Semantics for Comp 311

Walid Taha

October 25, 2004

Syntax of the lambda calculus with integer constants:

$$\begin{array}{rcll} x \in \mathcal{X} & & \text{Any infinite set of names} \\ i \in \mathcal{Z} & & \text{Integers} \\ e \in \mathcal{E} & ::= & i \mid x \mid \lambda x.e \mid e\ e \end{array}$$

Substitution:

$$\begin{array}{rl} i[z := e] & = i \\ x[z := e] & = x \text{ if } x \neq z \\ z[z := e] & = e \\ \lambda x.e_1[z := e] & = \lambda x'.(e_1[x := x'][z := e]) \text{ with } x' \notin FV(e) \\ e_1\ e_2[z := e] & = (e_1[z := e])\ (e_2[z := e]) \end{array}$$

Big-step substitution semantics:

$$\frac{}{i \hookrightarrow i} \quad \frac{}{\lambda x.e \hookrightarrow \lambda x.e} \quad \frac{\begin{array}{l} e_1 \hookrightarrow \lambda x.e_3 \\ e_2 \hookrightarrow e_4 \\ e_3[x := e_4] \hookrightarrow e_5 \end{array}}{e_1\ e_2 \hookrightarrow e_5}(CBV) \quad \frac{\begin{array}{l} e_1 \hookrightarrow \lambda x.e_3 \\ e_3[x := e_2] \hookrightarrow e_4 \end{array}}{e_1\ e_2 \hookrightarrow e_4}(CBN)$$

Small-step substitution semantics:

$$v \in \mathcal{V} ::= i \mid \lambda x.e \quad \frac{}{(\lambda x.e)\ v \mapsto e[x := v]} \quad \frac{e_1 \mapsto {e_1}'}{e_1\ e_2 \mapsto {e_1}'\ e_2} \quad \frac{e \mapsto e'}{v\ e \mapsto ve'}$$

Big-step environment semantics:
Syntax of the lambda calculus with integer constants:

$$\begin{array}{rcllrcl} E \in \mathcal{E}_{CBV} & ::= & [] \mid E :: (x := v) & E \in \mathcal{E}_{CBN} & ::= & [] \mid E :: (x := b) \\ v \in \mathcal{V} & ::= & i \mid \{E, \lambda x.e\} & b \in \mathcal{B} & ::= & i \mid \{E, \lambda.e\} \end{array}$$

$$\frac{}{E, i \hookrightarrow i} \quad \frac{}{E, x \hookrightarrow E(x)}(CBV) \quad \frac{}{E, \lambda x.e \hookrightarrow \{E, \lambda x.e\}} \quad \frac{\begin{array}{l} E, e_1 \hookrightarrow \{E', \lambda x.e_3\} \\ E, e_2 \hookrightarrow v_4 \\ E' :: (x := v_4), e_3 \hookrightarrow v_5 \end{array}}{E, e_1\ e_2 \hookrightarrow v_5}(CBV)$$

$$\frac{E(x) \hookrightarrow v}{E, x \hookrightarrow v}(CBN) \quad \frac{\begin{array}{l} E, e_1 \hookrightarrow \{E', \lambda x.e_3\} \\ E' :: (x := (E, e_2)), e_3 \hookrightarrow v_4 \end{array}}{E, e_1\ e_2 \hookrightarrow v_4}(CBN)$$