Note:

1. This assignment is \textit{OPTIONAL}!

2. The submitted assignment needs to be typeset in \LaTeX{}.

3. \textbf{Teamwork}: You may complete this assignment in teams. By signing your name on the assignment you are asserting that the work submitted was done collaboratively. At the very least, this entails:
   
   \begin{itemize}
   \item discussing each problem and agreeing on a sketch of the solution,
   \item reviewing the written solutions for technical correctness and typographical errors,
   \item doing a joint codewalk over all written programs,
   \item dividing labor equally (roughly), and
   \item being able to explain every answer as if it is your own answer.
   \end{itemize}

4. Note on the submission the total number of hours put in by each partner separately and by the team members together.

1. Let $\mathcal{A}$ be the standard model of arithmetics, with the set $\mathbb{N}$ of natural numbers as the domain of $\mathcal{A}$, and with the binary relation $<$, the constants 0 and 1, and the functions $+$ and $\ast$ getting their standard interpretations in $\mathcal{A}$.

   Show that there is a structure $\mathcal{A}'$ that is elementarily equivalent to $\mathcal{A}$ but not isomorphic to $\mathcal{A}$. (Hint: Start with the set of all sentences that hold in $\mathcal{A}$ and extend them with an infinite set of sentences that forces a new constant to be greater than all natural numbers. Use compactness to show that this set of sentence is satisfiable.)

   What can you conclude from this?

2. Prove the Skolem-Normal-Form Theorem in its full generality: Every prenex-normal-form first-order sentence is equisatisfiable with a universal sentence.

   Hint: Eliminate existential quantifiers from the inside out. Formulate the induction hypothesis carefully.
3. Formulate in first-order logic: “for every boy, there is a girl who loves only him”.

What can you say about the number of boys and girls?

4. Formulate and prove the validity of Drinker’s Principle: in every group of people one can point to one person in the group such that if that person drinks then all the people in the group drink.

Prove validity first by direct semantical reasoning and then by appealing to Herbrand’s Theorem.

5. Can you formulate in first-order logic the sentence ”There are some critics who admire one another”?

6. The vocabulary of Set Theory includes the single binary relation symbol $\in$, which is intended to express membership, and the usual identity relation symbol.

Express in first-order logic the following axioms:

- There exists an empty set.
- Two sets are identical iff they have precisely the same member.
- For every set $S$, there is a set $S'$ that contains $S$ strictly.