axiom $(ISC2)$ from above),*
and π denote $[\mathtt{Now} \rightarrow \mathtt{Before}(\mathtt{Now})]$*, (the replacement of* Before(Now) *for* Now*),*
for any sentence ϕ:

$$\mathcal{T} \cup \{\psi\} \models \phi$$

if and only if

$$\pi(\mathcal{T}) \cup \{\psi\} \models \pi(\phi)$$

3 Reformulations

Now that we have the apparatus for talking about *Now* and *Before*, we consider formulations which are particularly well suited for one-step planning. We start with a blocks world axiom from [Genesereth and Nilsson, 1987]:

$$\forall \mathtt{s}\ \mathtt{x}\ \mathtt{y} \qquad \mathtt{T}(\mathtt{On}(\mathtt{x}, \mathtt{y}), \mathtt{s}) \wedge \mathtt{T}(\mathtt{Clear}(\mathtt{x}), \mathtt{s}) \Rightarrow \mathtt{T}(\mathtt{Clear}(\mathtt{y}), \mathtt{Do}(\mathtt{U}(\mathtt{x}, \mathtt{y}), \mathtt{s})) \qquad (1)$$

For one-step planning, we also need some notion of the relation between goals and what an agent ought to do now. We introduce an axiom of rationality.

$(ISC7)$. $\forall \mathtt{a\ p\ s\ T(p, Do(a, Now))} \wedge \mathtt{Goal(p, Now)}$
$\wedge \neg \mathtt{T(p, Now)} \Rightarrow \mathtt{Must(Now) = a}$

Which just says that if doing a would result in p, if one has the goal p, and if p doesn't hold now then one must do a.

In one step planning the *Goal* and properties of *Now* are the relevant aspects of the situation, and so the axiom should be reformulated in those terms.

Resolving (ISC7) and (1), so that we are talking about goals,

$$\begin{aligned} \forall \mathtt{x\ y} \quad & \mathtt{T(On(x, y), Now)} \wedge \mathtt{T(Clear(x), Now)} \\ & \wedge \neg \mathtt{T(Clear(y), Now)} \wedge \mathtt{Goal(Clear(y), Now)} \\ & \Rightarrow \mathtt{Must(Now) = U(x, y)} \end{aligned} \tag{2}$$

Introducing the time indexical Now has propositionalized the situation component of the planning axiom. We can carry this one step further, by instantiating the variables x and y with particular blocks.

$$\begin{aligned} & \mathtt{T(On(A, B), Now)} \wedge \mathtt{T(Clear(A), Now)} \\ & \wedge \neg \mathtt{T(Clear(B), Now)} \wedge \mathtt{Goal(Clear(B), Now)} \\ & \Rightarrow \mathtt{Must(Now) = U(A, B)} \end{aligned} \tag{3}$$

And finally introducing propositional symbols to make the sentence propositional,

$$\begin{aligned} & \mathtt{On\text{-}A\text{-}B} \wedge \mathtt{Clear\text{-}A} \\ & \wedge \neg \mathtt{Clear\text{-}B} \wedge \mathtt{Goal\text{-}Clear\text{-}B} \\ & \Rightarrow \mathtt{Must\text{-}U\text{-}A\text{-}B} \end{aligned} \tag{4}$$

Formula (4) is good for one-step planning because determining the truth value of a set of literals about *Now* determines what the agent must do.

4 Analysis

The class of theories in the situation calculus that we can transform to the indexical form is restricted by the requirement that the theory be consistent with $ISC1$-$ISC4$. This restriction rules out theories that fix the number of situations or that make two situations with distinct histories equivalent.

Claim 1 *A theory $\mathcal{T}$ can be indexicalized if $\mathcal{T} \cup \{ISC1\text{-}ISC4\}$ is consistent.*

In Section 3 we used the axioms of the indexical situation calculus to generate propositional theories that are suited for plan monitoring from a theory in the situation calculus. The previous theorem establishes the class of theories for which this conversion is possible. We now show that the compilation methods we use are sound.

Let I be a transformation performed on a theory $\mathcal{T}$ in the situation calculus that obeys the limitation in Claim 1. I is sound if

Soundness. $\mathcal{T} \cup \mathtt{ISC1\text{-}ISC6} \models \mathtt{I}(\mathcal{T})$

.

We further require that the theory $I(\mathcal{T})$ be propositional, because we wish to gain the computational advantages that reasoning within the propositional fragment of logic gives us.

Claim 2 *The transformations in Sections 3 are sound.*

Proof: Follows from the fact that the transformation I is deduction. □

We examine the computational consequences of indexicalization and propositionalization. First we consider space requirements. To propositionalize a theory $\mathcal{T}$ in clausal form in a typed logic, one must augment the language until there are object constants for each object in each type. The resulting theory $\mathcal{T}'$ is the set of all ground instances of all clauses in the original theory. The cardinality of this set of clauses is expressed by the following formula.

$$|\mathcal{T}'| = \sum_{c \in \mathcal{T}} \prod_{v \in Variables(c)} |type(v)|$$

Considering one type, $\mathcal{D}$, a clause replicates exponentially in the number of variables occurring in it.

$$|\mathcal{T}'| = \sum_{c \in \mathcal{T}} |\mathcal{D}|^{|Variables(c)|}$$

Even a theory with one clause containing one literal with one situation variable in propositional form would contain infinitely many clauses.

Here is where the power of the indexical approach lies. If the agent can do all its reasoning with a few situation terms such as Now and Before(Now), and a few world terms This-Block and That-Block, this explosion may not be significant. Essentially we reduce the size of the theory to

$$|\mathcal{T}'| = \sum_{c \in \mathcal{T}} \prod_{v \in Variables(c)} |type(v)|$$

where type$(v) = 1$ (Now) when v is a situation variable and type$(v) = n$ (the number of propositional object referents, like the-bee-that-is-chasing-me) when v is an object variable.

Claim 3 *Indexicalization is a win in terms of space only if the number of situation referents and object referents (indexical-functional aspects) is much smaller than the total number of situations and objects described in the theory $\mathcal{T}$ in the situation calculus.*

Suppose we have a theory of cars in the United States expressed in situation calculus. The straightforward propositionalization of this theory would result in another theory whose size is proportional to 200 million (number of cars in the United States). This theory allows us to name every car. If however, we restricted our attention to cars of interest to the agent: *The-car-behind-me*, *The-car-passing-me* etc.., we would have a propositional theory whose size is proportional to the number of such referents. The power of using these indexical terms is that it gives us implicit quantification: the actual referent of the `The-car-behind-me` is determined by the perceptual machinery of the agent. These indexical terms allow access to that part of the immense propositional theory that is actually needed by the agent.

A really significant performance improvement can be achieved by transforming the propositional theory into a set of equations. This can be done if the propositional theory can be "directionalized". We first subdivide the literals in the theory into the perceptions or the givens (examples are `On-A-B-Before-Now` and `On-A-B-Now`) and the actions or the conclusions (e.g. `Clear-B-Now` and `Must-Unstack-A-B`). To facilitate the directionalization of the reasoning chains in this theory from perceptions to actions, we will assume that the conclusion literals occur in unnegated form.[2] Now, for each conclusion, we collect all minimal conjunctions of the givens which imply it, disjoin the set of conjunctions, and construct a definition of the form *Conclusion* $\equiv$ *Disjunction of Conjunctions.*

Claim 4 *The worst case complexity of concluding the truth of the action literals in the directionalized theory constructed above is linear in their number.*

Proof: Since the truth assignments to the givens can be determined in constant time by the perceptual equipment, the complexity of concluding each action literal in the constructed theory is also a constant. If there are n action literals, we need $O(n)$ time to determine their truths.

The definitions for the action literals can be straightforwardly compiled into combinational circuits (AND gates for conjunctions and OR gates for disjunctions), then the time complexity for concluding *all* the action literals goes to $O(1)$.

Claim 5 *By compiling the definitions of the action literals into combinational circuits, we can determine their truths in constant time.*

Note that the two results above depend on the fact that perception can be done in constant time. This is actually an assumption that Pengi makes.

5 Conclusions

This paper provides a principled account of several forms of indexicality. The introduction of *Now* is a necessary condition for introducing other indexical terms non-rigidly referring to objects. We present a method for generating indexical theories from a domain description in situation calculus. Determining the truth of literals involving indexicals requires that perception be able to determine their referents. Indexical theories can be made into propositional ones with good computational properties. This benefit comes at the cost of losing the ability to rigidly refer to arbitrary objects and situations.

We have shown that indexical-functionals can be explained in terms of a well understood formalism: the situation calculus. In so doing we have exposed some of the expressiveness and efficiency trade-offs resulting from using them.

[2] This is to avoid problems that arise from negation as failure.

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