Deduction and Refutation:

1. Use Hilbert’s Deductive System to prove that $\vdash (p \rightarrow p)$.

2. Convert the following formulas to clausal form (CNF):
   
   - $((p \lor q) \rightarrow (s \lor t))$
   - $(\neg(p \land (q \land \neg s)))$
   - $(\neg((p \land q) \lor ((q \lor r) \lor (p \land r))))$

3. Show that the following set of clauses is refutable:
   
   \{ p \lor \neg q \lor r, q \lor r, \neg p \lor r, q \lor \neg r, \neg q \}.

4. Use resolution to show that the following formulas are not satisfiable:
   
   - $((p \leftrightarrow (q \rightarrow r)) \land ((p \leftrightarrow q) \land (q \leftrightarrow \neg r)))$
   - $((p \rightarrow q) \rightarrow (r \rightarrow (q \rightarrow (\neg s))))$

5. Use resolution to show that the formula $((\neg r \lor (p \land q)) \rightarrow ((r \rightarrow p) \land (r \rightarrow q)))$ is valid.

6. Use resolution to prove the soundness of Meredith’s Axiom: $(((p \rightarrow q) \rightarrow (r \rightarrow (\neg s))) \rightarrow r) \rightarrow (u \rightarrow (u \rightarrow (s \rightarrow p)))$

7. Show that if a set $C$ of clauses is refutable then it has a refutation whose length is at most exponential in $|C|$.

8. A clause is Krom if it contains at most two literals, i.e., it is a 2-clause. A Krom formula is a 2-CNF formula, i.e., set of Krom clauses. Use resolution to develop a polynomial-time algorithm for satisfiability of Krom formulas.

9. Consider the $n \times n$ discrete grid. A curve from $(0,0)$ to $(n,n)$ is a set of grid points that includes $(0,0)$ and $(n,n)$ and each point $(i,j)$ on the curve has as next point either $(i+1,j)$ or $(i,j+1)$ but not both. Similarly, a curve from $(0,n)$ to $(n,0)$ is a set of grid points that includes $(0,n)$ and $(n,0)$ and each point $(i,j)$ on the curve has as next point either $(i+1,j)$ or $(i,j-1)$ but not both. A curve from $(0,0)$ to $(n,n)$ and from $(0,n)$ to $(n,0)$ must meet. Express this in propositional logic, and prove it for $n = 3$ using resolution.