

Comp 411
Principles of Programming Languages
Lecture 9
Meta-interpreters III

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Major Challenge

LC does not include a recursive binding operation (like Scheme **letrec** or **local**). How would we define **eval** for such a construct?

- Key problem: the closure structure for a recursive **lambda** must include an environment that refers to itself!
- In imperative Java, how would we construct such an environment. Hint: how did we build “circular” data structures in Comp 211/212? Imperativity is *brute force*. But it works. We will use it in Project 3 and thereafter.

Minor Challenge

How could we define an environment that refers to itself in *functional* Scheme?

- Key problem: observe that in **let** and **lambda** the expression defining the value of a variable cannot refer to itself.
- Solution: does functional Scheme contain a recursive binding construct?
- How can we use this construct to define a recursive environment?
- What environment representation must we use?

A Bigger Challenge

Assume that we want to write LC in a purely functional language without a recursive binding construct (say functional Scheme without **define** and **letrec**)?

- Key problem: must expand **letrec** into **lambda**
- No simple solution to this problem. We need to invoke syntactic magic or (equivalently) develop some sophisticated mathematical machinery.

Key Intuitions

- Computation is incremental—not monolithic
- Slogan: general computation is successive approximation (typically in response to successive demands for more information).
- Familiar example: a program mapping a potentially infinite input stream of characters to a potentially infinite output stream of characters. Generalization: infinite trees mapped to infinite trees.

Key Mathematical Concepts

Domains of computations (like streams, trees):

- partially ordered set (**po**)
- finitary basis (set finite approximations)
 - countable
 - closed under LUBs on finite bounded subsets
- chain
- chain-complete
- complete partial order (**cpo**)
- “home-plate” **cpo** (not domain; finite elements not a finitary basis)
- bottom (\perp)
- flat domain (monolithic set of values formulated as domain)
 - integers, booleans, strings, conventional finite lists, ASTs

Key Mathematical Concepts

Computable functions:

- monotonic (universal)
- continuous (universal)
- strict (typical)

Examples

Domains

- flat domains
- strict function spaces on flat domains
- lazy trees of boolean (of D where D is flat)
- factorial functional