COMP 515: Advanced Compilation for Vector and Parallel Processors

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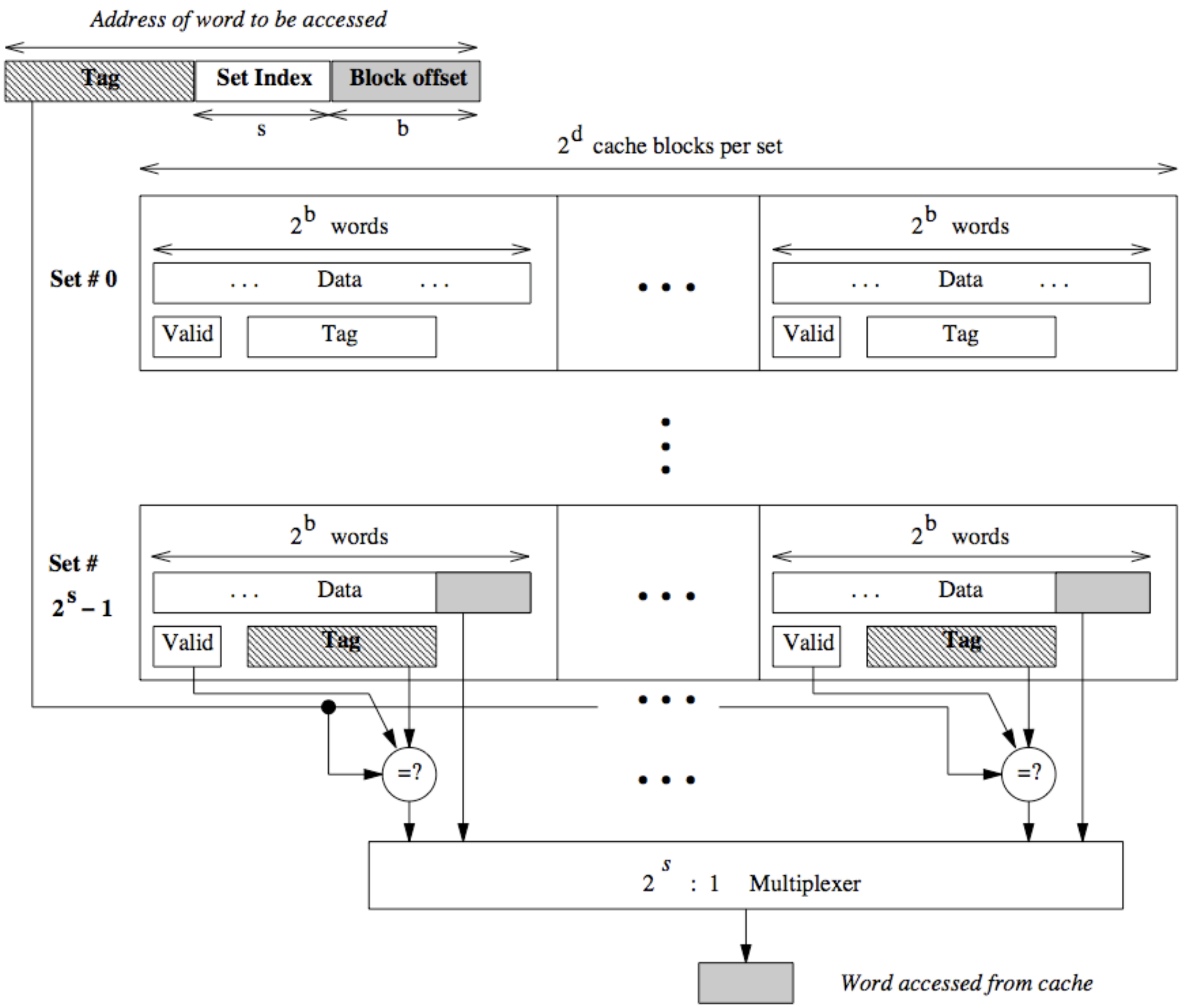
Review: How do set-associative caches work?



Acknowledgments

- Slides from previous offerings of COMP 515 by Prof. Ken Kennedy
  - http://www.cs.rice.edu/~ken/comp515/

Compiling Array Assignments

Allen and Kennedy, Chapter 13

Fortran 90

- Fortran 90: successor to Fortran 77
- Slow to gain acceptance:
  - Need better/smarter compiler techniques to achieve same level of performance as Fortran 77 compilers
- This chapter focuses on a single new feature - the array assignment statement: `A[1:100] = 2.0`
  - Intended to provide direct mechanism to specify parallel/vector execution
- This statement must be implemented for the specific available hardware. In an uniprocessor, the statement must be converted to a scalar loop: *Scalarization*

Fortran 90

- Range of a vector operation in Fortran 90 denoted by a triplet: <lower bound: upper bound: increment>

```
A[1:100:2] = B[2:51] + 3.0
```

- Semantics of Fortran 90 require that for vector statements, all inputs to the statement are fetched before any results are stored

Outline

- Simple scalarization
- Safe scalarization
- Techniques to improve on safe scalarization
  - Loop reversal
  - Input prefetching
  - Loop splitting
- Multidimensional scalarization
- A framework for analyzing multidimensional scalarization

Scalarization

- Replace each array assignment by a corresponding DO loop
- Is it really that easy?
- Two key issues:
  - Wish to avoid generating large array temporaries
  - Wish to optimize loops to exhibit good memory hierarchy
  - performance

Simple Scalarization

- Consider the vector statement:

```
A(1:200) = 2.0 * A(1:200)
```

- A scalar implementation:

```
S1  DO I = 1, 200
S2      A(I) = 2.0 * A(I)
          ENDDO
```

- However, some statements cause problems:

```
A(2:201) = 2.0 * A(1:200)
```

- If we naively scalarize, we get incorrect code:

```
DO i = 1, 200
      A(i+1) = 2.0 * A(i)
ENDDO
```

Scalarization Faults

- Why do scalarization faults occur?
- Vector operation semantics: All values from the RHS of the assignment should be fetched before storing into the result
- If a scalar operation stores into a location fetched by a later operation, we get a scalarization fault

- Principle 13.1: A vector assignment generates a scalarization fault if and only if the scalarized loop carries a true dependence.

- These dependences are known as scalarization dependences
- To preserve correctness, compiler should never produce a scalarization dependence

Safe Scalarization

- Naive algorithm for safe scalarization: Use temporary storage to make sure scalarization dependences are not created
- Consider:

```
A(2:201) = 2.0 * A(1:200)
```

- can be split up into:

```
T(1:200) = 2.0 * A(1:200)
A(2:201) = T(1:200)
```

- Then scalarize using *SimpleScalarize*

```
DO I = 1, 200
     T(I) = 2.0 * A(I)
ENDDO
DO I = 2, 201
     A(I) = T(I-1)
ENDDO
```

Safe Scalarization

- Procedure *SafeScalarize* implements this method of scalarization

- Good news:
  - Scalarization always possible by using temporaries

- Bad News:
  - Substantial increase in memory use due to temporaries
  - More memory operations per array element
  - Akin to overheads incurred in implementing functional languages

- We shall look at a number of techniques to reduce the effects of these disadvantages

Loop Reversal

```
A(2:256) = A(1:255) + 1.0
```

- A scalarization approach using loop reversal that avoids the need for a temporary:

```
DO I = 256, 2, -1
    A(I) = A(I-1) + 1.0
ENDDO
```

Loop Reversal

- When can we use loop reversal?
  - Loop reversal maps true dependences into antidependences
  - But may also map antidependences into dependences

```
A(2:257) = ( A(1:256) + A(3:258) ) / 2.0
```

- After scalarization:

```
DO I = 2, 257
   A(I) = ( A(I-1) + A(I+1) ) / 2.0
ENDDO
```

- Loop Reversal gets us:

```
DO I = 257, 2
   A(I) = ( A(I-1) + A(I+1) ) / 2.0
ENDDO
```

- Thus, cannot use loop reversal in presence of antidependences
- Goal: ensure that scalarized loop has no loop-carried true dependences

Input Prefetching

```
A(2:257) = ( A(1:256) + A(3:258) ) / 2.0
```

- Causes a scalarization fault when naively scalarized to:

```
DO I = 2, 257
   A(I) = ( A(I-1) + A(I+1) ) / 2.0
ENDDO
```

- Problem: Stores into first element of the LHS in the previous iteration
- Input prefetching: Use scalar temporaries to store elements of input and output arrays

Input Prefetching

- A first-cut at using temporaries:

```
DO I = 2, 257
   T1 = A(I-1)
   T2 = ( T1 + A(I+1) ) / 2.0
   A(I) = T2
ENDDO
```

- `T1` holds element of input array, `T2` holds element of output array
- But this faces the same problem. Can correct by moving assignment to T1 into previous iteration...

Input Prefetching

```
T1 = A(1)
DO I = 2, 256
   T2 = ( T1 + A(I+1) ) / 2.0
   T1 = A(I)
   A(I) = T2
ENDDO
T2 = ( T1 + A(257) ) / 2.0
A(I) = T2
```

- Note: We are using scalar replacement, but the motivation for doing so is different than in Chapter 8

Input Prefetching

- Already seen in Chapter 8, we need as many temporaries as the dependence threshold + 1.
- Example:

```
DO I = 2, 257
   A(I+2) = A(I) + 1.0
  ENDDO
```

- Can be changed to:

```
T1 = A(1)
T2 = A(2)
DO I = 2, 255
   T3 = T1 + 1.0
   T1 = T2
   T2 = A(I+2)
   A(I+2) = T3
ENDDO
T3 = T1 + 1.0
T1 = T2
A(258) = T3
T3 = T1 + 1.0
A(259) = T3
```

Input Prefetching

- Can also unroll the loop and eliminate register to register copies

- Principle 13.2: Any scalarization dependence with a threshold known at compile time can be corrected by input prefetching.

Input Prefetching

- Sometimes, even when a scalarization dependence does not have a constant threshold, input prefetching can be used effectively

```
A(1:N) = A(1:N) / A(1)
```

- which can be naively scalarized as:

```
DO i = 1, N
   A(i) = A(i) / A(1)
ENDDO
```

- true dependence from first iteration to every other iteration
- antidependence from first iteration to itself
- Via input prefetching, we get:

```
tA1 = A(1)
DO i = 1, N
   A(i) = A(i) / tA1
ENDDO
```

Multidimensional Scalarization

- Vector statements in Fortran 90 in more than 1 dimension:

```
A(1:100, 1:100) = B(1:100, 1, 1:100)
```

- corresponds to:

```
DO J = 1, 100
  A(1:100, J) = B(1:100, 1, J)
ENDDO
```

- Scalarization in multiple dimensions:

```
A(1:100, 1:100) = 2.0 * A(1:100, 1:100)
```

- Obvious Strategy: convert each vector iterator into a loop:

```
DO J = 1, 100, 1
  DO I = 1, 100
    A(I,J) = 2.0 * A(I,J)
  ENDDO
ENDDO
```

Multidimensional Scalarization

- What should the order of the loops be after scalarization?
  - Familiar question: We dealt with this issue in Loop Selection /Interchange in Chapter 5
- Profitability of a particular configuration depends on target architecture
  - For simplicity, we shall assume shorter strides through memory are better
  - Thus, optimal choice for innermost loop is the leftmost vector iterator

Multidimensional Scalarization

- Extending previous results to multiple dimensions:
  - Each vector iterator is scalarized separately, starting from the leftmost vector iterator in the innermost loop and the rest of the iterators from left to right

- Once the ordering is available:
  1. Test to see if the loop carries a scalarization dependence. If not, then proceed to the next loop.
  2. If the scalarization loop carries only true dependences, reverse the loop and proceed to the next loop.
  3. Apply input prefetching, with loop splitting where appropriate, to eliminate dependences to which it applies. Observe, however, that in outer loops, prefetching is done for a single submatrix (the remaining dimensions).
  4. Otherwise, the loop carries a scalarization fault that requires temporary storage. Generate a scalarization that utilizes temporary storage and terminate the scalarization test for this loop, since temporary storage will eliminate all scalarization faults.

Outer Loop Prefetching

```
A(1:N, 1:N) =
            (A(0:N-1, 2:N+1) + A(2:N+1, 0:N-1)) / 2.0
```

- If we try to scalarize this (keeping the column iterator in the innermost loop) we get a true scalarization dependence (<, >) involving the second input and an antidependence (>, <) involving the first input

- Cannot use loop reversal...

Outer Loop Prefetching

```
A(1:N, 1:N) =
      (A(0:N-1, 2:N+1) + A(2:N+1, 0:N-1)) / 2.0
```

- We can use input prefetching on the outer loop. The temporaries will be arrays:

```
T0(1:N) = A(2:N+1, 0)
DO j = 1, N-1
   T1(1:N)=( A(0:N-1, j+1) + T0(1:N) ) / 2.0
   T0(1:N) = A(2:N+1, j)
   A(1:N, j) = T1(1:N)
ENDDO
T1(1:N) = ( A(0:N-1, N) + T0(1:N) ) / 2.0
A(1:N, N) = T1(1:N)
```

- Total temporary space required = 2 rows of original matrix
- Better than storage required for copy of the result matrix

Loop Interchange

- Sometimes, there is a tradeoff between scalarization and optimal memory hierarchy usage

```
   A(2:100, 3:101) = A(3:101, 1:201:2)
```

- If we scalarize this using the prescribed order:

```
DO I = 3, 101
  DO 100 J = 2, 100
    A(J,I) = A(J+1,2*I-5)
  ENDDO
ENDDO
```

- Dependences (<, >) (I = 3, 4) and (>, >) (I = 6, 7)
- Cannot use loop reversal, input prefetching
- Can use temporaries

Loop Interchange

- However, we can use loop interchange to get:

```
DO J = 2, 100
  DO I = 3, 101
    A(J,I) = A(J+1,2*I-5)
  ENDDO
ENDDO
```

- Not optimal memory hierarchy usage, but reduction of temporary storage
- Loop interchange is useful to reduce size of temporaries
- It can also eliminate scalarization dependences

General Multidimensional Scalarization

- Goal: To vectorize a single statement which has m vector dimensions
  - Given an ideal order of scalarization (l_1, l_2, ..., l_m)
  - (d_1, d_2, ..., d_n) be direction vectors for all true and antidependences of the statement upon itself
  - The scalarization matrix is a n × m matrix of these direction vectors
- For instance:

```
A(1:N, 1:N, 1:N) = A(0:N-1, 1:N, 2:N+1) +
                             A(1:N, 2:N+1, 0:N-1)
```

$$\begin{pmatrix} > & = & < \\ < & > & = \end{pmatrix}$$

General Multidimensional Scalarization

- If we examine any column of the direction matrix, we can immediately see if the corresponding loop can be safely scalarized as the outermost loop of the nest:
  - If all entries of the column are = or >, it can be safely scalarized as the outermost loop without loop reversal.
  - If all entries are = or <, it can be safely scalarized with loop reversal.
  - If it contains a mixture of < and >, it cannot be scalarized by simple means.
    - Loop skewing could work

General Multidimensional Scalarization

- Once a loop has been selected for scalarization, the dependences carried by that loop, any dependence whose direction vector does not contain a = in the position corresponding to the selected loop may be eliminated from further consideration.
- In our example, if we move the second column to the outside, we get:

$$\begin{pmatrix} > & = & < \\ < & > & = \end{pmatrix} \Rightarrow \begin{pmatrix} = & > & < \\ > & < & = \end{pmatrix}$$

- Scalarization in this way will reduce the matrix to:

$$\begin{pmatrix} > & < \end{pmatrix}$$

Scalarization Example

```
DO J = 2, N-1
   A(2:N-1,J) = A(1:N-2,J) + A(3:N,J) + &
   A(2:N-1,J-1) + A(2:N-1,J+1)/4.
ENDDO
```

- Loop carried true dependence, antidependence
- Naive compiler could generate:

```
DO J = 2, N-1
  DO i = 2, N-1
    T(i-1) = (A(i-1,J) + A(i+1,J) + A(i,J-1) + A(i,J+1) )/4
  ENDDO
  DO i = 2, N-1
    A(i,J) = T(i-1)
  ENDDO
ENDDO
```

- 2 × $(N-2)^2$ accesses to memory due to array T

Scalarization Example

- However, can use input prefetching to get:

```
DO J = 2, N-1
  tA0 = A(1, J)
  DO i = 2, N-2
    tA1 = (tA0+A(i+1,J)+A(i,J-1)+A(i,J+1))/4
    tA0 = A(i-1, J)
    A(i,J) = tA1
  ENDDO
  tA1 = (tA0+A(N,J)+A(N-1,J-1)+A(N-1,J+1))/4
  A(N-1,J) = tA1
ENDDO
```

- If temporaries are allocated to registers, no more memory accesses than original Fortran 90 program

Post Scalarization Issues

- Issues due to scalarization:
  - Generates many individual loops
  - These loops carry no dependences. So reuse of quantities in registers is not common

- Solution: Use loop interchange, loop fusion, unroll-and-jam, and scalar replacement

Announcements

- Schedule for take-home final exam
  - Scope of exam: Chapters 1 - 11
  - Will be handed at last class, on April 21st
  - Due by 5pm on April 29th
- Lecture by Prof. Sadayappan on Friday, April 24th
  - "A polyhedral loop transformation framework for automatic parallelization and data locality optimization"
  - 11am – 12noon, DH 1049
- Today is the last lecture!
- Additional references for today's lecture
  - Scalarization using loop alignment and loop skewing, Yuan Zhao, Ken Kennedy. The Journal of Supercomputing, Volume 31, Issue 1 (January 2005).
  - Optimized Execution of Fortran 90 Array Language on Symmetric Shared -Memory Multiprocessors. Vivek Sarkar. Eleventh Workshop on Languages and Compilers for Parallel Computing (LCPC), August 1998.