**COMP 409**
**Assignment No. 4**

**Due date:** Oct. 20, 2022.

**Note:**

1. The submitted assignment needs to be typeset in LaTeX.

2. **Teamwork:** You are expected to complete this assignment in teams. By signing your name on the assignment you are asserting that the work submitted was done collaboratively, and that you have adhered to Rice’s Honor Code. At the very least, this entails:
   - discussing each problem and agreeing on a sketch of the solution,
   - reviewing the written solutions for technical correctness and typographical errors,
   - doing a joint codewalk over all written programs,
   - dividing labor equally (roughly), and
   - being able to explain every answer as if it is your own answer.

3. Note on the submission the total number of hours put in by each partner separately and by the two partners together.

**Deduction and Refutation:**

1. Use Hilbert’s Deductive System to prove that $\vdash (p \rightarrow p)$.

2. Convert the following formulas to clausal form (CNF):
   - $((p \lor q) \rightarrow (s \lor t))$
   - $(\neg(p \land (q \land \neg s)))$
   - $(\neg((p \land q) \lor ((q \lor r) \lor (p \land r))))$

3. Show that the following set of clauses is refutable:
   $$\{p \lor \neg q \lor r, q \lor r, \neg p \lor r, q \lor \neg r, \neg q\}.$$ 

4. Use resolution to show that the following formulas are not satisfiable:
   - $((p \leftrightarrow (q \rightarrow r)) \land ((p \leftrightarrow q) \land (q \leftrightarrow (\neg r))))$
   - $(\neg(((p \rightarrow q) \rightarrow (\neg q)) \rightarrow (\neg q)))$

5. Use resolution to show that the formula $((\neg r \lor (p \land q)) \rightarrow ((r \rightarrow p) \land (r \rightarrow q)))$ is valid.

6. Use resolution to prove the soundness of Meredith’s Axiom: $(((p \rightarrow q) \rightarrow (\neg r \rightarrow \neg s)) \rightarrow (r \leftrightarrow u) \rightarrow ((u \rightarrow p) \rightarrow (s \rightarrow p))$
7. Show that if a set $C$ of clauses is refutable then it has a refutation whose length is at most exponential in $|C|$.

8. A clause is *Krom* if it contains at most two literals, i.e., it is a 2-clause. A Krom formula is a 2-CNF formula, i.e., set of Krom clauses. Use resolution to develop a polynomial-time algorithm for satisfiability of Krom formulas.

9. Consider the $n \times n$ discrete grid. A curve from $(0, 0)$ to $(n, n)$ is a set of grid points that includes $(0, 0)$ and $(n, n)$ and each point $(i, j)$ on the curve has as next point either $(i + 1, j)$ or $(i, j + 1)$ but not both. Similarly, a curve from $(0, n)$ to $(n, 0)$ is a set of grid points that includes $(0, n)$ and $(n, 0)$ and each point $(i, j)$ on the curve has as next point either $(i + 1, j)$ or $(i, j - 1)$ but not both. A curve from $(0, 0)$ to $(n, n)$ and from $(0, n)$ to $(n, 0)$ must meet. Express this in propositional logic and prove it, for the $3 \times 3$-grid using resolution.