Note:

1. The submitted assignment needs to be typeset in LaTeX.

2. **Teamwork:** You are expected to complete this assignment in pairs. By signing your name on the assignment you are asserting that the work submitted was done collaboratively. At the very least, this entails:
   - discussing each problem and agreeing on a sketch of the solution,
   - reviewing the written solutions for technical correctness and typographical errors,
   - doing a joint codewalk over all written programs,
   - dividing labor equally (roughly), and
   - being able to explain every answer as if it is your own answer.

3. Note on the submission the total number of hours put by each partner separately and by the two partners together.

Syntax of Propositional Logic:

1. Show that there are no formulas of length 2, 3, or 6, but that every other length is possible.

2. Show that between every left parenthesis and right parenthesis in a formula $\varphi$ there is at least one occurrence of a propositional connective.

3. Let $\varphi$ be a formula, let $p$ be the number of parentheses (left and right) in $\varphi$ and let $l$ be the length (number of symbols) of $\varphi$. What are the minimum and maximum values of $p/l$?

4. Let $\varphi$ be a formula, let $c$ be the number of occurrences of binary connectives ($\land$, $\lor$, $\rightarrow$, and $\leftrightarrow$) in $\varphi$, and let $s$ be the number of occurrences of atomic propositions in $\varphi$. Show that $s = c + 1$.

5. Give examples of formulas $\alpha$ and $\beta$ and expressions $\gamma$ and $\delta$ such that $(\alpha \land \beta)$ and $(\gamma \land \delta)$ are identical, but $\alpha$ is different than $\gamma$.

6. The BNF for formulas in *prefix notation* (also called *Polish notation*) is

$$Form := AP | \neg Form | \bigcirc Form Form,$$

where $\bigcirc$ is a binary propositional connective. Note that this syntax does not use parentheses. For example, the prefix notation for $((p \land q) \rightarrow p)$ is $\rightarrow \land p q p$. Formulate and prove a unique-readability theorem for formulas in prefix notation.
7. We say that \( \theta \) is a subformula of \( \varphi \) if

- \( \theta \) is \( \varphi \),
- \( \varphi \) is \( (\neg \psi) \) and \( \theta \) is a subformulas of \( \psi \), or
- \( \varphi \) is \( (\psi_1 \circ \psi_2) \) and \( \theta \) is either a subformulas of \( \psi_1 \) or a subformula of \( \psi_2 \).

Let \( \alpha \) and \( \beta \) be expressions. We say that \( \beta \) is a subexpression of \( \alpha \) if \( \alpha = \gamma \beta \delta \), for some expressions \( \gamma \) and \( \delta \). Prove that if \( \alpha \) and \( \beta \) are formulas and \( \beta \) is a subexpression of \( \alpha \), then \( \beta \) is a subformula of \( \alpha \).

8. Well Formedness:

- Describe an algorithm that checks whether a given expression is a formula.
- Analyze the time and space requirements of your algorithm.
- Implement the algorithm in your favorite programming language and use the program to check which of the following expressions is a formula (attach a listing of the code).
  
  (a) \(((\neg (A \lor B)) \land C)\)
  
  (b) \((A \land B) \lor C\)
  
  (c) \((A \rightarrow (B \land C))\)
  
  (d) \(((A \leftrightarrow B) \rightarrow (\neg A))\)
  
  (e) \(((\neg A) \rightarrow B \lor C)\)
  
  (f) \(((C \lor B \land A) \leftrightarrow D)\)
  
  (g) \(((\lor A) \land (\neg B))\)
  
  (h) \((A \land (B \land C)))\)