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Efficient Optimization of Memory Accesses in Parallel Programs

by

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ABSTRACT

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The power, frequency, and memory wall problems have caused a major shift in mainstream computing by introducing processors that contain multiple low power cores. As multi-core processors are becoming ubiquitous, software trends in both parallel programming languages and dynamic compilation have added new challenges to program compilation for multi-core processors. This thesis proposes a combination of high-level and low-level compiler optimizations to address these challenges.

The high-level optimizations introduced in this thesis include new approaches to May-Happen-in-Parallel analysis and Side-Effect analysis for parallel programs and a novel parallelism-aware Scalar Replacement for Load Elimination transformation. A new Isolation Consistency (IC) memory model is described that permits several scalar replacement transformation opportunities compared to many existing memory models.

The low-level optimizations include a novel approach to register allocation that retains the compile time and space efficiency of Linear Scan, while delivering runtime performance superior to both Linear Scan and Graph Coloring. The allocation phase is modeled as an optimization problem on a Bipartite Liveness Graph (BLG) data structure. The assignment phase focuses on reducing the number of spill instructions by using register-to-register move and exchange instructions wherever possible.
Experimental evaluations of our scalar replacement for load elimination transformation in the Jikes RVM dynamic compiler show decreases in dynamic counts for getfield operations of up to 99.99%, and performance improvements of up to 1.76× on 1 core, and 1.39× on 16 cores, when compared with the load elimination algorithm available in Jikes RVM. A prototype implementation of our BLG register allocator in Jikes RVM demonstrates runtime performance improvements of up to 3.52× relative to Linear Scan on an x86 processor. When compared to Graph Coloring register allocator in the GCC compiler framework, our allocator resulted in an execution time improvement of up to 5.8%, with an average improvement of 2.3% on a POWER5 processor.

With the experimental evaluations combined with the foundations presented in this thesis, we believe that the proposed high-level and low-level optimizations are useful in addressing some of the new challenges emerging in the optimization of parallel programs for multi-core architectures.
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Chapter 1

Introduction

The computer industry is at a major inflection point in its hardware roadmap due to the end of a decades-long trend of exponentially increasing clock frequencies. Unlike previous generations of hardware evolution, the shift towards multicore and manycore computing will have a profound impact on software — not only will future applications need to be deployed with sufficient parallelism for manycore processors, but the parallelism must also be energy-efficient. For decades, caches have helped bridge the memory wall for programs with high spatial and temporal locality. Unfortunately, caches come with an energy cost that limits their use as on-chip memory in future manycore processors. It is therefore desirable for programs to use more energy-efficient storage structures such as registers and local memories (scratchpads) instead of caches, as far as possible.

Energy-efficient storage structures offer lower latencies and are faster to access. However, they are smaller in size and number due to architectural complications involved in their design. For example, the Intel x86 architecture offers only 8 fixed registers for integer valued data items. A compiler that converts a higher-level program into an optimized machine level instruction sequence performs several optimizations in order to improve the execution performance of a program. One such optimization that focuses on improving memory accesses in the program is memory-access optimization. The goal of a memory-access optimization is to promote frequently executed data values from memory (with higher latency of access) to more efficient structures like registers and local memories in order to take advantage of their lower latencies and faster accesses. Several compiler optimizations have been proposed in the literature.
that address optimization for memory accesses such as scalar replacement [33, 34], load elimination [52, 75, 102], redundant memory operation analysis [45], and register promotion [83]. The compiler community has studied these techniques extensively over three decades and have shown benefits of performing them inside a compiler.

Gordon Moore predicted in 1965 that the number of transistors on a machine would double every eighteen months. This trend has been observed for a long period of time. In the past, this increase in the number of transistors (and decrease in transistor sizes) has led to a corresponding increase in clock frequency. However, recently, the \textit{power wall} has caused a trend shift from serial to parallel computing by introducing more and more low power cores in a processor. All hardware vendors now ship systems with multi-core processors. The performance gain by the introduction of multi-core processors is strongly dependent on the software algorithms and their implementation. For example, in order to achieve speedup on a quad-core machine, it is necessary to exploit the four cores in software. Hence, new programming languages like MPI [109], UPC [51], OpenMP [95], Cilk [18], X10 [38], and Titanium [63] have been developed to expose the available parallelism on a multi-core processor to the application programmer. Along with new programming languages for parallelism, there is a need for new compiler techniques to analyze the parallel constructs of the language and optimize programs keeping parallelism in mind. Currently, most compilers make conservative assumptions for parallel constructs and hence, miss several opportunities for code optimization including memory-access optimization. For example, the Jikes RVM [66] prevents code motion around parallel constructs.

Parallelism poses another challenge to compiler transformations in the form of \textit{interferences} among shared data accesses of multiple cores. The legality of a compiler transformation in the presence of interferences is typically dictated by the underlying memory model. A memory model determines the set of possible observable behaviors of the program. A compiler transformation is said to be correct if the set of possible observable behaviors of the transformed program is a subset of the possible observable
behaviors of the original programs. All memory models have the same semantics for a data-race-free program. However, without prior knowledge, a compiler does not know if the input program is data-race free or not. Hence, it is desirable to define a memory model for parallel programs that is both programmer and compiler friendly and at the same time allows for more opportunities for compiler optimizations, which is critical for program performance. Note that memory-access compiler optimizations are often viewed as a variant of code reordering transformations, because they can result in a reordering of load and store instructions, and hence, are correct to perform under a given memory model.

In conjunction with the hardware trend shift from serial computing to parallel computing, in dynamic compilation, program execution and compilation can be interleaved. Dynamic compilation is also referred to as Just-In-Time (JIT) compilation and runtime compilation. For example, the platform-independent bytecodes of a Java program are usually compiled and executed by a virtual machine that invokes a JIT compiler. A dynamic compiler shares the common goal of producing optimized code with that of an offline/static compiler. However, a key difference is that in a dynamic compiler the compilation time overhead adds to the runtime performance. The optimizations performed in a dynamic compiler must strike a balance between performing deeper analysis (with higher complexity) and runtime benefits achieved from them. In practice, the optimizations must be performed as close to linear time and space as possible. For example, the Linear Scan register allocation algorithm proposed by Poletto and Sarkar [100] is performed by many Java virtual machine JIT compilers due to its linear time and space complexity instead of the Graph Coloring register allocation approach [25, 28, 35] used in static compilation. However, Linear Scan is known to lag in runtime performance compared to Graph Coloring approaches.

**Thesis Statement:** Recent trends in hardware with multi-core processors as well as software with parallel languages and dynamic compilation have added new challenges to the Memory Wall problem. Our thesis is that a combination of high-level
and low-level compiler optimizations can be effective in addressing these challenges. The high-level optimizations introduced in this thesis include new approaches to *May-Happen-in-Parallel* analysis, *Side-Effect* analysis, and *Scalar Replacement for Load Elimination* transformation for explicitly parallel programs. The low-level dynamic optimizations include a *Space-efficient* register allocation algorithm that incurs an order-of-magnitude smaller compile-time and space overhead than Graph Coloring, while delivering run-time performance that matches or surpasses that of Graph Coloring.

1.1 Research Contributions

This dissertation highlights the challenges in memory-access optimization for parallel programs, using X10 as an example parallel programming language. The X10 v1.5 language \cite{38} builds on a subset of Java language constructs and adds new constructs like async, finish, atomic, places, region, distribution, and distributed arrays for supporting fine-grained locality, parallelism and synchronization. Since, version 1.7, X10 has adopted a Scala-like syntax for source code and has introduced new advances in the type system relative to Java. The Habanero-Java (HJ) programming language that is being developed in the Habanero Multicore Software Research project at Rice University focuses on addressing the implementation challenges for the core constructs of X10 v1.5 language on multi-core processors, with programming model extensions as needed (such as phasers and isolated blocks). A significant part of the research results presented in this thesis were obtained for HJ programs.

The dissertation makes the following contributions:

1. a novel *May-Happen-in-Parallel* (MHP) algorithm for HJ programs that identifies pairs of execution instances of statements that may execute in parallel. Compared to past work for other concurrent languages like Java and Ada, we introduce a more precise definition of the MHP by adding *condition vectors* that distinguishes execution instances of statements for which the MHP holds,
instead of just returning a single true/false value for all pairs of executing instances. The availability of basic concurrency control constructs such as async, finish, isolated and places in HJ enables the use of more efficient and precise analysis algorithms based on simple path traversals in a Program Structure Tree.

2. a side-effect analysis for the core parallel constructs of HJ. The side-effect analysis is designed for dynamically compiling HJ programs and hence, is compile-time efficient.

3. a novel parallelism-aware scalar replacement transformation for memory load elimination. The legality of the transformation is established by a new Isolation Consistency (IC) memory model. Like many relaxed memory models, the IC memory model provides sequentially consistent behavior for data-race-free programs. At the same time, IC allows many compiler transformations via weak-atomicity for programs with data-races.

4. a space-efficient register allocation algorithm that bridges the performance gap between Linear Scan and Graph Coloring register allocation algorithms while maintaining the compile-time efficiency of Linear Scan. We model the allocation phase of a register allocation algorithm as an optimization problem on Bipartite Liveness Graphs (BLG’s), a new data structure introduced in this thesis. The assignment phase focuses on reducing the number of spill instructions by using register-to-register move and exchange instructions wherever possible to maximize the use of registers. The register assignment that includes register-to-register moves, exchanges, coalescing as well as register class constraints is modeled as another optimization problem, and we provide a heuristic solution to this problem as well.

5. an enhanced bitwidth-aware register allocation algorithm that packs several narrow-width data items onto the same physical register to reduce register pressure of
the program. We present an enhanced bitwidth analysis that performs more detailed scalar analysis and array analysis than past work. We describe an enhanced packing algorithm that includes more accurate packing and performs less conservative (more aggressive) coalescing than past work.

1.2 Thesis Organization

- Chapter 2 introduces necessary backgrounds, definitions and notations used in the thesis. The overall code optimization framework used in the thesis is also described in this chapter.

- Chapter 3 describes the May-Happens-in-Parallel algorithm for HJ programs.

- Chapter 4 presents the Side-Effect Analysis for parallel constructs and function calls.

- Chapter 5 describes the Isolation Consistency (IC) memory model and scalar replacement for load elimination transformation for parallel programs.

- Chapter 6 describes the space-efficient register allocation algorithm and compares it with the graph coloring register allocation.

- Chapter 7 presents our enhancements to bitwidth-aware register allocation algorithm.

- Chapter 8 presents our experimental results.

- Chapter 9 concludes the thesis with a summary and future directions.
Chapter 2

Background

In this chapter, we introduce notations and terminologies used in the rest of the dissertation. First, we describe some basic compiler terminologies. Next, we describe the HJ parallel programming language. Next, we present our overall code optimization framework for parallel programs. Finally, we describe the background, foundations and notations for each of the analyses and optimizations described in our code optimization framework.

2.1 Basics of a Compiler

A compiler (static or dynamic) typically consists of two components: a front-end and a back-end. In the front-end, the input program is parsed, represented as an intermediate representation (IR), and transformed. Typical transformations performed in the front-end are deadcode elimination, constant propagation, copy propagation, and inlining. After the front-end pass is complete, the back-end component performs additional transformations that are specific to the target architecture. Typical transformations performed in the back-end are register allocation, instruction scheduling, and instruction selection. Sometimes compilers add a middle-end that consists of the transformations of the front-end.

An IR captures the compiler’s knowledge of the input program. It consists of a set of instructions that correspond to the original input program.

Definition 2.1.1 An instruction defines an operation that possibly reads some variables and possibly writes some other variables. The variables that are read at an
a = ...

b = true

for (i=2; i<a-1; i++) {
    c = a % i
    if (c == 0) {
        b = false
        break
    }
}

Figure 2.1: An example control flow graph. The code snippet is shown on the left. The corresponding control flow graph is shown in the right. Note that, the special basic blocks START and END are added to demarcate entry and exit to the procedure.

An IR can be represented in various ways. Some dominant IR representations are: 1) a linear IR consisting of a linear ordering of instructions, e.g., Java bytecode; 2) a structural IR consisting of graphical representations of instructions, e.g., abstract syntax trees; 3) linear+structural IR consisting of a combination of graphical representation and linear ordering, e.g., control flow graph (CFG). A CFG-based representation is widely used for compiler analyses and transformations.

Definition 2.1.2 A control flow graph is a graph, $G = (V, E)$, where $V$ consists of basic blocks and $E$ consists of possible execution paths. A basic block is a maximal sequence of instructions where the execution enters at the first instruction and exits at the last instruction of the sequence, i.e., there exists no intermediate instruction in a basic block where an execution can enter or exit. Two special basic blocks START and END are added to a CFG to indicate the unique entry and unique exit of a procedure.
Consider the example program shown in Figure 2.1. The example program computes if \( a \) is a prime number or not. The control flow graph (CFG) is shown on the right. It consists of seven basic blocks, i.e., B0–6, including two special entry and exit basic blocks B0 and B6, respectively. The basic block B3 consists of two instructions 
\[ c = a \% i \text{ and } c == 0. \]
For instruction \( c = a \% i \), variables \( a \) and \( i \) are used whereas \( c \) is defined.

Often it is useful to define \( \text{dom} \), \( \text{idom} \), and \( \text{postdom} \) relationships between two nodes of a CFG.

**Definition 2.1.3** Given a CFG, a node \( x \) is said to dominate (dom) another node \( y \) if every path from START to \( y \) passes through \( x \). Similarly, node \( x \) is said to postdominate (postdom) node \( y \) if every path from \( y \) to EXIT passes through \( x \). A node \( x \) is the immediate dominator (idom) of another node \( y \) if \( x \) dominates \( y \) and there is no intervening node \( p \) such that \( x \) dom \( p \) and \( p \) dom \( y \). The idom relation forms a dominator tree.

For precision, it is often necessary to represent information in between two instructions. For example, the liveness of a variable needs to be defined at a program point rather than at an instruction level.

**Definition 2.1.4** A program point is a point between two consecutive instructions.

**Definition 2.1.5** A variable \( v \) is live at a program point \( p \) if \( \exists \) a path in the CFG (indicating a possible execution) from \( p \) to some use of \( v \) along which \( v \) is not defined again. As we will discuss in Definition 2.7.5, sometimes it is desired to split a program point into two sub-program points.

A popular intermediate representation used in the literature is static single assignment (SSA) form [47]. In SSA form representation, each variable is defined in exactly one place in the code. New \( \phi \) instructions are inserted in the CFG to ensure that each use of a variable sees exactly one definition. An IR is converted into SSA
form using two simple steps: (1) \(\phi\)-insertion phase: \(\phi\) statements are inserted at the iterated dominance frontiers of assignment statements [48]; (2) renaming phase: the renaming phase assigns unique names using version ids to each variable definition. Several efficient transformations have been proposed in literature that exploit the single-assignment property of SSA form such as sparse-conditional constant propagation [122], strength reduction [46], partial redundancy elimination [41] and SSA based register allocation [29, 59]. We will describe SSA based register allocation in Section 2.7.2.3.

A common compiler transformation is to find redundant expressions in a program. An expression \(a + b\) is said to be redundant at a program point \(p\) if it has already been computed in every path starting from the START block to \(p\), and no intervening operation kills either \(a\) or \(b\). If the compiler can find such redundant expressions, it can save the value in a scalar variable at the previous computation and replace any subsequent computations with the scalar variable. The classic approach to accomplish this is to use Value Numbering [6]. Value numbering assigns distinct numbers to each value computed during run time. Two expressions, \(e_1\) and \(e_2\), have same value number iff they always compute the same value. We denote the value number of an expression \(e\) as \(V(e)\). If the value numbers of two expressions are same, then they are redundant.

An ordering-based compiler transformation such as redundant expression elimination is said to be correct if it does not violate any dependences. A control dependence arises from the control flow in the program, whereas a data dependence arises from the flow of values between statements in the program.

**Definition 2.1.6** The following types of data dependences exist:

1. Statements \(S_1\) and \(S_2\) are said to have a flow dependence between them (denoted as \(S_1\delta_f S_2\)) if \(S_2\) uses the value written at \(S_1\).

2. Statements \(S_1\) and \(S_2\) are said to have an anti dependence between them (denoted as \(S_1\delta^{-1}_f S_2\)) if \(S_1\) uses a value from a location to which \(S_2\) writes.
3. Statements $S_1$ and $S_2$ are said to have an output dependence between them (denoted as $S_1\delta^o S_2$) if both $S_1$ and $S_2$ write to the same location.

4. Statements $S_1$ and $S_2$ are said to have an input dependence between them (denoted as $S_1\delta^i S_2$) if both $S_1$ and $S_2$ use a value from the same location.

A succinct way of capturing dependences for statements inside a loop is to use distance and direction vectors.

**Definition 2.1.7** Given a dependence from statement $S_1$ on iteration $i$ to statement $S_2$ on iteration $j$ of a common loop nest $l$, the direction vector $\mathcal{D}(i,j)$ is defined as a vector of length $l$ such that,

$$
\mathcal{D}(i,j)_k = \begin{cases} 
< & \text{if } j_k - i_k > 0 \\
= & \text{if } j_k - i_k = 0 \\
> & \text{if } j_k - i_k < 0 
\end{cases} \quad (2.1)
$$

Various dependences between the statements in a program are represented using a program dependence graph (PDG). PDG's are used as the foundation for many compiler reordering transformations such as *vectorization*, *scalar replacement*, and *scheduling*.

### 2.2 The HJ Parallel Programming Language

The HJ programming language offers several constructs to improve programmability in high-performance computing for parallel systems that includes multi-core processors, symmetric shared-memory multiprocessors (SMPs), commodity clusters, high-end supercomputers like BlueGene [1], and even embedded processors like Cell [99]. The key features of HJ include:

- Lightweight *activities* embodied in `async`, `future`, `foreach`, and `ateach` constructs which subsume communication and multithreading operations.
• A **finish** construct for termination detection and rooted exception handling of descendant activities.

• Support for lock-free synchronization with **isolated** blocks.

• Explicit reification of locality in the form of **places**, with support for a *partitioned global address space* (PGAS) across places.

• Support for collective and point-to-point communication using **phaser** constructs.

**HJ** uses a serial subset of the **Java** v1.4 language as its foundation, but replaces the **Java** language’s current support for concurrency by new constructs that are motivated by high-productivity high-performance parallel programming. For further details, the reader is referred to “An overview of X10 v1.5” [38]. The scope of this dissertation focuses on four core constructs: **async**, **finish**, **isolated**, and **places**. Extensions for the **foreach**, **ateach**, and **future** constructs follow naturally from the approach described in this thesis, and have been omitted for simplicity. An important safety result in **HJ** is that any program written with **async**, **finish**, and **isolated** can never deadlock.

### 2.2.1 Single Place HJ Language Constructs

In a single-place **HJ** program, all activities execute within the same logical **place** and have uniform read and write access to all shared data, as in multithreaded **Java** programs where all threads operate on a single shared heap.

**async** ⟨**stmt**⟩: Async is the **HJ** construct for creating or forking a new asynchronous activity. The statement, **async** ⟨**stmt**⟩, causes the parent activity to create a new child activity to execute ⟨**stmt**⟩. Execution of the async statement returns immediately *i.e.*, the parent activity can proceed immediately to the statement following the async.
Consider the following HJ code example of an async construct. The goal of this example is to use two activities to compute in parallel the sums of $f(i)$ for odd and even values of $i$ in the range $1 \ldots n$. This is accomplished by having the main program activity use the `async for-i` statement to create a child activity to execute the `for-i` loop and print `oddSum`, while the parent (main program) activity proceeds in parallel to execute the `for-j` loop and print `evenSum`.

```java
public static void main(String[] args) {
    final int n = 10000;
    async { // Compute oddSum in child activity
        double oddSum = 0;
        for (int i=1 ; i<=n ; i+=2) oddSum += f(i);
        System.out.println("oddSum = " + oddSum);
    }
    // Compute evenSum in parent activity
    double evenSum = 0;
    for (int j=2 ; j<=n ; j+=2) evenSum += f(j);
    System.out.println("evenSum = " + evenSum);
}
```

HJ permits the use of async to create multiple nested activities in-line in a single method, unlike Java threads where the body of the thread must be specified out-of-line in a separate `Runnable` class. Also, note that the child activity uses the value of local variable $n$ from the parent activity, without the programmer having to pass it explicitly as a parameter. HJ provides this sharing of local variables for convenience, but requires that any local variables in the parent activity that are accessed by a child activity must be defined as final (constant) in the parent activity so as to ensure that no data races can occur on local variables.

**finish (stmt):** The HJ statement, `finish (stmt)`, causes the parent activity to execute `(stmt)` and then wait till all sub-activities created within `(stmt)` have terminated.

---

1 Function $f$ is assumed to be a pure function of its input $i$, and to involve sufficient computation granularity to ensure that the async overhead is insignificant in these examples.
globally. There is an implicit finish statement surrounding the main program in an HJ application. If async is viewed as a fork construct, then finish can be viewed as a join construct. However, the async-finish model is more general than the fork-join model [38].

HJ distinguishes between local termination and global termination of a statement. The execution of a statement by an activity is said to terminate locally when the activity has completed all the computation related to that statement. For example, the creation of an asynchronous activity terminates locally when the activity has been created. A statement is said to terminate globally when it has terminated locally and all activities that it may have spawned (if any) have, recursively, terminated globally.

Consider a variant of the previous example in which the main program waits for its child activity to finish so that it can print the total sum obtained by adding oddSum and evenSum:

```java
public static void main(String[] args) {
    final int n = 10000;
    final BoxedDouble oddSum = new BoxedDouble();
    double evenSum = 0;
    finish {
        async {
            // Compute oddSum in child activity
            for (int i=1; i<=n; i+=2)
                oddSum.val += f(i);
        }
        // Compute evenSum in parent activity
        for (int i=2; i<=n; i+=2)
            evenSum += f(i);
    } // finish
    System.out.println("Sum = " +
                      (oddSum.val + evenSum));
} // main()
```

The finish statement guarantees that the child activity terminates before the print statement is executed. Note that the result of the child activity is communicated to
the parent in a shared object, *oddSum*, since HJ does not permit a child activity to update a local variable in its parent activity.

In addition to waiting for global termination, the finish statement plays an important role with regard to exception semantics. An HJ activity may terminate normally or abruptly. A statement terminates abruptly when it throws an exception that is not handled within its scope; otherwise it terminates normally. While it may seem that an obvious solution is to propagate exceptions from a child activity to a parent activity, doing so is problematic when the parent activity terminates prior to the child activity. Since we want to permit child activities to outlive parent activities in HJ, the finish construct is a more natural collection point for exceptions thrown by descendant activities. HJ requires that if statement S or an activity spawned by S terminates abruptly, and all activities spawned by S terminate, then finish S terminates abruptly and throws a single exception formed from the collection of all exceptions thrown by S or its descendant activities. Exceptions thrown by this statement are caught by the runtime system and result in an error message printed on the console. This provides more robust exception handling support for multithreaded programs compared to the Java model in which an exception is simply propagated from a thread to the top-level console instead of propagation to an appropriate handler in an ancestor thread.

**isolated (stmt)**, **isolated (method-decl)**: An isolated block is executed by an activity as if in a single step during which all other concurrent activities within the same place are suspended. The isolated construct is our renaming of X10's *atomic* construct. As stated in [38], an atomic block in X10 is intended to be “executed by an activity as if in a single step during which all other concurrent activities in the same place are suspended”. This definition implies a *strong atomicity* semantics for the atomic construct. However, all X10 implementations that we are aware of (including the one used in this paper) use a single lock per place to enforce mutual exclusion of atomic blocks. This approach supports weak atomicity, since no mutual exclusion guarantees are enforced between computations within and outside an atomic block.
As advocated in [73], we use the isolated keyword instead of atomic to make explicit the fact that the construct supports weak isolation rather than strong atomicity. An isolated block may include method calls, conditionals, and other forms of sequential control flow. Parallel constructs such as async and finish are not permitted in an isolated block. Isolated blocks may be nested and the isolated modifier on method definitions are permitted as a shorthand for enclosing the body of the method in an isolated block. The isolated construct is semantically equivalent to X10’s atomic construct.

Consider the following example in which each iteration of a loop executes in parallel and accumulates its result in a shared location, Sum.val:

```java
public static void main(String[] args) {
    final int n = 10000;
    final BoxedDouble Sum = new BoxedDouble();
    finish
    for (int i = 1; i <= n; i++)
        async { // Compute oddSum in child activity
            double result = f(i);
            isolated Sum.val += result;
        }
    System.out.println("Sum = " + Sum.val);
} // main()
```

In the previous example, the finish construct was used to ensure that shared location oddSum.val was computed by the child activity before it was read by the parent activity. In this example, the shared location Sum.val can be updated in parallel by multiple activities, and the isolated block is used to ensure that the read-modify-write operations are performed in a consistent manner.

### 2.2.2 Multi-Place Programming in HJ

Current programming models use two separate levels of abstraction for shared-memory thread-level parallelism (e.g., Java threads, OpenMP, pthreads) and distributed-memory
communication (e.g., Java messaging, RMI, MPI, UPC) resulting in significant complexity when trying to combine the two. The three core HJ constructs introduced earlier can be extended to multiple places, as illustrated in Figure 2.2. A place is a collection of resident (non-migrating) mutable data objects and the activities that operate on the data. Every HJ activity runs in a place; the activity may obtain a reference to this place by evaluating the constant $here$.

HJ takes the conservative decision that the number of places ($\text{MAX\_PLACES}$) is fixed at the time an HJ program is launched. Thus, there is no construct to create new places. This is consistent with current programming models, such as MPI, UPC, and OpenMP, that require the number of processes to be specified when an application is launched.

Places are virtual — the mapping of places to physical locations is performed by a deployment step that is separate from the HJ program [37, 125]. Though objects and activities do not migrate across places in an HJ program, an HJ deployment is free to migrate places across physical locations based on affinity and load balance considerations. While an activity executes at the same place throughout its lifetime, it may dynamically spawn activities in remote places.

HJ supports a partitioned global address space (PGAS) that is partitioned across places. Each mutable location and each activity is associated with exactly one place,
and places do not overlap. A scalar object in HJ is allocated completely at a single place. In contrast, the elements of an array, may be distributed across multiple places. We now discuss how the async and finish constructs discussed earlier in a single-place context, extend directly to the multi-place case.

2.2.2.1 Remote Asyncs

The statement, \texttt{async (place-expr) (stmt)}, causes the parent activity to create a new child activity to execute \texttt{(stmt)} at the place designated by \texttt{(place-expr)}. The async is local if the destination place is same as the place where the parent is executing, and remote if the destination is different. Local async’s are like lightweight threads, as discussed earlier in the single-place scenario. A remote async can be viewed as an active message, since it involves communication of input values as well as remote execution of the computation specified by \texttt{(stmt)}. The semantics of the HJ finish operator is identical for local and remote async’s viz., to ensure global termination of all asyncs created in the scope of the finish.

HJ supports a Globally Asynchronous Locally Synchronous (GALS) semantics for reads/writes to mutable locations. We say that a mutable variable is local for an activity if it is located in the same place as the activity; otherwise it is remote. An activity may read/write only local variables (this is called the Locality Rule, and it may do so synchronously. Any attempt by an activity to read/write a remote mutable variable results in a \texttt{BadPlaceException}. As mentioned earlier, isolated blocks are used to ensure atomicity of groups of read/write operations among multiple activities located in the same place. However, an activity may read/write remote variables only by spawning activities at their place. Thus a place serves as a coherence boundary in which all writes to the same datum are observed in the same order by all activities in the same place. In contrast, inter-place data accesses to remote variables have weak ordering semantics. The programmer may explicitly enforce stronger guarantees by using sequencing constructs such as finish.
2.3 Code Optimization Framework

Figure 2.3 depicts the overall compiler analysis and optimization framework assumed in this thesis. The overall compiler infrastructure consists of a static optimization component and a dynamic optimization component. The input parallel programming language considered is HJ, however the techniques described in this thesis can be applied to other parallel programming languages as well. The input parallel program is parsed by the front-end of the static optimizer and is translated into a parallel intermediate representation (PIR). PIR is an intermediate representation in which the parallel constructs such as async, and finish are explicitly represented in a hierarchical manner. Like other intermediate representations, several analyses and transformations are performed at the PIR level.

One of the key foundations of analyzing parallel programs is to determine statement instances that may execute in parallel with each other. This is known as May-Happen-in-Parallel (MHP) analysis. The MHP information can be used in other compiler analysis and optimization of parallel programs e.g., the constant
propagation described in [77] using concurrent-SSA form representation needs to know the interfering data values and these can be determined using the MHP analysis. In this thesis, we present a precise definition of MHP using condition vectors that identify execution instances of statements for which the MHP holds, instead of just returning a single true/false value for all pairs of executing statement instances. Based on this definition, we present an efficient algorithm for computing MHP information for HJ parallel programs. Compared to the MHP analysis of other languages, our approach [2] is based on a simple walk over the program structure tree which is an abstraction of the abstract syntax tree. The MHP analysis analyzes async, finish, isolated, and places constructs of HJ.

Traditionally, procedure calls hinder the precision of compiler transformations in the absence of interprocedural analysis. Side-effect analysis is an interprocedural analysis that summarizes the modified and referenced data items for each procedure. For parallel programs, the parallel constructs themselves embed inherent side-effects. To enable PIR transformations across procedure boundaries and parallel constructs, we present a unified side-effect analysis in this thesis that summarizes side-effects of procedure calls in the presence of parallel constructs. The side-effect analysis [12] uses a heap-array representation for faster side-effect computation. It computes side-effects for unique features of HJ programs like global termination using finish and escaping-async. The side-effects can be used by other code reordering transformations such as code motion.

After PIR analysis is performed, several PIR transformations are performed. One such PIR transformation is scalar replacement for load elimination that replaces memory load operations of object references by scalar variables, thereby enabling the back-end to generate register accesses instead of load instructions. In this thesis, we describe a parallelism-aware scalar replacement transformation for eliminating memory load operations. The legality of such a transformation in parallel programs is strongly dependent on the underlying memory model supported by the programming
language. We describe an Isolation Consistency (IC) memory model [12] for HJ parallel programs. IC is a weak memory model that allows more opportunities for code reordering than other existing weaker memory models described in past work [21, 53, 64, 84]. After transformations are applied at the PIR level, platform-independent bytecode is produced for the input HJ parallel program.

The other component in our optimization framework is the dynamic optimizer. The bytecodes produced in the static optimizer are subsequently processed within the dynamic optimizer framework. Additional higher level and low-level optimizations are performed at the bytecode level within the dynamic optimizer framework. A key low-level optimization is *register allocation*. This thesis makes a contribution to register allocation optimization by providing an *space-efficient* register allocation algorithm [105] that is compile-time efficient and produces comparable executable code quality as a Graph Coloring based register allocation. The space-efficient register allocation builds on the notion of intervals with holes used in Linear Scan register allocation.

One approach to moderate register pressure in a program is to pack several narrow width data variables into the same physical register. A register allocation algorithm that is aware of the bitwidth information and performs such a packing is known as a *Bitwidth-aware register allocation* algorithm. This thesis makes contributions to bitwidth-aware register allocation by proposing several enhancements to the computation of bitwidth information and variable packing heuristic [11].

Finally, the bytecodes are converted to the machine code within the dynamic optimizer and executed on the target machine.

### 2.4 May-Happen-in-Parallel (*MHP*) Analysis

Parallel programming languages offer many high level parallel constructs to create, synchronize, communicate, and join parallel tasks. All these parallel constructs indicate the relative progress and interactions of parallel tasks during execution.
Further, the interactions among parallel tasks indicate their possible ordering of execution. For example, the end of a `finish` scope in HJ ensures the completion of any parallel task, *i.e.*, `async`, created within its scope. This implies any `async` created after this `finish` scope will never synchronize/communicate with the `asyncs` created with the `finish` scope.

Knowledge of the possible ordering of parallel tasks has a variety of uses in the compilation and debugging of parallel programs. These uses include program debugging tools, data-flow analysis, detecting synchronization anomalies like data-races and deadlocks [32]. The possible ordering among tasks leads to a problem of determining the actions that can occur in parallel. This is known in the literature by several different terms: *Concurrency* analysis [50, 86], *B4* analysis [32], and *May-Happen-in-Parallel* analysis [90, 92]. In this thesis, we will use the *May-Happen-in-Parallel (MHP)* term. Note that, *MHP* analysis determines actions that *may* happen during execution, *i.e.*, it is *may* information rather than *must* information, hence any query for *MHP* information can conservatively return *true*.

**Definition 2.4.1** *May-Happen-in-Parallel (MHP) analysis statically determines if it is possible for execution instances of two statements (or the same statement) to execute in parallel.*

The complexity of *MHP* analysis is highly dependent on the underlying parallel constructs supported by the programming language. For example, let us consider the asynchronous parallel loop constructs consisting of `parallel DO`, `parallel case`, `POST`, and `WAIT` constructs (described in [68]). Callahan and Sublok [32] have shown that for a program using the above constructs and without any loop construct, the *MHP* computation is NP-hard. Similar complexity results have been proved for *Ada*’s rendezvous model of synchronization [118], which is similar to *Java*’s wait-notify model of synchronization.

In general, it is safe for a compiler to compute a conservative approximation of *MHP* information. For example, Callahan and Sublok [32] proposed a data flow
algorithm to compute a conservative approximation of the sets of statements that must be executed before a given statement (B₄ analysis). Most recently, Naumovich et al. [92] proposed a similar data-flow based algorithm for concurrent Java programs.

### 2.4.1 MHP Analysis for Java Programs

Java offers parallelism in the form of explicit creation, synchronization, and termination of threads. Threads can be created using `start()` method call. Similarly, threads can be terminated using `join()`, which is a blocking method call that blocks the parent thread until the child thread terminates. Interaction among threads can also occur via synchronized blocks and methods that allow exclusive access to a thread. Monitors are represented at a higher level using synchronized blocks and are implemented using locks. Execution inside monitor sections can be interrupted using low-level synchronization primitives such as `wait`, `notify`, and `notifyAll`.

As discussed in the previous section, MHP analysis of Java programs is NP-hard. A conservative approximation of MHP analysis for Java programs is provided by Naumovich et al. [92]. Their approach is based on a data-flow analysis framework over an interprocedural Parallel Execution Graph (PEG). Below, we summarize their data flow analysis algorithm.

**Definition 2.4.2** A Parallel Execution Graph (PEG), $G = \langle N, E \rangle$, where $N$ consists of the set of nodes and $E = E_{control} \cup E_{thread} \cup E_{sync}$. $E_{control}$ consists of the interprocedural control flow edges. $E_{thread}$ consists of the thread creation edges. $E_{sync}$ consists of the synchronization edges.

Let $O$ denote the set of objects in the program and $T$ denote the set of threads in the program. $N_t$ comprises of the set of nodes belonging to a thread $t \in T$. $M(n)$ denotes the MHP information for a $n \in N$, i.e., the set of nodes that may execute in parallel with $n \in N$. Further, each node $n \in N$ has an associated node type, i.e., $C(n) = \{FORK, START, END, JOIN, LOCK, UNLOCK, WAIT, NOTIFY\}$. $tsucc(n)$
denote the thread creation edge of \( n \), i.e., it comprises of the thread \( \text{START} \) node (for the first \( \text{CFG} \) node of a \text{run} method) corresponding to a thread \( \text{FORK} \) node (for a thread \text{start} node). The edges from \( \text{FORK} \) nodes to \( \text{START} \) nodes constitute \( \mathcal{E}_{\text{thread}} \). \( \text{nsucc}(n) \) denotes all the synchronization successors of a \( \text{NOTIFY} \) node. Note that a \text{notifyAll} construct in \text{Java} is translated into a \( \text{NOTIFY} \) node with multiple successors. These edges constitute \( \mathcal{E}_{\text{notify}} \). \( \mathcal{W}(o) \) stands for the set of \( \text{WAIT} \) nodes corresponding to an object \( o \in \mathcal{O} \). \( \text{lnodes}(o) \) denotes the set of nodes \( n \in \mathcal{N} \) such that \( n \) gets executed under a lock on \( o \in \mathcal{O} \). \( \text{notifies}(n) \) for a \( \text{NOTIFY} \) node \( n \in \mathcal{N} \) consists of the object to which \( n \) notifies, e.g., for a node "\( n.o.\text{notify()} \)”, \( o \in \text{notifies}(n) \). \( \text{thread}(n) \) returns the current thread corresponding to \( n \).

Unlike traditional data-flow analysis at a basic block level, the \( \text{MHP} \) information is computed at a node/statement level. The basic data flow equations for \( \text{GEN} \) and \( \text{KILL} \) for a node \( n \) are defined as follows:

\[
\begin{align*}
\text{GEN}(n) &= \begin{cases} 
  \text{tsucc}(n) & \text{if } C(n) = \text{FORK} \\
  \text{nsucc}(n) & \text{if } C(n) = \text{NOTIFY} \\
  \phi & \text{otherwise}
\end{cases} \\
\text{KILL}(n) &= \begin{cases} 
  \mathcal{N}_t & \text{if } C(n) = \text{JOIN} \land n \text{ joins } t \\
  \text{lnodes}(o) & \text{if } C(n) = \text{LOCK} \land n \text{ locks } o \\
  \text{lnodes}(o) & \text{if } \exists p, p \in \text{npred}(n) \land o \in \text{notifies}(p) \\
  \mathcal{W}(o) & \text{if } C(n) = \text{NOTIFY} \land o \in \text{notifies}(n) \\
  \phi & \text{otherwise}
\end{cases}
\end{align*}
\tag{2.2, 2.3}
\]

New nodes are added to the \( \text{GEN} \) set of a node \( n \) that correspond to thread start and notify nodes (as shown in the Equation 2.2). If a thread joins, it removes all the nodes of the joined thread from the data flow analysis (shown in the first condition of the Equation 2.3). For entering a new monitor section on \( o \), \( \text{KILL} \) removes all the nodes under the same monitor from the data flow equations since monitors provide
exclusive access. Similarly, the statements following a \textit{WAIT} node can not execute in parallel with any other node under the same monitor on \(o\). Also, none of the \textit{WAIT} nodes execute in parallel with any \textit{NOTIFY} node for the same monitor on \(o\). All these conditions are shown in Equation 2.3.

Once the \textit{GEN} and \textit{KILL} information for all the nodes are computed, the \textit{MHP} information is obtained using the following two equations for \(\mathcal{M}\) and \(\text{OUT}\):

\[
\text{OUT}(n) = (\mathcal{M}(n) \cup \text{GEN}(n)) - \text{KILL}(n) \tag{2.4}
\]

\[
\mathcal{M}(n) = \mathcal{M}(n) \cup \begin{cases} \\
\cup_{p \in \text{pred}(n)} \text{OUT}(p) - \mathcal{N}_t & \text{if } C(n) = \text{START} \\
\cup_{p \in \text{npred}(n)} (\text{OUT}(p) \cap \text{OUT}(\text{pred}(n)) \cup \{m\}) & \text{if } \exists p, p \in \text{npred}(n) \\
\cup_{p \in \text{npred}(n)} (\text{OUT}(p) \cap \text{OUT}(\text{pred}(n))) & \text{if } \exists p, p \in \text{npred}(n) \\
\cup_{p \in \text{pred}(n)} \text{OUT}(p) & \text{otherwise}
\end{cases} \tag{2.5}
\]

The \(\mathcal{M}\) computation for \(n\) consists of several conditions as shown in Equation 2.5. The first condition states that if \(n\) is a \textit{START} node for thread \(t\), then all the nodes for \(t\), \textit{i.e.}, \(\mathcal{N}_t\), are removed from the predecessor’s \textit{OUT} set, \textit{i.e.}, nodes of the same thread can not execute in parallel with the first node of the thread. The second condition states the case that two successors of a \textit{NOTIFY} node may execute in parallel with respect to each other. The third condition states that if \(n\) is a successor of a \textit{NOTIFY} node, then the successor of the \textit{NOTIFY} node will propagate \(\mathcal{M}\) information that is coming from both \textit{NOTIFY} node and the \textit{WAIT} node. The fourth condition propagates \textit{MHP} information along normal control flow edges. Finally, \(\text{OUT}\) of a node \(n\) (as shown in Equation 2.4) is computed by adding and removing appropriating
information from $\mathcal{M}$ based on $\text{GEN}$ and $\text{KILL}$.

**Theorem 2.4.3** The data flow equations for $\mathcal{M}$ terminates.

**Proof:** Refer to [92]. □

**Theorem 2.4.4** The worst-case time complexity of computing $\mathcal{M}$ sets for all nodes in the program is $\mathcal{O}(N^3)$ where $N$ denotes the set of nodes in the PEG.

**Proof:** Refer to [92]. □

A practical implementation of the above data flow equations is provided in [81]. Even though the data flow equations are an elegant way of solving the $\text{MHP}$ problem, it has several efficiency and precision problems in the context of Java: 1) the analysis is closely dependent on an interprocedural alias analysis for thread objects, lock objects and virtual method calls; 2) the analysis needs explicit enumeration of runtime threads during compilation time to precisely compute $\mathcal{E}_{\text{sync}}$; 3) the analysis has $\mathcal{O}(N^3)$ complexity. If we closely look at the limitation (1), the interprocedural alias analysis can also benefit from $\text{MHP}$ information by eliminating aliases arising from statements that do not execute in parallel with each other. This causes a cyclic dependency between $\text{MHP}$ analysis and alias analysis. A solution to break the cyclic dependency may require an incremental analysis between the two causing the overall complexity to increase and become less practical to perform. To overcome limitation (2), an abstraction of runtime threads is needed that is aware of runtime threads created within loops and recursion. This is presented in [10]. Since $\text{MHP}$ analysis involves propagating information at parallel construct boundaries, it’s complexity can be reduced by computing $\text{MHP}$ information at multiple levels, e.g., thread-level and node-level. Using this approach, a quadratic $\text{MHP}$ algorithm is presented in [10].

Chapter 3 of this thesis focuses on $\text{MHP}$ analysis for HJ programs. We provide a precise definition of $\text{MHP}$ for statements executed in loops and recursions. Using the high-level constructs of HJ like finish, async, places, and isolated, an efficient
MHP algorithm [2] is described that is linear in complexity and does not involve any interprocedural alias analysis.

2.5 Side-Effect Analysis

Subroutines (also known as methods, functions and procedures) are a key programming tool in today’s programming languages. They offer several software engineering benefits including the reduced cost of development and maintenance. For example, the object-oriented programming in Java consists of two core constructs: objects and methods. Typically a method consists of a set of parameters, a body, and an optional return value. When one procedure (the caller) calls another procedure (the callee), following actions take place in order: 1) binding between formal and actual parameters; 2) execution of the body of the callee; 3) binding of the return value; 4) return of control to the caller after callee executes. The effect of the callee is visible to the caller after the call. Side-Effect analysis is a compiler analysis that determines the effects of a procedure call in an attempt to enhance the opportunities for optimization. For example, an expression inside a loop containing procedure call can only be identified as loop-invariant if we knew the side-effects of the procedure call.

The side effect of a callee consists of the side effect of each statement in the body of the callee. The term “side effect” was introduced by Spillman [113] for PL/I programming language. Later on Banning [9] formalized the notion of side-effects for statements and procedures. We summarize Banning’s side-effect analysis for method calls as follows.

For a statement $s$, there are four common types of side effects:

1. $MOD(s)$ consists of the set of variables whose value may be modified by executing $s$.

2. $REF(s)$ consists of the set of variables whose value may be inspected or refer-
Figure 2.4: Control flow graph structures depicting distinction between four kinds of side effects \textit{MOD, REF, USE, and DEF}

3. \textit{USE}(s) consists of the set of variables whose value \textit{may} be inspected or referenced by executing \textit{s} before being defined by \textit{s} again.

4. \textit{DEF}(s) consists of the set of variables whose value \textit{must} be defined by executing \textit{s}.

The difference between “modified” and “defined” is that “defined” refers to complete overwriting of values where as “modifies” refers to partial overwriting of values like modifying an element of a structure.

Figure 2.4 shows two control flow graph structures \(G_1\) and \(G_2\). The side effects for \(G_1\) and \(G_2\) are as follows:
\[
\begin{align*}
\text{MOD}(G_1) &= \text{MOD}(G_2) = \text{MOD}(S_1) \cup \text{MOD}(S_2) \\
\text{REF}(G_1) &= \text{REF}(G_2) = \text{REF}(S_1) \cup \text{REF}(S_2) \\
\text{USE}(G_1) &= \text{USE}(S_1) \cup (\text{USE}(S_2) - \text{DEF}(S_1)) \\
\text{USE}(G_2) &= \text{USE}(S_1) \cup \text{USE}(S_2) \\
\text{DEF}(G_1) &= \text{DEF}(S_1) \cup \text{DEF}(S_2) \\
\text{DEF}(G_2) &= \text{DEF}(S_1) \land \text{DEF}(S_2)
\end{align*}
\]

As can be seen from the above equations, \textit{MOD} and \textit{REF} are flow-insensitive problems since they only use the union (\(\cup\)) operation. However, \textit{USE} and \textit{REF} are flow-sensitive since they use the meet (\(\land\)) operation. For a statement \(s\) inside procedure \(p\) that invokes another procedure \(q\), the \textit{MOD} and \textit{REF} for \(s\) involves analyzing \(q\), all of its subsequent callee, and binding the formal and actual parameters at \(s\). Let \(\text{DMOD}(s)\) for a call site \(s\) be the set of variables directly modified by invoking the callee at \(s\). Let \(\text{ALIAS}(p,v)\) denote the set of aliases of \(v\) on entry to \(p\). The set of aliases for \(v\) indicate the set of memory locations that \(v\) can point to. We can formally define \textit{MOD} and \(\text{DMOD}\) of a call site \(s\) as follows:

\[
\begin{align*}
\text{MOD}(s) &= \text{DMOD}(s) \cup_{v \in \text{DMOD}(s)} \text{ALIAS}(p,v) \\
\text{DMOD}(s) &= \{v|s \text{ invokes } p, \text{ for } b \in \text{GMOD}(p), v \text{ bindsto } b\}
\end{align*}
\]

As shown above, the \(\text{DMOD}\) of a call site is defined in terms of the \(\text{GMOD}\) of the called procedure and the parameter bindings at the call site. \(\text{GMOD}(p)\) of a procedure \(p\) denotes the generalized modification set for \(p\) irrespective of the call sites for \(p\). Since \(\text{DMOD}\) includes the parameter binding, \(\text{GMOD}(p)\) analyzes each
individual statement in the procedure body using \( IMOD \) and each call site in \( p \) using \( DMOD \) (recursively). \( IMOD(p) \) denotes the set of variables \textit{immediately} modified by \( p \) (without analyzing the call sites) and is an initial approximation to \( GMOD \). Formally,

\[
GMOD(p) = IMOD(p) \cup \bigcup_{s \text{ invokes } q \text{ from } p} DMOD(s) \tag{2.8}
\]

The equation for \( GMOD \) is solved using the \textit{reverse call graph} of the whole program until a fixed point is reached. Reverse call graph edges emanate from the callee and are connected to the caller. Note that if a callee is invoked multiple times from a caller, multiple edges are added for each invocation of the callee since the parameter binding in \( DMOD \) computation might vary for each invocation of the callee. Each recursive call path in the program must belong to a strongly connected component (SCC) in the reverse call graph which can be detected using Tarjan’s depth-first search algorithm [117]. The SCC is iterated until a fixed point is reached.

The complexity of solving the above data-flow equations for \( GMOD \) is \( O(NE\alpha(E, N)) \), where \( E \) is the number of call sites in the program, \( N \) is the number of procedures in the program and \( \alpha \) represents the inverse Ackermann’s function. The complexity holds true for all reducible call graphs. The intuition behind the Ackermann’s function is that the size of the \( GMOD \) set grows linearly with respect to the size of the program.

The side-effect computation is a monotonic data flow framework as the \( GMOD \) and \( GREF \) sets grow monotonically. According to Kam and Ullman’s theory for reducible flow graphs [67], the complexity of side-effect computation should have been bounded by the loop connectedness of the reverse call graph. However, Cooper and Kennedy [43] observed that for a single recursive procedure program where the first parameter is modified before the recursive call with rest of the parameters (as shown in Figure 2.5), the complexity of the side-effect computation is bounded by
```c
void foo (int x1, int x2, ..., int xn) {
    int y;
    ...
    x1 = ...;
    ...
    foo (x2, x3, ..., xn, y);
}
```

Figure 2.5: Termination of Side-effect analysis

the number of parameters. They proposed a decomposed method for computing
side-effects using the binding multi-graph.

In Chapter 4 of this thesis, we present a fast side-effect analysis for programs
under dynamic compilation environment that builds on the foundations of GMOD and
GREF. Further, it demonstrates how to compute side effects for parallel constructs
and present a combined side-effect analysis [12] of procedure calls and parallel con-
structs for HJ programs. Note that, there exists a natural interplay between procedure
calls and parallel constructs, especially when parallel constructs are translated to low-
level runtime procedure calls as in common practice.

2.6 Scalar Replacement Transformation for Load Elimina-
tion

To ameliorate the Memory Wall problem in recent computer systems, compilers need
to perform transformations that promote values from memory to lower levels of the
memory hierarchy, i.e., registers or scratchpads. For example, for scientific programs
in Fortran, scalar replacement transformation [33] is used to convert array references
in the program to scalar references so that the scalar references can be allocated in
machine registers. Additional transformations like unroll-jam, loop unrolling, and loop
fusion are proposed to expand opportunities for scalar replacement of array references.

Modern programming models such as Java primarily focus on objects. Objects
are allocated dynamically and are indirectly accessed through references. Objects can point to other objects via indirect memory load operations (also known as \textit{path expressions}), such as \texttt{o.f}. These kinds of indirection using path expressions is a common practice in \texttt{Java}. Inspired by the principle of scalar replacement for arrays, the memory operations on objects can also be promoted to lower level of memory hierarchy to address the \texttt{Memory Wall} problem. This approach of eliminating memory load operations of array and object references via scalar replacement is known as \textit{Load Elimination} transformation. Load elimination is increasing in importance for multi-core and many-core architectures as it reduces the gap between memory and cpu speed.

Figures 2.6(a) and Figures 2.6(b) demonstrate the load elimination transformation for object fields. For the original program in figure 2.6(a), introducing a scalar temporary \texttt{T1} for the store (def) of \texttt{p.x} can enable the load (use) of \texttt{p.x} to be eliminated, \textit{i.e.}, to be replaced by a use of \texttt{T1}. Figure 2.6(b) contains an example in
which a scalar temporary (T2) is introduced for the first load of p.x, thus enabling the second load of p.x to be eliminated, i.e., replaced by T2. The load elimination transformations in Figures 2.6(a) and 2.6(b) are correct because p and q have different values thus ensuring that the store of q.x does not interfere with p.x.

Scalar replacement [33] studied in the context of register reuse leads to load elimination as the two scenarios described above are exactly same as the reuse due to flow and input dependencies that a scalar replacement addresses. However, load elimination needs additional mechanisms for disambiguating object references that scalar replacement did not address.

There has been much past work on load elimination via scalar replacement including [19, 20, 33, 45, 52, 75, 82, 83, 102]. We now summarize the load elimination algorithm presented by Fink et al. [52] as it is assumed as a baseline for our work on load elimination in parallel programs.

2.6.1 Unified Modeling of Arrays and Objects

As described in Fink et al. [52], accesses to object fields and array elements in the program can be represented using hypothetical heap arrays that are compile-time abstractions of the runtime heap. Each object field x in the program is abstracted by a distinct heap array, \( \mathcal{H}^x \). \( \mathcal{H}^x \) represents all the instances of field x in the heap. A use of a.x is represented as a use of element \( \mathcal{H}^x[a] \), and a definition of b.x is represented as a def of element \( \mathcal{H}^x[b] \). The use of heap arrays ensures that field x is considered to be the same across instances of two different static types \( T_1 \) and \( T_2 \), if (say) \( T_1 \) is a subtype of \( T_2 \). It also ensures disambiguation of memory accesses to distinct fields, since they will be converted to accesses to distinct heap arrays.

Likewise, each array is abstracted as a two dimensional heap array with one dimension representing object reference and the second dimension represented by the subscript. We use the notation \( \mathcal{H}^{T\text{||}\mathcal{R}} \) to denote a heap array whose dimensionality (rank) is \( \mathcal{R} \) and element type is \( T \). Note that distinct heap arrays are created for
each distinct array type in the source program, e.g., $\mathcal{H}^i_R$ represents integer arrays.

### 2.6.2 Extended Array SSA form

The arrays and object references in the IR can be renamed with the heap array representation described above to build an extended version of Array SSA form [69]. This involves inserting two specialized $\phi$ functions for use and def of heap arrays apart from the standard SSA form based $\phi$ functions. Each definition of a heap variable is replaced with a definition $\phi$ ($d\phi$) that indicates a merge function to merge the old values with the partial modification in the current definition statement. For example, for $a[i] = ...$, only $i$-th element is modified keeping other elements of array $a$ intact. Similarly, each use of a heap variable is replaced with a use $\phi$ ($u\phi$) to link multiple loads for the same heap array in control flow order. For each $d\phi$ and $u\phi$ instructions that are added to the IR, new $\phi$ instructions are added at their respective iterated dominance frontier to keep the program in SSA form.

### 2.6.3 Load Elimination Algorithm

For eliminating redundant loads, we need a way of distinguishing heap variables $\mathcal{H}^x[a]$ for $a.x$ and $\mathcal{H}^x[b]$ for $b.x$. This is described using the definitely-same and definitely-different relations.

**Definition 2.6.1** $\mathcal{H}^x[a]$ and $\mathcal{H}^x[b]$ are definitely same ($DS$) if $a$ and $b$ have same values at all program points that are dominated by the definition of $a$ and dominated by the definitions of $b$. This information can be obtained using value numbers of $a$ and $b$, i.e., $V(a) = V(b)$, where $V(a)$ represents the value number associated with $a$. Note that $DS$ is a transitive relation.

**Definition 2.6.2** $\mathcal{H}^x[a]$ and $\mathcal{H}^x[b]$ are definitely different ($DD$) if $a$ and $b$ have distinct values at all program points that are dominated by the definition of $a$ and dominated by the definitions of $b$. Note that $DD$ is not a transitive relation.
While $\mathcal{DS}$ can be computed using a global value numbering pass, $\mathcal{DD}$ can be computed using alias information or can be conservatively approximated using allocation sites and a reaching definition analysis.

The complete algorithm for load elimination [52] is provided in Algorithm 2.7. To compute if a load instruction is redundant, we need to propagate the heap variables and their associated value numbers in the extended array SSA $IR$. The propagation essentially performs a def-use chaining of heap variables that indicate which value numbers are already available from previous instructions and hence are redundant at the use. This is performed using an index propagation system that consists of a lattice over the value number set for heap variables. The details of the lattice and its operations are provided in [52]. The core idea of the algorithm is to propagate the value numbers over extended array SSA form until a fixed point is reached. Finally, the load operations are replaced with scalar temporaries based on the availability of their value numbers and the code is transformed using the scalars.

```
function LoadElim()
    Input: Method $m$ and its $IR$
    Output: Transformed $IR$ after load elimination
    1. Construct extended array SSA form for each heap operand access;
    2. Perform global value numbering to compute definitely-same (DS) and
definitely-different (DD) relations;
    3. Perform data flow analysis to propagate uses to defs;
    4. Create data flow equations for $\phi$, $d\phi$, and $u\phi$ nodes;
    5. Iterate over the data flow equations until a fixed point is reached;
    6. Perform load elimination;
    7. For a load of a heap operand, if the value number of the associated heap
    operand is available, then replace the load instruction;

Figure 2.7: Load elimination algorithm
```

Even if the target processor has a limited number of registers, replacing a general heap load access by a read of a compiler-generated temporary can be profitable in future many-core processors because it enables the use of more energy-efficient
and scalable storages like registers and local memories (scratchpads). Performing such scalar replacement for load elimination in parallel programming languages for multi/many-core processors pose additional challenges in the form of interferences among shared data accesses across parallel tasks. These interferences are commonly known as *data races*. Analyzing programs with or without data races is strongly tied to the underlying memory model supported by the language. Chapter 5 of this dissertation describes a parallelism-aware scalar replacement algorithm for load elimination transformation [12] whose legality is provided using a new weak memory model called Isolation Consistency (IC).

### 2.7 Register Allocation

The *Register file* is the most critical storage resource in a computer’s processing unit. It contains a limited number of physical machine registers and provides faster access to operands than any other storage resource in a computer. For example, the x86 architecture provides 8 fixed machine registers and multiple of these registers can be accessed in one cycle. Hence, it is important from an optimizing compiler’s perspective that the utilization of the register file be controlled. *Register Allocation* is a compiler back-end phase that maps operands to physical registers at various program points. Operands are either program variables or compiler generated temporaries. We use variables, symbolic registers, and operands, interchangeably. Since the number of physical registers is usually smaller than the number of simultaneously live variables, it is almost always the case that some of the operands need to be *spilled*, *i.e.*, allocated in other resources like cache, memory, or scratchpads.

Given \( k \) physical registers, a register allocation problem can be formally stated as follows:

**Definition 2.7.1** Given a set of variables \( V \) and \( k \) physical registers, determine if it is possible to assign each variable \( v \in V \) to a physical register at each program point where \( v \) is live. If so, rewrite the code using physical registers. Otherwise, rewrite the
Typically, a register allocator consists of two tasks: *allocation* and *assignment*. *Allocation* determines which operands should be held in physical machine registers at various program points and which operands should be “spilled”. *Assignment* identifies which specific machine registers of the target machine should be used at different program points to hold which operands. While allocation ensures that no more than \( k \) variables are residing in registers at any program point (where \( k \) is the total number of physical registers available in the target machine), assignment produces the actual register names required in the executable code. Both these tasks are equally difficult, *i.e.*, NP-hard to perform at all levels of compilations including *local* (basic block level), *global* (procedure level) and *interprocedural* (whole-program level). Modern architectural innovations like *register classes*, *register aliasing*, and *register pairs* further complicate the register allocation problem.

In the rest of the section, we first summarize common terminology used in register allocation and describe several existing techniques for register allocation.

### 2.7.1 Terminology

#### 2.7.1.1 Liveness, Live-ranges and Interference Graph

For register allocation, it is necessary to know which variables can be allocated to the same physical register and which can not. This information is usually abstracted away in the form of *live-ranges* and *interference*. Based on the notion of *liveness* and *program point* described in Section 2.1, we can define the live-range of a variable and the interference of two variables as follows:

**Definition 2.7.2** The live-range of a variable \( v \) denoted as \( lr(v) \) is the set of program points where \( v \) is live.

---

\(^2\)It is possible to perform optimal allocation for a single basic block \([14, 44, 88]\)
Definition 2.7.3 Two variables \(a\) and \(b\) are said to be interfering (or conflicting) with each other if \(lr(a) \cap lr(b) \neq \emptyset\).

The live-ranges in a program can be computed using the algorithms presented in [8, 24]. The interference among all program variables can be represented using an undirected graph known as the interference graph.

Definition 2.7.4 The interference graph (IG) is a graph, \(G = \langle V, E \rangle\), where \(V\) consists of variables and \(E\) consists of edges between variables arising from interference, i.e., two interfering variables \(a\) and \(b\) will have an edge between them.

The IG serves as the main data structure of all register allocation algorithms based on graph coloring. Hence it is critical to represent an interference graph efficiently. Usually IG is implemented using two representations: 1) bit matrix: that supports constant time implementation of determining interference between two variables; 2) adjacency list: consists of a list of adjacent neighbors for every node in IG and supports fast iteration over the neighbors. The worst-case space complexity of an IG is \(O(n^2)\), where \(n\) is the total number of variables in the IR.

2.7.1.2 Spilling

Spilled variables incur the additional cost of load and store memory operations for transferring values between registers and memory. These memory operations constitute spill code and are usually expensive. Hence, a register allocator should always aim to keep the frequently-used values in registers to minimize the impact of spill code.

2.7.1.3 Coalescing

The IR undergoes several front-end transformations before back-end register allocation pass of the compiler. One particular instruction that occurs frequently in the IR after compiler transformation is a move instruction of the form “\(dest = src\)” as move
src, dest. From a register allocation perspective, if src and dest are assigned to the same physical register (also known as coalesced), then the move instruction can be removed from the IR. However, when the live-ranges of src and dest interfere with each other, it may not be possible to assign src and dest the same physical register. Since the ultimate goal of register allocation is to produce efficient code, a register allocation algorithm needs to optimize away as many move IR instructions as possible. Several coalescing techniques have been proposed in the literature: aggressive coalescing [35], conservative coalescing [28], optimistic coalescing [96], and iterated coalescing [57]. Recently, [23] have shown that all the above coalescing techniques are NP-hard.

2.7.1.4 Live-range splitting

Sometimes it may be beneficial to split the live-range of a variable into two or more smaller live-ranges. The smaller live-ranges can be separately allocated to memory or physical registers after variable renaming. This is helpful especially when a live-range can be assigned a physical register in some program points instead of the whole live-range. The down-side of live-range splitting is that it incurs extra cost of inserting move and load/store instructions among smaller live-ranges.

2.7.1.5 Architectural Considerations

Register Classes: Typically, a register allocation problem is stated using a set of k uniform physical registers. These k registers are assumed to be independent and interchangeable. Independent means that modifying one physical register does not modify another and interchangeable means that they can be exchanged with each other in a particular context. However, modern systems come with physical registers which may not necessarily be interchangeable. For example, the Intel x86 architecture provides eight integer physical registers, of which six are usable by the compiler. These six physical registers are further divided into four high level register classes based on calling conventions and 8-bit operand accesses. Similarly, for most architectures the
physical registers in a floating point register class can not be interchanged with the physical registers of the integer register class. If the register classes were disjoint, then we can state the register allocation problem independently for each class and solve them independently. However, the register classes are not necessarily disjoint. For example, the four integer register classes of x86 architecture overlap with each other. To produce high quality machine code, a register allocator must take into account these register classes.

**Register Aliases:** The independence assumption between physical registers is violated using register aliases. Aliasing indicates that modifying one physical register can affect another. An example demonstrated in [26] is to use two single precision floating point registers for one double-precision register. Similarly the integer physical registers of x86 architecture can be accessed as 8-bit operands using AL, BL, CL and DL registers and the same is aliased with 16-bit operands using AX, BX, CX, and DX. A register allocation algorithm must consider these features to produce reasonable machine code.

**2.7.2 Register Allocation Techniques**

**2.7.2.1 Graph Coloring Register Allocation**

Starting with the seminal paper by Chaitin [35], the dominant approaches for global register allocation have been based on the idea of building the IG for variables, and employing Graph Coloring (GC) heuristics to perform the allocation. If the machine has $k$ physical registers, Graph Coloring looks for a $k$-coloring of IG, i.e., $k$ colors are assigned to the node of IG such that neighboring nodes always have different colors. Graph Coloring is shown to be NP-complete [54]. Chaitin presents a heuristic to find the $k$-coloring of an IG. If a $k$-coloring is not found, some variables are spilled onto memory.

Figure 2.8 illustrates Chaitin’s register allocator. It consists of the following phases:
1. *Renumber* renames live ranges. It creates a new live range for each definition of a variable. At each use of a variable, it combines the live ranges that reach the use.

2. *Build* constructs the *IG*.

3. *Coalesce* combines two non-interfering live ranges that are part of a move instruction. When two live ranges are coalesced, new live ranges are created and hence, *IG* is updated. Chaitin’s coalescing approach did not consider the decrease/increase of the colorability of *IG* after coalescing. Hence, Chaitin’s coalescing is commonly referred to as *aggressive coalescing* [35].

4. *Spillcost* computes the compiler’s estimation of run-time cost of a live range when the live range is spilled onto memory.

5. *Simplify* removes nodes from the *IG* and creates an ordering of the nodes using a stack. If the degree of a node is less than *k*, the node and its edges are removed from *IG* and pushed onto the stack. If there are no nodes in the *IG* that have degree less than *k*, then a node is chosen for spilling. After all the nodes are removed from the *IG*, spill code is added for the spilled nodes and the register allocation process is repeated from the beginning.

6. *Spill code* inserts appropriate *load* and *store* memory operations for spilled live ranges.

7. *Select* assigns colors to the nodes in the order specified by the stack in *Simplify* pass.

The above heuristic leverages the observation that when a node with degree less than *k* is removed from the *IG*, the *k*-colorability of *IG* is not changed. Nodes with degree greater than or equal to *k* are spilled. The node selection for spilling is based on the smallest ratio of spill cost divided by the degree.
Figure 2.8: Chaitin’s Register Allocator

Briggs et al. [25, 28] showed that Chaitin’s heuristic does not always find a $k$-coloring even if one exists. For example, a diamond interference graph has a 2-coloring that can not be recognized by Chaitin’s heuristic. They proposed a modification to the Simplify phase that repeatedly removes nodes of smallest current degree in the IG and pushes them onto the stack. Actual spill decisions are taken in the Select phase. Apart from this modification, they proposed a conservative coalescing scheme that coalesces nodes in the IG such that the colorability of the resulting IG after coalescing is not increased.

Since the inception of Graph Coloring register allocation, significant advances have been achieved over these years through the introduction of new coloring, spilling, and coalescing heuristics based on the IG, e.g., [30, 31, 40, 57, 96, 104, 110]. However, a key limitation that underlies all register allocation algorithms based on Graph Coloring is that the number of variables that can be processed by the register allocation phase in an optimizing compiler is limited by the size of the IG. The number of edges in the IG can be quadratic in the number of nodes in the worst case, and is usually observed to be super-linear in practice. Though it is used widely in practice, Coloring-based Register Allocation is usually the scalability bottleneck phase in an optimizing back-end. Recently, a study by Cooper and Dasgupta [42] to investigate the most expensive component of Graph Coloring register allocator reported that the Build phase consumes almost 72% of the total allocator time. This non-linear complexity in space and time of Graph Coloring limits the code size that can be
optimized and thereby has a damping effect on aggressive use of code transformations that can potentially increase opportunities for register allocation, such as variable renaming, loop unrolling and procedure inlining, but which also have the side effect of increasing the size of the IG. Finally, the non-linear complexity makes it prohibitive to use Graph Coloring for register allocation in just-in-time and dynamic compilers, where compile-time overhead contributes directly to runtime.

2.7.2.2 Linear Scan Register Allocation

Just-in-Time (JIT) compilation performed by dynamic compilers aim at compiling codes on the fly, i.e., compilation happens while the code executes. This adds extra constraints in terms of time and space than static compilation. Past work by [100, 101] introduce a new register allocation algorithm, the “linear scan” register allocation. Their proposed algorithm is fast as it makes a single pass over the IR instructions and requires very little space since it does not build the interference graph explicitly. Due to its lightweight nature, Linear Scan has been used in many state-of-the-art Virtual Machines such as HotSpot Client Compiler [70], LLVM [74], and Jikes RVM [66].

Linear scan assumes a linear ordering of the IR instructions. The choice of ordering affects the quality of allocation but not the correctness. There are several possible orderings, such as the original order in which instructions appear in IR (linear order) and a depth-first ordering over the control flow graph. It has been observed by Poletto and Sarkar [100] that both linear and depth-first order produce similar code. In the rest of the thesis, we will consider depth-first ordering.

Definition 2.7.5 Each program point \( i \) is represented with \( i^- \) and \( i^+ \), where \( i^- \) consists of the variables that are read at \( i \) and \( i^+ \) consists of the variables that are written at \( i \). \( i^- \) and \( i^+ \) are represented in integer numbers based on the order of the IR instructions.

Like live ranges in graph coloring register allocation, the central data structure in
linear scan is the notion of a *live interval*. According to the original notion described in [100], a live interval is defined as follows:

**Definition 2.7.6** An integer range \([x, y]\) is a live interval for a variable \(v\) iff \(\not\exists p, \text{ such that}

1. \(p < x\) and \(v\) is live at \(p\), or,

2. \(p > y\), \(v\) is live at \(p\).

For \([x, y]\), \(x\) is referred to as the unique start point (denoted as \(\text{Lo}(v)\)) and \(y\) is referred to as the unique end point (denoted as \(\text{Hi}(v)\)).

The above definition permits a live interval to include program points where a variable \(v\) may not be live. The sub-interval during which a variable is not live is known as a *life-time hole* [119]. It is important to consider life-time holes in linear scan as it is possible that two overlapping intervals (according to the above definition of live intervals) can be allocated to the same physical register if both of them are not live simultaneously. To distinguish between live intervals and live intervals with holes, let us define *basic interval* (BI) and *compound interval* (CI).

**Definition 2.7.7** \([x, y]\) is a basic interval (BI) for variable \(v\) iff \(\forall p, x \leq p \leq y\), \(v\) is live at \(p\). Note that a BI does not allow any life-time hole. \(\text{Lo}(b)\) and \(\text{Hi}(b)\) denote the unique start and end points of interval \(b\), respectively.

**Definition 2.7.8** A compound interval (CI) for a variable \(v\) consists of a set of disjoint basic intervals for \(v\). Note that, a CI permits life-time holes.

From a register allocation perspective, it is required to decide those BIs that need to be in physical registers and those CIs that need to be spilled\(^3\). Register

\(^3\)Spilling at a compound interval level permits *all-or-nothing* or *spill everywhere* approach. In the rest of the dissertation, we will focus on this approach instead of *partial spills*. 
assignment decides the exact mapping of BI to physical registers. Given that each BI is represented using a unique start and end point, the register allocation problem can be viewed as an interval graph coloring problem. Formally,

**Definition 2.7.9** An interval graph is a graph, \( G = \langle V, E \rangle \), where \( V \) consists of the set of BIs and \( E \) consists of the intersecting BIs.

**Definition 2.7.10** Two BIs, \([x_1, y_1]\) and \([x_2, y_2]\), are said to be intersecting if one of the following holds:

1. \( x_2 \geq x_1 \) and \( x_2 \leq y_1 \)
2. \( y_2 \geq x_1 \) and \( y_2 \leq y_1 \)

**Theorem 2.7.11** An interval graph \( G = \langle V, E \rangle \) can be colored optimally in linear time.

**Proof:** Refer to [88, 94]. \( \square \)

The optimal interval graph coloring described in [94] consists of the following steps: 1) find an ordering of the intervals based on increasing \( Hi \) values\(^4\); 2) Assign color to an interval by looking at the already colored adjacent nodes (or neighbors) in the interval graph. Readers are encouraged to see [88, 94] for more details.

Linear Scan register allocation [100] follows the basic theme of interval graph coloring described above. However, it brings important implementation efficiencies to ensure smaller time and space requirements. Like interval graph coloring, linear scan sorts the basic intervals in increasing order of \( Lo \). Instead of building the interval graph explicitly, it maintains an active list of basic intervals that orders the basic intervals in increasing order of \( Hi \). Note that, the active list plays the role of already colored adjacent nodes (or neighbors) in interval graph coloring. The reason the active list is sorted in increasing order of end points is two fold: 1) For efficiency

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\(^4\)One particular order either increasing or decreasing should be considered
reasons, linear scan performs allocation and assignment in a single pass over the interval start points. Hence, it is necessary to decide spilled intervals while allocation is being performed. Using the farthest-use approach of Belady [14], linear scan decides intervals that have largest end point in active list for spilling; 2) While basic intervals expire (create life-time holes), they are removed from active list. The scan to remove basic intervals from active list stops as soon as the current intervals start point is reached.

The complete linear scan algorithm is presented in Algorithm 2.9. Step 2 builds a sorted set of BIs based on increasing start points. Step 3 initializes the active list ActiveSet to φ. Step 4 iterates over sorted basic intervals and at each BI’s start point, it makes the allocation and assignment decisions. In Step 5, the function Expire removes basic intervals from ActiveSet whose end points (i.e., Hi) are less than the current BI’s start point. If at any point the size of the ActiveSet is equal to the number of physical registers (k), then a spill candidate is chosen based on interval end points (this is shown in Step 6). Step 9 assigns the specific physical register to an interval. Finally, the current BI is added to the ActiveSet.

Now let us discuss about the complexity of Algorithm 2.9. The overall complexity is bounded by the following steps: 1) Step 4; 2) the time to add an interval to ActiveSet; 3) Step 13. Let us assume that there are |BI| number of basic intervals that arise from V variables. Let there be k number of physical registers. Further, let the ActiveSet be implemented using a binary search tree, i.e., the time to add an interval to ActiveSet is log k. Step 4 takes $O(|BI|)$ time. Step 13 takes $O(k)$ time. So the overall complexity is $O(|BI| \times (k + \log k))$.

Let us consider the example program shown in Figure 2.10. Variable a is initialized before the if-else construct and is used inside both the branches but not after the merge point. This gives rise to two contiguous basic intervals for a when the code is ordered in the sequence shown on the left, i.e., CI(a) = \{[1^+, 4^-], [8^-, 11^-]\}. Note that, CI(a) includes hole in between the basic intervals. The compound intervals of b and c consist
function LinearScan()
IntervalSet = sorted set of BIs in increasing start points i.e., $Lo(v)$;
ActiveSet = φ;
foreach $b \in \text{IntervalSet}$ do
  Expire(b);
  if $|\text{ActiveSet}| == k$ then
    Spill(b);
  else
    Assign an available color to $b$;
    ActiveSet = ActiveSet $\cup \{b\}$;
function Expire(b)
foreach $b'$ in ActiveSet do
  if $Hi(b') \geq Lo(b)$ then
    return;
  Remove $b'$ from ActiveSet;
  Make the physical register of $b'$ available;
function Spill(b)
SpillCandidate = last basic interval in ActiveSet;
if $Hi(\text{SpillCandidate}) > Hi(b)$ then
  Assign the physical register of SpillCandidate to $b$;
  Spill SpillCandidate and remove it from ActiveSet;
  ActiveSet = ActiveSet $\cup \{b\}$;
else
  Spill $b$;

Figure 2.9 : Linear Scan register allocation algorithm
Figure 2.10: Example program demonstrating linear scan register allocation. The source code is shown on the left. The corresponding compound intervals are shown on the right. We can observe that $\text{CI}(a) \cap \text{CI}(b) = \phi$. Hence $a$ and $b$ can be assigned in the same physical register.

of a single basic interval, \textit{i.e.}, $\text{CI}(b) = \{[5^+, 6^-]\}$ and $\text{CI}(c) = \{[2^+, 13^-]\}$. The hole created by $\text{CI}(a)$ is large enough to contain the entire compound interval of $\text{CI}(b)$ thereby ensuring $\text{CI}(a) \cap \text{CI}(b) = \phi$. Hence, $\text{CI}(a)$ and $\text{CI}(b)$ can be assigned to the same physical register. $\text{CI}(c)$ needs a separate physical register as it intersects with both $\text{CI}(a)$ and $\text{CI}(b)$.

Both register assignment and spilling decisions in Poletto and Sarkar [100] are performed at a compound interval level, \textit{i.e.}, a compound interval is either assigned the same physical register throughout the entire program or is spilled throughout the entire program. In other words, all the basic intervals of the same compound intervals are either assigned the same physical register or spilled. There is no notion of partial spills or live-interval splitting. Additionally, in [100], a separate code rewrite pass after register assignment is introduced to rewrite physical register names and generate spill codes.

A variant of linear scan proposed by Traub et al. [119] known as \textit{second-chance bin packing} addresses the above concerns to some extent. Specifically: 1) it combines allocation, assignment and code rewrite in a single pass; 2) allows compound intervals be
split multiple times (i.e., a CI can be assigned to a physical register in some program points and be assigned to a memory location in some other program points, giving it a second chance for allocation). Their algorithm walks over the IR instructions in a linear order. When a new CI is encountered, it rewrites the output code with an available physical register. If no such physical register is available, it spills a CI based on a next-use distance metric [14] and inserts spill code appropriately. When a spilled CI is encountered later on, it tries to give a second chance to the CI by finding an available physical register at the current program point. If an available physical register is found and the current reference of CI is a read, a memory load instruction is added. If the current reference is a write, then no memory store instruction is added until some other CI evicts the physical register. Further memory store instructions can be avoided during eviction of a CI that is held in a physical register \( r \), if the value held in \( r \) matches that of the memory location of CI\(^5\). Since CIs can be allocated in multiple physical registers at various program points, control flow needs to be accounted, i.e., appropriate load, store, and move instructions are added on control flow edges for generating correct code. For example, if the same CI was spilled at the source end of a control flow edge and was in a physical register at the destination end of a control flow edges, then a memory load instruction is added along the control flow edge.

As can be seen above, the second-chance binpacking algorithm [119] spends more time in compilation as it makes repetitive decisions of spill or register assignment at every reference of an interval. In comparison, Poletto and Sarkar [100] make decisions of spill or register assignment at interval start points and it does not offer opportunities for second-chance.

Like second chance binpacking, more recently Wimmer and Mössenböck [123] present a linear scan algorithm for x86 architectures that allows splitting of live

\(^5\)This requires a data-flow analysis to determine memory consistency and is performed before register allocation.
intervals by allowing some part of the CI to be in a register and some other part be in memory. Like memory store optimizations of [119], they allow split positions be placed in low-frequency basic blocks.

One of the key observations made while comparing Linear Scan with Graph Coloring is that Linear Scan should be used when compile-time space and time overhead is at a premium (as in dynamic compilation), but an algorithm based on graph coloring should be used when the best runtime performance is desired. Let us discuss some of the key reasons why this is the case. One of the limitations of current linear scan is that it combines allocation and assignment in a single pass for improved compilation time. This leads to local decisions for allocations and assignments at a given instruction or at an interval start point based on active list. Instead, the spilling decisions of a graph coloring is global because the spilling decisions are made based on the interference graph that represents a global view of program. The global decisions usually yield improved spilling and register assignment.

Chapter 6 of this dissertation addresses the limitations of current linear scan register allocation algorithms and presents a space-efficient register allocation algorithm [105] that retains the compile-time efficiency of linear scan while delivering performance that can match or surpass that of Graph Coloring. The proposed register allocation algorithm performs allocation and assignment in a compile-time efficient manner in two separate passes.

2.7.2.3 SSA-based Register Allocation

Recently, much of the attention in register allocation has shifted to performing register allocation in SSA representation. The key property of a SSA program is that every variable is defined exactly once. This leads to the fact that an interference graph built from the live ranges of a SSA program is a chordal graph [22, 29, 59]. A graph is chordal if every cycle with four or more edges has a chord, that is, an edge which is not part of the cycle but which connects two vertices on the cycle. For example, the
Figure 2.11: Examples of chordal and non-chordal graphs; Case (a) is a chordal graph and Case (b) is not a chordal graph.

graph in Figure 2.11(a) is chordal as the edge \((v_1, v_3)\) is a chord in the cycle comprising of \(v_1, v_2, v_3,\) and \(v_4\). In contrast, the graph in Figure 2.11(b) is not chordal as it does not have a chord for the cycle comprising of \(v_1, v_2, v_3,\) and \(v_4\).

Chordal graphs have the property that they can be colored in polynomial time [55]. Optimal coloring of a chordal graph \(G = \langle V, E \rangle\) can be performed in \(O(|E| + |V|)\) time.

A SSA based register allocation follows the same basic theme of a register allocation using Graph Coloring. The overall SSA register allocation framework is depicted in Figure 2.12. Given an input SSA IR, live ranges and the interference graph is built in the Build phase. Using the interference graph, spill candidates are chosen so as to reduce the register pressure of the program to the available physical registers \(k\). MCS order phase finds a node ordering of the interference graph that can be colored optimally using Maximum Cardinality Search algorithm [16]. Potential Select phase assigns colors to live ranges. Coalesce phase recolors the live ranges (using Actual Select) so as to get rid of the move instructions in the IR. Note that Actual Select and Coalesce phases are repeated until all the nodes are assigned a color and most of the frequently executed move instructions are removed from the IR. The SSA program is then translated out of SSA form and spill code is added. The advantage
of SSA based register allocation is that the Spill phase does not need to be in a loop with the Coalesce phase like Graph Coloring approach as the spill decisions are taken independently. Once spilled variables are determined in Spill phase, there is no need to spill any further variables during Coalesce or Select phase. This simplifies the register allocation process compared to Graph Coloring approach. However, as we will see in Chapter 6 of this thesis, the recoloring during Coalesce phase and the out-of-SSA translation makes the allocator less attractive than other register allocators.

### 2.8 Bitwidth-aware Register Allocation

Registers are few but provide fastest access to a computer system. They must be allocated efficiently to achieve maximum benefits. Several techniques like coalescing and live-range splitting (as described in preceding section) are used to reduce register pressure in a program. Another recent approach of reducing register pressure in a program is to pack multiple conflicting narrow sized variables onto the same physical register which otherwise would have occupied more than one physical register. For example, two 16 bit conflicting variables can be co-located in a single physical register of 32-bit size. Since two 16 bit variables are conflicting, they would have occupied two physical registers, thereby wasting 32 bits. A register allocation algorithm that is sensitive to the widths/sizes of program variables is called a *Bitwidth-aware register allocation* algorithm. While packing and unpacking subwords in registers can be a source of overhead, it is expected that the locality benefits of bitwidth-aware register allocation will outweigh the overhead in future processors.
Consider the following example illustrating how registers can be under-utilized in a 32-bit machine due to allocation of narrow sized variables onto physical registers. First, if a variable is declared as boolean, char, or short, then it will occupy the complete 32-bits of a physical register even though it only needs access to a subword. Second, if a variable is declared as a 32-bit integer, but the application uses it to store values which use less than 32 bits, then some bits of the physical register are wasted. The second example is quite widespread in applications from embedded systems. For example, Networking and Multimedia applications in embedded systems typically hold values both in packed and unpacked form. Specifically, the data in a network packet is unpacked in a program into various components such as header and control. The packing and unpacking operations are typically seen as bitwise operations in a program. A bitwidth aware register allocation algorithm can use this information to pack several unpacked items onto the same physical register to make efficient use of the physical register.

Let us consider the code fragment shown in Figure 2.13. This code is part of the kernel code of the adpcm multimedia benchmark [76]. We can observe that the variable delta on line 4 can have values in the range 0 · · · 15 since it is used to access indexTable and the size of indexTable is a compile-time constant having value 16. This bounds delta to occupy atmost 4 bits on a physical register. The variable index on line 4 can have any integer value and hence, may end up using the complete 32 bits. However, after line 6, index can only have values in the range 0-88, i.e., it needs atmost 7 bits. Similarly, on line 7, the variable step can have values in the range 7 · · · 32767 as the values in stepTable are compile-time constants that lie in the range 7 · · · 32767. Hence step needs atmost 15 bits. We can also observe that bufstep is a boolean variable and hence needs only 1 bit. The assignment on line 9 masks the value to 0xf0. This implies that outbuff can be represented in 4 useful bits. Finally, *outp on line 11 needs only 8 bits.

It can be observed that almost all the variables in the code fragment shown in
Figure 2.13: Code fragment from `adpcm` benchmark showing the benefits of bitwidth-aware register allocation

```c
int stepTable[89]; // values lie in range [7...32767]
int indexTable[16]; // values lie in range [-1...8]
...
index += indexTable[delta]
if (index < 0) index = 0
if (index > 88) index = 88
step = stepTable[index]
if (bufstep)
  outbuf = (delta << 4) & 0xf0
else
  *outp++ = (delta & 0x0f) | (outbuf & 0xf0)
bufstep = !bufstep
...
```

Figure 2.14: Overall Bitwidth aware register allocation framework

Figure 2.13 contribute to wasting bits when they reside in physical registers. Even if the variables are spilled onto memory, they waste bits while accessing memory. It would be ideal to perform packing of variables for both memory access and register allocation. Memory access packing was studied by Davidson and Jinturkar [49]. Stephenson et al. [115] studied the impact of bitwidth analysis on silicon compilation, i.e., programs that are directly compiled onto hardware. More recently, Tallam and Gupta [116] introduced a bitwidth-aware register allocation algorithm that focuses on packing of variables in physical registers.

We will now summarize the foundations of bitwidth-aware register allocation algorithm described in [116]. Figure 2.14 depicts the overall bitwidth aware register allocator. It consists of four key steps: 1) Bitwidth analysis; 2) Variable packing; 3) Move insertion; 4) Register allocation. Let us describe each of them in detail.
2.8.1 Bitwidth Analysis

*Bitwidth analysis* is a static analysis that determines the actual widths (or bits) of variables at various program points. We denote the width of a variable \(v\) at a program point \(p\) as \(B(v, p)\). Before we describe how to compute \(B(v, p)\) information, it is necessary to have a representation for \(B(v, p)\). There exists two representations in the literature: 1) value range based representation proposed by Stephenson et al. [115] that determines minimum and maximum value assigned to a variable; 2) dead bit representation proposed by Tallam and Gupta [116] that divides the width of a variable into three contiguous sections: (a) leading dead bits representing the bits having zeros in the leading part when the variable is represented in binary; (b) middle live bits representing the actual bit used; (c) trailing dead bits representing the bits having zeros in the trailing part. Even though the value range based representation is more precise, the dead bit representation is better suited for use by the register allocator. The dead bit representation can be stated formally as:

**Definition 2.8.1** The width of a variable \(v\) at program point \(p\) is a pair of leading and trailing dead bits, i.e., \(B(v, p) = (l(v, p), t(v, p))\), where \(l(v, p)\) denotes the size of leading dead bits and \(t(v, p)\) denotes the size of trailing dead bits when \(v\) is represented in binary.

Now let us discuss how bitwidth information \(B(v, p)\) for variables are computed. The definition points of variables in the program generate new width information that needs to be propagated to their uses. This involves a forward data flow analysis that propagates bitwidth information along control flow edges on a lattice over all possible pairs of \((l(v, p), t(v, p))\) with merge functions that take appropriate *max* or *min* actions. Further, the type of usage of a variable at a program point (arising from bitwise operations like shifting, masking, or-ing) can improve the precision of the forward data flow analysis. These new bitwidth information need to be propagated back from use program points to the definition program points using a backward data
flow analysis. Using both the forward and backward data flow algorithms, \( B(v, p) \) for every variable at every program point is computed.

### 2.8.2 Variable Packing

*Variable packing* is the process of packing multiple variables onto the same physical register. We would like to distinguish between the terms *coalescing* and *packing* which are sometimes used inconsistently in the literature. *Coalescing* (as described in Section 2.7.1.3) attempts to combine two non-interfering variables so as to remove any *move* instruction between them. In contrast, *packing* attempts to combine two interfering variables that can fit onto the same physical registers. Packing is shown to be an NP-complete problem by Tallam and Gupta [116] using a simple reduction from bin-packing.

Packing is usually performed on an interference graph whose interfering edges are annotated with bitwidth information between two variables having maximum width. The maximum width is represented using *maximum interference width* (MIW). Formally,

**Definition 2.8.2** \[ \text{MIW}(v_1, v_2) = |B(v_1, p)| + |B(v_2, p)| \text{ such that } \exists n, |B(v_1, n)| + |B(v_2, n)| > |B(v_1, p)| + |B(v_2, p)|, \text{ where } |B(v, p)| \text{ denotes the middle section of useful bits in dead bit representation.} \]

Initially, each interfering edge between \( v_1 \) and \( v_2 \) is annotated with \( (|B(v_1, n)|,|B(v_2, n)|) \) where \( n \) is the program point having maximum interference width. The packing algorithm proceeds by packing variables in the interference graph as long as \( \text{MIW}(v_1, v_2) \) is less than the size of a physical register. One subtlety that occurs while packing is that every step of packing needs recomputation of MIW as new packed variables are formed. This can be an expensive operation as it needs to walk over the IR every time. Tallam and Gupta [116] proposed the use of estimates of MIW (known as estimated maximum interference width, *i.e.*, EMIW) that are conservative approximation of MIW and can be computed in constant time without traversing the IR.
Figure 2.15: Two scenarios for variable packing: Case (a) and Case (b) demonstrating updated MIW after nodes $A$ and $B$ are packed into a single node $AB$. In these two scenarios, there is no imprecision since the MIW can be computed directly from the given edge labels.

**Definition 2.8.3** $EMIW(v_1, v_2) \geq MIW(v_1, v_2)$ for all pairs of interfering variables $v_1$ and $v_2$.

Figure 2.15 depicts two scenarios that occur while computing MIW during packing of variables in the interference graph. These two cases propagate precise MIW information after packing since only two of the variables are simultaneously live. The third scenario is shown in Figure 2.16. Since all the three variables are simultaneously live, the precise MIW computation would require a recomputation of maximum interference width of the three variables (and hence, needs a pass over the $IR$). This is an expensive process. To ameliorate this, $EMIW$ estimates are used that propagate an *intermediate value* that is proven safe using an *intermediate mean value theorem* (described below). Note that, the $EMIW$ estimates are computed on-the-fly and do not require any $IR$ pass.

**Theorem 2.8.4** If $E_{min} \leq E_{int} \leq E_{max}$, such that $E_{min} = \min(E_A, E_B, E_C)$ and $E_{max} = \max(E_A, E_B, E_C)$, $EMIW(A, B, C) = E_{int}$ is safe.

**Proof:** Refer to [116].

One of the side effects of variable packing is that it may increase the colorability of the interference graph. For example, consider the diamond interference graph
which is 2-colorable. If two adjacent neighbors of the diamond graph are packed during variable packing, then the diamond interference graph reduces to a triangle that requires 3 colors now. Hence, it is desirable that variable packing be guided by the techniques used in conservative coalescing [28] and a priority function based on spillcost.

2.8.3 Move Insertion

After variable packing is performed, the IR code is rewritten with new names for the packed variables. Since the variables are in packed form now, they need to be unpacked for their individual uses and definitions. If the underlying architecture provides hardware instructions for bit-level referencing of a physical register [80], then unpacking instructions are not needed. Additionally, new move instructions may be needed to perform intra-register data transfer. This essentially takes care of bit-fragmentation due to packing.
2.8.4 Register Allocation

The final step of bitwidth aware register allocation algorithm is to perform register allocation of the packed variables. Since the code was rewritten in the Move Insertion phase, the interference graph needs to be rebuilt and a standard Graph Coloring register allocation is performed.

Chapter 7 of this dissertation makes several contributions [11] to the bitwidth-aware register allocation. First, it proposes a limit study on bitwidth analysis that indicates the opportunities available for improving the bitwidth aware register allocator. Second, it proposes several enhancements to bitwidth analysis that closes the gap between runtime bitwidth analysis and static bitwidth analysis. Finally, it proposes a number of EMIW estimates that enhances the precision of variable packing compared to Tallam-Gupta variable packing.
Chapter 3

May-Happen-in-Parallel (MHP) Analysis

In this chapter, we describe a May-Happen-in-Parallel (MHP) analysis for HJ programs that determines if two statements can execute in parallel. As described in Section 2.2, HJ is a modern object-oriented programming language designed for high performance and high productivity programming of parallel and multi-core computer systems. HJ offers various concurrency control constructs to the programmers: multiple parallel activities can be created using the async construct, their termination can be coordinated using the finish construct, mutual exclusion can be enforced using isolated blocks, and barrier based synchronization among activities can be performed using the phaser construct \cite{107}. HJ also inherits from X10 the partitioning of data and activities across places through the use of distributions. In this chapter we describe a MHP analysis for HJ programs that consists of the async, finish, isolated, and place constructs.

3.1 Introduction

May-Happen-in-Parallel (MHP) analysis statically determines if it is possible for execution instances of two given statements (or the same statement) to execute in parallel. This analysis serves as foundations for static analysis of concurrent programs and debugging tools for a concurrent program \cite{39, 71, 87, 91}. Static analysis techniques that may benefit from MHP analysis include detection of synchronization anomalies like data-races and deadlock, and improving the accuracy of compiler analysis by removing infeasible def-use pairs.

Several MHP analyses have been proposed in the literature, e.g., \cite{10, 92}. However,
the precision of these approaches is severely limited by the fact that Java’s concurrency constructs are tied to objects. For example, the `synchronized`, `wait`, `notify`, `start`, and `notifyAll` operations are all invoked on specific target objects. Objects are created dynamically and may escape method and thread boundaries. This implies that we need a precise interprocedural alias analysis [7, 114] to model the interactions among concurrent tasks. Since precise alias analysis is expensive to perform, many compilers including dynamic compilers prefer faster approximations. These approximations lead to over-approximating the MHP information, i.e., assuming that two statement instances may execute in parallel when in fact, they can not.

Compared to Java, the concurrency constructs in HJ (described in Section 2.2) are simpler yet powerful. They are powerful enough to cover all aspects of parallel programming as evidenced by the range of benchmark suites that have been ported to HJ including SPECJBB [112], Java Grande [65], Nas-Parallel Benchmark [93], and Shootout [108]. They are simpler because several concurrency constructs of HJ are not attached to objects and do not cross method boundaries. Hence, they do not need any interprocedural alias analysis. This simplicity allows us to obtain more precise MHP information using linear-time algorithms.

### 3.2 Steps for MHP analysis of HJ programs

The high level steps involved in MHP analysis for HJ programs are:

1. Create a Program Structure Tree (PST) representation of the HJ method/procedure, as described in Section 3.3.

2. Perform a Never-Execute-In-Parallel (NEP) analysis is performed as described in Section 3.4. This analysis considers the occurrences of `finish` and `async` nodes in the PST and identifies pairs of statement instances that can never execute in parallel. For soundness, the NEP analysis conservatively errs on the side of returning `NEP = false` when it is unable to perform a precise analysis of
the input HJ program. In the case of statements in a loop-nest, we use condition vectors (defined in Section 3.4) to qualify the instances of execution that can never happen in parallel.

3. Perform a Place-Equivalence (PE) analysis as described in Section 3.5. The output of this analysis is a predicate, \( PE(S1,S2) \), which is set to true if selected instances of \( S1 \) and \( S2 \) are guaranteed to execute at the same place. \( PE(S1,S2) = false \) indicates that the instances may or may not execute in the same place. For soundness, the PE analysis conservatively errs on the side of returning \( PE = false \) when it is unable to perform a precise analysis of the input HJ program. Similar to NEP analysis, we use condition vectors to qualify the instances of execution of statements that are place equivalent.

4. In the final step of MHP analysis as defined in Section 3.6, we combine NEP and PE analyses to further refine MHP information for isolated constructs. The basic intuition is that for all instances of statement pairs where \( NEP \) is true, MHP is assigned false. In addition, if the statements are executed in isolation, then MHP is assigned false for all those instances of execution which happen at the same place.

Each of the above steps is described in detail in the following sections. An earlier version of these results was presented in [2].

### 3.3 Program Structure Tree (PST) Representation

We introduce the Program Structure Tree (PST) representation for HJ procedures, which will be used in later sections as the foundation for performing MHP analysis.

**Definition 3.3.1** A Program Structure Tree \( PST(N,E) \) for a procedure is a rooted tree where
1. \( N \) is a set of nodes such that each node \( n \in N \) has one of the following types: root, statement, loop, async, finish, isolated. The root node designates the start of the procedure. Each async node is annotated with a place expression that indicates the HJ place executing the async. Likewise, each isolated node can be annotated with a set of places (default is at the current place, i.e., here). Note that a statement node does not contain any loop, but may contain other control flow structures such as if (represented as IF-STMT in the PST).

2. \( E \) is set of tree edges obtained by collapsing the abstract syntax tree representation of the procedure into the six node types listed above. The tree edges define the parent-child relationships in the PST.

An PST is directly obtained from an abstract syntax tree and is linear in size with respect to the program. Note that HJ language semantics ensures that an isolated node will not be an ancestor of finish or async node. In addition, all statement nodes must be leaf nodes in the PST.

### Definition 3.3.2
For a PST, parent\((N)\) returns the parent of node \( N \) as defined by \( E \). parent\((\text{root})\) is defined to be null.

### 3.3.1 Example

Figure 3.1 contains a simple example of an HJ code fragment. The code fragment operates on a three dimensional array \( A \) whose values are distributed across places. Parallel tasks are created at multiple HJ places on line 5 based on the underlying data distribution of the array \( A \). We will describe details of array distributions later when we discuss PE analysis. The array elements are exclusively referenced and modified in statement \( \text{S1} \) and statement \( \text{S2} \) respectively within a single place using the isolated constructs. Note that the read of \( A[i - 1, j, k] \) on \( \text{S1} \) uses a value written in \( \text{S2} \), i.e., there exists a flow dependence from \( \text{S2} \) to \( \text{S1} \). PST for the example program is shown
for ( i = 1 ; i <= n ; i++ )
finish
  for ( j = 1 ; j <= n ; j++ )
    for ( k = 1 ; k <= n ; k++ )
      async (A.distribution[i,j,k])
        isolated {
          S0 : temp = 0;
          if (...) {
            S1 : temp = f(A[i-1,j,k]);
          }
          S2 : A[i,j,k] = temp;
        }

Figure 3.1 : Example HJ program to demonstrate the computation of $MHP(S1,S2)$.

Figure 3.2 : PST for example program in Figure 3.1
in Figure 3.2. As can be seen, the PST has a direct correspondence with the source level program constructs.

3.4 Never-Execute-in-Parallel Analysis

In this section, we describe the approach for determining if two statements will *never execute in parallel* (NEP). The NEP relation is the complement of the *May-Happen-in-Parallel* (MHP) relation that has been introduced in past work for Java and other concurrent programming languages. That is, NEP between two statements holds true if no instances of the two statements can never occur simultaneously. NEP is used instead of MHP in this section for the sake of convenience in presentation. In addition, the NEP relation will be used to compute a refined MHP relation later in Section 3.6.

The algorithm for computing the NEP relation is given in Figure 3.3. The algorithm takes two inputs: the PST for the HJ procedure being analyzed, and two statements, $S_1$ and $S_2$, for which we want to compute whether NEP is true or false. Note that the algorithm also accepts the case where $S_1 = S_2$. The first step is to find the least common ancestor of the two statements, denoted by $A = LCA(S_1,S_2)$. This gives us the common scope of execution of the two statements. In Steps 4 and 10, it is established for $S_1$ and $S_2$ respectively whether they execute within an “unfinished” async created within $A$. Depending on this information, there are 4 cases that arise for NEP analysis as described in Steps 20 - 26:

- **Case 1 (Step 20):** If both $S_1$ and $S_2$ do not execute in an async construct under $A$ then we can conclude they will never execute in parallel.

- **Case 2 and 3 (Steps 22 and 24):** If exactly one of $S_1$ or $S_2$ executes in an async scope, then the dominator relation (as defined in Section 2.1.3) can be used to compute the NEP($S_1$, $S_2$) relation. In the algorithm, the dominator relation is checked on ancestors of $S_1$ and $S_2$ ($AS_1$ and $AS_2$ respectively) that
function BasicNEP()
    Input: PST and two statement nodes $S_1$ and $S_2$ in the PST
    Output: NEP($S_1, S_2$), a boolean value
    $A := LCA(S_1, S_2)$, the Lowest Common Ancestor of $S_1$ and $S_2$ in the PST;
    //Determine if an instance of $S_1$ can be executed in a new async activity that escapes a given execution instance of $A$
    async$_{S_1} := false$;
    for $N := S_1$ ; $N \neq A$ ; $N := parent(N)$ do
        if $N$ is an async node then
            async$_{S_1} := true$;
        if $N$ is a finish node then
            async$_{S_1} := false$;
    //Repeat the previous step for $S_2$
    async$_{S_2} := false$;
    for $N := S_2$ ; $N \neq A$ ; $N := parent(N)$ do
        if $N$ is an async node then
            async$_{S_2} := true$;
        if $N$ is a finish node then
            async$_{S_2} := false$;
    flag := false;
    if $S_1 \neq S_2$ then
        //Analyze four cases for async$_{S_1}$ and async$_{S_2}$
        AS$_1 :=$ PST ancestor of $S_1$ that is a child of $A$; //Note that AS$_1 := S_1$
            if $S_1$ is a child of $A$
                AS$_2 :=$ PST ancestor of $S_2$ that is a child of $A$; //Note that AS$_2 := S_2$
                    if $S_2$ is a child of $A$
                        if $\neg$ async$_{S_1} \land \neg$ async$_{S_2}$ then
                            flag := true; //Case 1
                        if $\neg$ async$_{S_1} \land$ async$_{S_2}$ then
                            flag := (AS$_2$ does not dominate AS$_1$); //Case 2
                        if async$_{S_1} \land \neg$ async$_{S_2}$ then
                            flag := (AS$_1$ does not dominate AS$_2$); //Case 3
                        if async$_{S_1} \land$ async$_{S_2}$ then
                            flag := false; //Case 4
        return NEP($S_1, S_2$) := flag;

Figure 3.3 : Algorithm for computing Never-Execute-in-Parallel (NEP) relations
Main thread:
----------
S1: ExternalHelper1.start();
S2: ...
S3: ExternalHelper1.join();
S4: ... // MHP algorithm concludes that
    // S11 and S12 may happen in parallel with S4

ExternalHelper1 thread:
-----------------------
S5: ...
S6: InternalHelper1_1.start();
S7: InternalHelper1_2.start();
S8: InternalHelper1_1.join();
S9: InternalHelper1_2.join();
S10: ... // MHP algorithm concludes that
      // S11 and S12 cannot happen in parallel with S10

InternalHelper1_1 thread:
------------------------
S11: ...

InternalHelper1_2 thread:
------------------------
S12: ...

Figure 3.4: Java example program to illustrate MHP algorithm

are immediate children of LCA(S₁, S₂). If the PST path from S₁ upto LCA(S₁, S₂) contains an async node which is not followed by any finish node and AS₁ dominates AS₂, then S₁ and S₂ will never execute in parallel.

- Case 4 (Step 26): If both S₁ and S₂ execute in a async scope, then we have to conservatively assume that NEP = false.

3.4.1 Comparison with MHP Analysis of Java programs

We compare the NEP algorithm from Figure 3.3 with the MHP data flow analysis algorithm developed by [92]. The later algorithm was designed to address all concurrency features in Java threads, including wait/notify/notifyAll operations in
S0: finish {
    S1: async { // ExternalHelperThread1.start()
        finish {
            S5: ...
            S6: async S11 // InternalHelperThread1_1.start()
            S7: async S12 // InternalHelperThread1_2.start()
            ...
        }
        S8: ... // finish subsumes InternalHelper1_1.join()
        S9: ... // finish subsumes InternalHelper1_2.join()
        S10: ... // NEP algorithm concludes that
            // NEP(S10,S11) = NEP(S10,S12) = false
    }
    S2: ...
}
S3: ... // S0's finish subsumes ExternalHelperThread1.join()
S4: ... // NEP algorithm concludes that
    // NEP(S4,S11) = NEP(S4,S12) = false

Figure 3.5: HJ example program to illustrate NEP algorithm

Figure 3.6: PST for example program in Fig 3.5
synchronized blocks. In this comparison, we will restrict our attention to the MHP algorithm’s handling of the start, join, and synchronized constructs in Java threads, which are comparable, but not equivalent, to async, finish, and isolated in HJ.

Figure 3.4 contains the skeleton of a Java program that represents the parallel control flow in the SplitRendererNested example used in [92]. As discussed in [92], their MHP algorithm is conservative in its analysis of nested parallelism and concludes that S11 and S12 may happen in parallel with S4, even though it is able to conclude that S11 and S12 cannot happen in parallel with S10.

As a comparison, Figure 3.5 contains the skeleton of an HJ program that is equivalent to the Java program in Figure 3.4. Its PST is shown in Figure 3.6. If the NEP algorithm from Figure 3.3 is invoked to compute NEP(S4,S11), it will perform the following steps to conclude that NEP(S4,S11) = true:

- Step 2: A := LCA(S4,S11) = ROOT
- Step 4: async_S4 := false
- Step 10: async_S11 := false
- Step 17: AS4 := S4
- Step 18: AS11 := S0
- Step 20: flag := true
- Step 27: NEP(S4,S11) := true

Thus, the NEP algorithm is able to establish that S11 and S12 cannot happen in parallel with S4, while the MHP algorithm from [92] conservatively concludes that S11 and S12 may happen in parallel with S4.

The above discussion focused on the conservativeness in analysis of nested parallelism in past work on MHP analysis. As mentioned earlier, another dimension of conservativeness in MHP analysis of Java programs is the necessity to perform
Main thread:
---------
int i,j,k; // Shared scalar variables
int A[][][] = ...; // Shared array A
for (i=1; i<n; i++) {
    for (j=0; j<n; j++) {
        for (k=0; k<n; k++) {
            t[j][k] = ...; // Allocate threads
t[j][k].start(); // start child threads
        }
    }
    for (j=0; j<n; j++) {
        for (k=0; k<n; k++) {
            t[j][k].join(); // join child threads
        }
    }
}

Child thread:
-----------
S1: ... = f(A[i-1,j,k];
S2: A[i,j,k] = ...;

Figure 3.7: Java code example that demonstrates that \( NEP(S1, S2) \) is not just a binary relation

interprocedural pointer alias analysis of thread objects to establish accurate parallel control flow relationships among threads. For example, the \( MHP \) analysis must establish that all thread objects (\( e.g., \) ExternalHelper1, InternalHelper1_1, InternalHelper1_2) are distinct before it can even conclude that \( S11 \) and \( S12 \) cannot happen in parallel with \( S10 \) in Figure 3.4. As observed in past work on static data race detection, interprocedural alias analysis of thread objects can pose a significant challenge in practice. In contrast, intra-procedural analysis of HJ’s \( \text{async}, \text{finish}, \) and \( \text{isolated} \) constructs is simpler because it does not rely on alias analysis of thread objects.

As defined thus far, \( NEP \) is a binary relation, \( i.e., \) it returns either \( \text{true} \) or \( \text{false} \). However, this is not precise enough to capture all possible scenarios. Consider the \text{Java} example program and the corresponding HJ example program shown in Figure 3.7
Main thread: 
-------------
int A[][][] = ...; // Shared Array
for (i=1; i<n; i++) {
    finish {
        for (j=0; j<n; j++) {
            for (k=0; k<n; k++) {
                async {
                    S1: ... = f (A[i-1,j,k]);
                    S2: A[i,j,k] = ...
                }
            }
        }
    }
}

Figure 3.8: HJ code example that demonstrates that $NEP(S1, S2)$ is not just a binary relation

and Figure 3.8, respectively. If we apply the algorithms presented in Figure 3.3 to Figure 3.8 or the algorithm from [92] to Figure 3.7, we will conclude that statements $NEP(S1, S2) := false$, i.e., they may execute in parallel. However, if we observe the example in Figure 3.7 closely, for threads executed with same value of $i$ and any value of $j$ and $k$, $NEP(S1, S2) := false$ because iterations of $j$ and $k$ loops can execute in parallel. However, for threads across multiple $i$ iterations, $NEP(S1, S2) := true$ as all the threads created using $j$ and $k$ loops are joined before next iteration of $i$. This indicates that the binary relation of $NEP$ definition is not precise enough to capture the above mentioned scenario and hence, conservatively reports $NEP(S14, S15) := false$.

What we need is a more precise definition that extends the $NEP$ relation to statement instances from individual loop iterations. There exists a large body of work in the domain of automatic parallelization that uses direction vectors and distance vectors [68] to distinguish arrays accessed across multiple iterations. Motivated by direction and direction vectors, we define the $NEP$ relation using a condition vector notation. Formally,
Definition 3.4.1 Two statements $S_1$ and $S_2$ are said to never execute in parallel, written as $\text{NEP}(S_1, S_2) = true$, with condition vector set $CS$ if the following conditions hold:

1. $S_1$ and $S_2$ have $k \geq 0$ loop nodes, $L_1, \ldots, L_k$ as common ancestors in the PST.
2. Each element $(C_1, \ldots, C_k)$ in $CS$ is a vector of $k$ functions of type $\text{int}\times\text{int} \rightarrow \text{boolean}$. In this definition, we will restrict our attention to three possible functions: “=”, “≠”, and “∗”. The symbol “∗” denotes the function that returns true for all inputs:\footnote{These three operators have been also used in past definitions for direction vectors [124], and can easily be extended to distance vectors.}
3. Let $S_1[i_1, \ldots, i_k]$ denote any execution instance of $S_1$ in iteration $i_1, \ldots, i_k$ of loops $L_1, \ldots, L_k$, and likewise for $S_2[j_1, \ldots, j_k]$. If $C_x(i_x, j_x) = true \ \forall 1 \leq x \leq k$ for some condition vector $(C_1, \ldots, C_k)$ in $CS$, then it is guaranteed that statement instances $S_1[i_1, \ldots, i_k]$ and $S_2[j_1, \ldots, j_k]$ cannot execute in parallel. □

To summarize Definition 3.4.1, if $\text{NEP}(S_1, S_2) = false$ then there are no pairs of instances of $S_1$ and $S_2$ that can be guaranteed to not execute in parallel. If $\text{NEP}(S_1, S_2) = true$ then the instances of $S_1$ and $S_2$ that can be guaranteed to not execute in parallel are determined by the condition vectors in $CS$. If $CS$ is $(\ast, \ldots, \ast)$ then no instance of $S_1$ can execute in parallel with any instances of $S_2$.

The refined algorithm for computing the NEP relation is given in Figure 3.9. The refined algorithm invokes the basic algorithm presented in Figure 3.3 and embeds condition vector set to NEP. Step 9 is performed in the case when $S_1$ and $S_2$ have $k \geq 1$ common loops. This step examines all nodes in the PST starting from $A$, the least common ancestor of $S_1$ and $S_2$, and ending at $L_1$, the outermost common loop that encloses $S_1$ and $S_2$. Note that the algorithm uses the fact whether a loop contains a finish or async node in the PST to restrict the set of iterations for which NEP
function RefinedNEP()

Input: PST and two statement nodes $S_1$ and $S_2$ in the PST with $k \geq 0$ common loop node ancestors in the PST, $L_1, \ldots, L_k$

Output: $NEP(S_1, S_2)$, a boolean value and $CS$, a set of condition vectors that is used only if $NEP(S_1, S_2) = true$. Given statement instances $S_1[i_1, \ldots, i_k]$ and $S_2[j_1, \ldots, j_k]$, if $C_x(i_x, j_x) = true \ \forall 1 \leq x \leq k$ for some condition vector $\langle C_1, \ldots, C_k \rangle$ in $CS$ then it is guaranteed that the two statement instances cannot execute in parallel.

A := LCA($S_1, S_2$);
flag = BasicNEP();
if $S_1 \neq S_2 \land flag$ then
$CS := CS \cup \{ \langle "=", \ldots, "=" \} \};$
if $k \geq 1$ then
  // $S_1$ and $S_2$ have at least one common loop
  seqloop := true;
  $x := k + 1$;
  for $N := A ; \ N \neq L_1 ; \ N := parent(N)$ do
    if $N$ is an async node then
      seqloop := false;
    if $N$ is a finish node then
      seqloop := true;
    if $N$ is a loop node then
      $x := x - 1$;
    if seqloop then
      $CS :=$
      $CS \cup \{ (C_1 = "=", \ldots, C_x = "\neq", C_{x+1} = "\ast", \ldots, C_k = "\ast") \} \};$
return $NEP(S_1, S_2) := ( CS \neq \emptyset )$; //Return $NEP = true$ if $CS$ is non-empty.

Figure 3.9: Algorithm for computing refined Never-Execute-in-Parallel ($NEP$) relations.
If (say) loop $L_x$ contains a finish node that is an ancestor of both $S_1$ and $S_2$ statements and there is no intervening async node in PST path from the finish node to $L_x$, we observe that instances of $S_1$ and $S_2$ from two distinct iterations of $L_x$ (but created in the same iteration of outer loops $L_1, \ldots, L_{x-1}$) can never execute in parallel. This property is captured by a condition vector in which $C_x$ is set to “≠”, $C_1, \ldots, C_{x-1}$ are set to “=”, and $C_{x+1}, \ldots, C_k$ are set to “*”.

### 3.4.2 Complexity

The algorithm in Figure 3.9 assumes that the PST has already been constructed, which is a one-time $O(N)$ cost. In addition, Steps 22 and 24 of Figure 3.3 use the dominator relation on the original control flow graph, which can be computed using algorithms that vary in execution time complexity from $O(N \log N)$, $O(N \alpha(N))$ [79] to $O(N)$ [61] as a one-time cost. We observe that the NEP algorithm takes $O(H)$ time to determine if a given pair of nodes, $S_1$ and $S_2$, satisfy $NEP(S_1, S_2) = true$, where $H$ is the height of the PST. Note that the condition vector set $CS$, can contain at most $L+1$ condition vectors — one contributed by Step 5 and $L$ by Step 17 — each of which has $O(L)$ size, where $L \leq H$ is the maximum nesting of loops in the PST. Step 14 can be considered a constant time operation. If used to compute the NEP relation for all pairs of statements, the total execution time will be $O(N^2H)$, which is more efficient than the $O(N^3)$ time of the MHP algorithm in [92]. However, we expect that the execution time overhead of the NEP algorithm will be much smaller than $O(N^2H)$ in practice, since it can be used in a demand-driven fashion to only compute the NEP relation for pairs of statements that are of interest to an interactive tool or compiler transformation system.

### 3.4.3 Example

We start by using the example program in Figure 3.1 to illustrate the algorithm. The example was intentionally chosen to be as simple as possible to illustrate the core
ideas. In this example, we are interested in determining which pairs of execution instances of statements \(S_1\) and \(S_2\) will never execute in parallel with each other, so the algorithm in Figure 3.9 will be invoked to compute \(NEP(S_1, S_2)\). The output of this algorithm will be \(NEP(S_1, S_2) = true\), with condition vector set \(CS= \{(=, =, =), (\neq, *, *)\}\). This implies that two instances of \(S_1\) and \(S_2\) are guaranteed to never execute in parallel if: 1) they belong to the same \(i-j-k\) iteration, or 2) they come from iterations with distinct values of \(i\). The first case is true because the statements are executed in order with respect to the same \texttt{async}. The second case is true because of the \texttt{finish} construct within each for-\(i\) loop iteration.

Now, let us use the following HJ code fragment to illustrate the four cases in Step 15 in the basic \(NEP\) algorithm provided in Figure 3.3:

\[
\{ \texttt{S1 ; async S2 ; S3 ; async S4 ; } \}
\]

Case 1 \(NEP(S_1, S_3) = true\), in accordance with Step 20.

Case 2 \(NEP(S_1, S_2) = true\), in accordance with Step 22.

Case 3 \(NEP(S_2, S_3) = false\), in accordance with Step 24.

Case 4 \(NEP(S_2, S_4) = false\), in accordance with Step 26.

To summarize, the significant differences between the \(NEP\) analysis presented in this chapter and past work on \(MHP\) analysis are as follows:

1. The availability of basic concurrency control constructs in HJ such as \texttt{async} and \texttt{finish} enables a more efficient and precise \(NEP\) analysis algorithm compared to past work on \(MHP\) analysis for Java. Our algorithm is based on simple path traversals in the \(PST\).

2. Past work on \(MHP\) analysis resulted in a simple true/false value for a given pair of statements. Our work makes the \(NEP\) relation more precise by adding
condition vectors that are able to identify execution instances for which the NEP relations hold.

3. As discussed later in Section 3.6, we show how the NEP information can be further refined by using the isolation properties of isolated blocks in HJ.

### 3.5 Place Equivalence Analysis

In an attempt to combine shared-memory programming (e.g., Java, OpenMP, pthreads) and distributed-memory programming (e.g., MPI, UPC), HJ inherits the place feature from X10. Places co-locate data objects and the activities that operate on them. This feature can also be used to mitigate some of the false-sharing issues that arise in parallel computing. The activities that execute within a single place can have mutually exclusive accesses using isolated HJ constructs. Before we analyze isolated constructs, we need to determine if two statements can execute within the same place or not.

In this section, we describe our approach for determining if two statements are place equivalent (PE), i.e., if they will definitely execute at the same place. Most parallel programming models that are currently used for distributed-memory multiprocessors follow a Single Program Multiple Data (SPMD) model in which one thread is executed per place. However, the HJ programming model is more general since it integrates thread-level parallelism and cluster-level parallelism by allowing multiple activities to be created within the same place and across different places. Place equivalence analysis therefore becomes important for more general parallel programming models such as HJ.

Similar to the NEP relation, we need to distinguish the PE relation within loop iterations using a condition vector set.

**Definition 3.5.1** Two statements $S_1$ and $S_2$ are said to be place equivalent, written as $PE(S_1, S_2) = true$, with condition vector set $CS$ if the following conditions hold:
1. $S_1$ and $S_2$ have $k \geq 0$ loop nodes, $L_1, \ldots, L_k$ as common ancestors in the PST.

2. Let $S_1[i_1, \ldots, i_k]$ denote any execution instance of $S_1$ in iteration $i_1, \ldots, i_k$ of loops $L_1, \ldots, L_k$, and likewise for $S_2[j_1, \ldots, j_k]$. If $C_x(i_x, j_x) = \text{true}$ for some condition vector $(C_1, \ldots, C_k)$ in $CS$, then it is guaranteed that statement instances $S_1[i_1, \ldots, i_k]$ and $S_2[j_1, \ldots, j_k]$ must execute at the same place. □

To summarize Definition 3.5.1, if $PE(S_1, S_2) = \text{false}$ then there are no pairs of instance of $S_1$ and $S_2$ for which place equivalence is guaranteed. If $PE(S_1, S_2) = \text{true}$ then the instances of $S_1$ and $S_2$ that can be guaranteed to execute at the same place are determined by the condition vectors in $CS$.

Before we describe the complete algorithm for $PE$ computation, let us describe two subtle issues that complicate the analysis. First, async statements in $HJ$ can be provided with an additional place expression to indicate the location/place of the activity (without any place expression, the activity is created at the default place i.e., 0). This implies that, given two statements $S_1$ and $S_2$, $PE(S_1, S_2)$ needs to determine if the place expression of the activity executing $S_1$ is equal to the place expression of the activity executing $S_2$. This boils down to determining if two expressions can have the same value. The idea is to perform a Definitely Same (as described in 2.6.1) equivalence analysis, $\mathcal{DS}$, on all place expressions in the procedure. For two place-valued expressions, $e_1$ and $e_2$, $\mathcal{DS}(e_1, e_2) = \text{true}$ indicates that they must evaluate to the same place. A global value numbering technique such as [6] can be used to assign each expression $e$ a value number $\mathcal{V}(e)$. $\mathcal{DS}$ can then be computed by using the lexical identity, $\mathcal{DS}(e_1, e_2) := (\mathcal{V}(e_1) = \mathcal{V}(e_2))$. More advanced techniques for place equivalence analysis are described in [3, 37].

Second, the distributions of the underlying arrays accessed within a loop are needed to determine place equivalence. Consider the following code fragment as an example:
for (i = 1; i <= n; i++) // L1
for (j = 1; j <= n; j++) // L2
for (k = 1; k <= n; k++) // L3
async (A.distribution[f(i,j),k]) S;

We need to know the data distribution of array A in the async statement in A.distribution[f(i,j),k]. In HJ and X10 [38], array A can be distributed using a wide range of standard and user-defined distributions such as UNIQUE, RANDOM, CYCLIC, and BLOCK. As an example, let us assume that A is distributed in (BLOCK, *) fashion so that A[m, *] is guaranteed to reside at the same place, where m is the index of the first dimension. A depiction of (BLOCK, *) is shown in Figure 3.10. In this figure, both (BLOCK, *) and (CYCLIC, *) will result in the same distribution because A has \( p \times p \) elements. The async activity in the above code fragment with distribution (BLOCK,*) will be mapped to different places based on indices i and j, but not k, i.e., place-variant with respect to loops L1 and L2. We term this kind of analysis as LoopSet analysis. LoopSet determines loops for which a place expression is place variant. The LoopSet for the above code fragment is LoopSet(A.distribution[f(i,j),k]) := \{L1, L2\}, where L3 is not in the LoopSet indicates that it is place invariant.

LoopSet analysis for other HJ distributions can be determined in a similar fashion to (BLOCK, *) and have been omitted for simplicity of presentation. Note that, it may be in general hard to perform LoopSet analysis e.g., use of advanced features like array views, and array projection. In such cases, we can always use a conservative approach to include a loop in the LoopSet.

The algorithm for computing the PE relation is given in Figure 3.11. As described above, the algorithm needs two additional pre-passes as inputs along with the PST. First, a global place-value numbering pre-pass for place expressions to determine place local information for statements. Second, a global loop-invariant analysis based LoopSet analysis pre-pass to determine loops for which a given place expression is place variant. Global value numbering can be performed using an SSA-based algorithm as
Figure 3.10: (BLOCK, *) distribution of array $A[p,p]$ that uses $p$ places

in [6]. Global loop-invariant analysis [5] can be used to compute LoopSet information.

The algorithm uses value numbering for each node in the PST. This is computed by propagating the value number of the async that is executing the given node. As shown in Step 7 of Figure 3.11, a pair of statements $S_1$ and $S_2$ associated with same global place-value numbers, i.e., $V(S_1) = V(S_2)$ are always going to execute at the same place. If $V(S_1) \neq V(S_2)$ and there are no intervening async nodes within the innermost common scope of $S_1$ and $S_2$, then these statements are also bound to execute at the same place.

Step 13 traverses the common ancestors (only loop and async PST nodes) to compute condition vector using LoopSet. For loops that are placeLocalLoops, the condition vector entries are set to *. Note that LoopSet keeps track of the place-variant loops and placeLocalLoops keeps track of place-invariant loops.
function PEAnalysis()

Input: (1) PST and two statement nodes $S_1$ and $S_2$ in the PST with $k \geq 0$ common loop node ancestors in the PST, $L_1, \ldots, L_k$; (2) A value number $V(e)$, for each place expression $e$ that is the target of an async $(e)$ statement. For convenience, we also assume the availability of $V(N)$ for each async node $N$ in the PST, where $V(N)$ denotes the value of here for the activity executing $S$; (3) For each place expression $e$, $\text{LoopSet}(e) = \text{subset of loops } \{L_1, \ldots, L_m\}$ for which the value of place expression $e$ is place-variant

Output: $PE(S_1, S_2)$, a boolean value and $CS$, a set of condition vectors that is used only if $PE(S_1, S_2) = \text{true}$

A := $\text{LCA}(S_1, S_2)$, the Least Common Ancestor of $S_1$ and $S_2$ in the PST;
Compute async$_S_1$, and async$_S_2$ as in Figure 3.3;
$CS := \emptyset$; //Initialize $CS$ to an empty set
if $S_1 \neq S_2$ then
  if $V(S_1) = V(S_2)$ then
    $CS := CS \cup \{(*, \ldots, *)\}$;
    $PE(S_1, S_2) := \text{true}; \text{return}$;
  else if $\lnot \text{async}_S_1 \land \lnot \text{async}_S_2$ then
    $CS := CS \cup \{ (=, \ldots, =) \}$; //Instances of $S_1$ and $S_2$ that come from the same iteration of $L_1, \ldots, L_k$ must execute in the same activity and hence at the same place

// $S_1$ and $S_2$ have at least one common loop
if $k \geq 1 \land \lnot \text{async}_S_1 \land \lnot \text{async}_S_2$ then
  $\text{placeLocalLoops} := \{L_1, \ldots, L_k\}; x := k + 1$;
  for $N := A ; N \neq L_1 ; N := \text{parent}(N)$ do
    if $N$ is an async node with destination place expression $e$ then
      $\text{placeLocalLoops} := \text{placeLocalLoops} - \text{LoopSet}(e)$;
    if $N$ is a loop node then
      $x := x - 1$;
      if $L_x \in \text{placeLocalLoops}$ then
        $C_x := "*"$;
      else
        $C_x := "= "$;
    if $\text{placeLocalLoops} \neq \emptyset$ then
      $CS := CS \cup \{ (C_1, \ldots, C_k) \}$;
  return $PE(S_1, S_2) := (\text{CS} \neq \emptyset)$ //Return $PE = \text{true}$ if $CS$ is non-empty

Figure 3.11: Algorithm for computing Place Equivalence (PE) relations
3.5.1 Complexity

The algorithm in Figure 3.11 assumes that the PST is constructed in $O(N)$ time. The pre-passes for the other inputs to the algorithm, Global Value Numbering\(^2\) and LoopSet analysis, can also be computed in linear time. We observe that Step 2 takes $O(H)$ time. The condition vector set $CS$, can at most have two entries – one obtained from Step 7 and another from Step 23 – each of which has $O(L)$ size, where $L \leq H$. For all pairs of statements in the HJ program, the overall complexity of PE analysis is bounded by $O(N^2H)$, which is the same complexity as that of NEP analysis. As before, this can be limited to $O(H)$ time for each statement pair queried in a demand-driven fashion after the initial data structures are constructed.

3.5.2 Example

Let us now see how the algorithm works for the example program in Figure 3.1, assuming that array $A$ has a (BLOCK,BLOCK,* \(\rightarrow\)) distribution. This means that elements $A[i, j, *]$ of array $A$ are guaranteed to be mapped to the same place, and the async statement in the example will follow the same distribution. Hence, the PE algorithm will compute $placeLocalLoops = \{L_3\}$, which in turn results in a place condition vector set of $CS= \{(=,=,\_), (=,=,*))\}$. This implies that $S_1$ and $S_2$ with same values of $i$ and $j$ are guaranteed to execute in the same place. Note that, in general, the algorithm in Figure 3.11 does not require that the number of dimensions in an array reference to match the number of loops in the loop nest or that the index ordering for the array access match the ordering of the loop nesting.

3.6 MHP Analysis using Isolated Sections

In this section, we show how the Never-Execute-in-Parallel (NEP) analysis from Section 3.4 can be combined with the Place-Equivalence (PE) information analysis

\(^2\)For an SSA-based algorithm such as [6], the complexity is technically linear in the size of the SSA form, which in turn is observed to be linear in the size of the input program in practice.
from Section 3.5 to obtain a more precise May-Happen-in-Parallel (MHP) analysis for HJ programs by using isolated sections. The simple approach to computing MHP would be to simply invert the NEP relation, i.e., to return $MHP(S_1, S_2) = false$ when $NEP(S_1, S_2) = true$. The key insight leveraged in this section is that two execution instances of statements $S_1$ and $S_2$ in an HJ program are guaranteed to not happen in parallel if they both occur in isolated sections that are executed at the same place. This enables us to broaden the number of executions for which we can assert that $MHP = false$. Note that instances of $S_1$ and $S_2$ can indeed happen in parallel if they occur in isolated sections that execute at different places.

Figure 3.12: Algorithm for computing May-Happen-in-Parallel (MHP) relation using place equivalence and isolated sections in HJ

```python
1 function FinalMHP()
2     Input : PST and two statement nodes $S_1$ and $S_2$ in the PST with $k \geq 0$
3            common loop node ancestors in the PST, $L_1, \ldots, L_k$
4     Output: $MHP(S_1, S_2)$, a boolean value and $CS$, a set of condition vectors
5            that is used only if $MHP(S_1, S_2) = false$
6     Compute $NEP(S_1, S_2)$ and its associated condition vectors, $CS_{NEP}$ using the
7            $NEP$ algorithm in Figure 3.9;
8     Set isolated$_1 := true$ if $S_1$ has an isolated node as an ancestor in the
9            PST;
10    Set isolated$_2 := true$ if $S_2$ has an isolated node as an ancestor in the
11            PST;
12        //Combine NEP and PE analysis results
13        if isolated$_1 \land isolated$_2 then
14            Compute $PE(S_1, S_2)$ and its associated condition vectors, $CS_{PE}$ using
15            the $PE$ algorithm in Figure 3.11;
16            $MHP(S_1, S_2) := \neg (NEP(S_1, S_2) \lor PE(S_1, S_2));$
17            $CS := CS_{NEP} \cup CS_{PE};$
18        else
19            //Just return NEP analysis results
20            $MHP(S_1, S_2) := \neg NEP(S_1, S_2);$
21            $CS := CS_{NEP};$
22    return
```

The algorithm for computing the $MHP$ relation is given in Figure 3.12. For a pair of statements $S_1$ and $S_2$, Steps 3 and 4 check if they are nested in isolated blocks. In case both $S_1$ and $S_2$ are nested in isolated blocks, $MHP(S_1, S_2)$ is computed by combining the $NEP$ and PE results in Step 7. Otherwise, the $MHP$ relation is computed directly from the $NEP$ relation.

3.6.1 Complexity

The complexity of computing the $MHP$ relation is bounded by $O(N^2H)$. This is due to the complexity of computing both $NEP$ relation and $PE$ relation. As before, this can be limited to $O(H)$ time for each statement pair queried in a demand-driven fashion after the initial data structures are constructed.

3.6.2 Example

As discussed earlier, the $NEP$ solution computed using the analysis described in Section 3.4 for the example in Figure 3.1 was $NEP(S_1, S_2) = true$ with condition vector set $CS_{NEP} = \{\langle =, =, = \rangle, \langle \neq, *, * \rangle\}$. This indicates that for different values of loop index $i$, $S_1$ and $S_2$ can not execute in parallel. However, this information can be refined using the $PE$ solution due to the presence of isolated PST node in the loop body. The $PE$ solution computed using the analysis described in Section 3.5 was $PE(S_1, S_2) = true$ with condition vector set $CS_{PE} = \{\langle =, =, = \rangle, \langle =, =, * \rangle\}$. This indicates that $S_1$ and $S_2$ will execute at the same HJ place for different values of loop index $k$ but with same values for loop indices $i$ and $j$. Using the above two results, the algorithm in this section is able to determine that $MHP(S_1, S_2) = false$ with condition vector set $CS = \{\langle =, =, = \rangle, \langle \neq, *, * \rangle, \langle =, =, * \rangle\}$, i.e., $MHP(S_1, S_2) = false$ if they belong to the same $i$-$j$-$k$ iteration, or if they come from iterations with distinct values of $i$ or with the same values of $i$ and $j$. 
3.7 Summary

In this chapter, we introduced a new demand-driven algorithm for May-Happen-in-Parallel (MHP) analysis that is applicable to any language that adopts the core concepts of places, async, finish, and isolated blocks from the HJ programming model. The main contributions of this work compared to past MHP analysis algorithms are as follows:

1. We introduced a more precise definition of the MHP relation than in past work by adding condition vectors that identify execution instances for which the MHP relation holds, instead of just returning a single true/false value for all pairs of executing instances.

2. Compared to past work, the availability of basic concurrency control constructs such as async and finish enabled the use of more efficient and precise analysis algorithms based on simple path traversals in the Program Structure Tree, and did not rely on interprocedural pointer alias analysis of thread objects as in MHP analysis for the Java language.

3. We introduced place equivalence (PE) analysis to identify execution instances that happen at the same place. The PE analysis helps us in leveraging the fact that two statement instances which occur in isolated blocks that execute at the same HJ place must have $MHP = false$.

MHP analysis described in this chapter can be extended in future to other advanced concurrency constructs of HJ such as future-force and phaser. Additionally, the analysis currently operates at an intraprocedural level leaving extensions for interprocedural level as future work [10], perhaps as extensions to the interprocedural side effect analysis of parallel programs discussed in the next chapter.
Chapter 4

Side-Effect Analysis for Parallel Programs

Side-effect analysis determines the effects of a procedure call at a call site. Knowledge of the side-effects of a procedure call has several important applications. For example, many program analyses need to understand the effects of a procedure call so as to enable program transformations across procedure boundaries. In Section 2.5, four different types of side-effects using Banning’s formulation, i.e., MOD, REF, USE, and DEF have been described. Both MOD and REF side-effects are flow-insensitive problems whereas USE and DEF are flow-sensitive problems. Flow-sensitive problems are solved by tracing through control-flow paths whereas flow-insensitive problems are solved by ignoring the control flow paths. The complexity of computing MOD side-effects using Banning’s approach (as described in Section 2.5) is $O(NE\alpha(E, N))$, where $E$ is the number of call sites in the program, $N$ is the number of procedures in the program and $\alpha$ represents the inverse Ackermanns’ function.

JIT and dynamic compilers perform program optimization and transformation in limited situations. In particular they do not typically perform interprocedural analyses. The reason behind this is two folds: 1) the whole program may not be available during compilation and programs may be loaded dynamically; 2) the complexity of interprocedural analyses is high. The former case is addressed by closed-world solutions presented in [36]. This chapter addresses the later case and presents a fast flow-insensitive and field-insensitive side-effect analysis. The side-effect analysis does not need any interprocedural alias analysis to propagate the exact object references modified or referenced. Instead, it unifies all the references of an object field using a heap-array representation. This representation does not need any
special attention for parameter bindings.

Due to the advent of parallel programming languages for multi-core systems, the analysis of parallel programs pose additional challenges to dynamic compilers due to parallel constructs in the program. These parallel constructs in general carry inherent side-effects in them and hence further add to the complexity of the side-effect analysis. Note that there is a natural interplay between the side-effect analysis and parallel constructs analysis, since parallel constructs are usually translated to low-level runtime library procedure calls in the intermediate representation level at which program transformations are performed. In this chapter, we propose an algorithm to perform side-effect analysis of programs with parallel constructs in a dynamic compiler.

Section 4.1 discusses a flow-insensitive and field-insensitive side-effect analysis algorithm suitable for analyzing method calls in a dynamic compilation environment. Section 4.2 describes how the side-effect analysis algorithm can be extended for the async, finish, and isolated core parallel constructs of HJ programming language.

4.1 Side-Effect Analysis of Method Calls

Let us describe the impact of side-effect analysis on program transformations. Consider the code fragment shown in Figure 4.1. Let us assume that we are performing scalar replacement for load elimination transformation (as described in Section 2.6) for method bar. We notice that there is a flow dependence from line 8 to line 10 indicating that the memory load on line 10 can be replaced by a scalar. However, the load statement on line 10 cannot be scalar-replaced without the knowledge of the side-effects of the method call setNothing() in line 9. In contrast, a scalar replacement for load elimination algorithm based on interprocedural side-effect analysis can determine that the method call setNothing() does not have any side-effects, thereby realizing the opportunity for eliminating the load in line 10 by scalar replacing the value assigned in line 8. We also observe that there is an input dependence from line 11
Figure 4.1: Example program: Interprocedural side-effect information can enable the load in lines 10 and 13 to be replaced by a scalar. The load in line 15 cannot be fully removed when condition \( C \) is statically unknown.

to line 13 indicating a potential target for scalar replacement for the memory load on line 13. However, due to the method call on line 12, the memory load on line 13 can be eliminated using an interprocedural analysis if we store the result of the prior load on line 11 in a scalar variable. Note that the memory load in line 15 cannot be replaced by a scalar by total redundancy elimination (also known as \textit{fully redundant} or \textit{available expression}), but is a good candidate for partial redundancy elimination (PRE). The example demonstrates the importance of side-effect analysis on compiler transformations.

Before we describe the algorithm for side-effect computation of HJ programs, we need a way to efficiently represent the side-effect information. Memory load and store operations in the bytecode of HJ programs are explicitly visible via \texttt{GETFIELD} and \texttt{PUTFIELD} memory operations. A \texttt{GETFIELD} (or \texttt{PUTFIELD}) operation receives an object reference and a field reference to load (or store) into a memory location.
We represent the field references using the *heap array* representation described in Section 2.6.1.

### 4.1.1 Heap Array Representation

To recap from Section 2.6.1, each field $x$ in the program is abstracted by a distinct heap array, $\mathcal{H}^x$. $\mathcal{H}^x$ represents all the instances of field $x$ in the heap during runtime. A *GETFIELD* of $a.x$ is represented as an use of element $\mathcal{H}^x[a]$, and a *PUTFIELD* of $b.x$ is represented as a def of element $\mathcal{H}^x[b]$. The use of heap arrays ensures that field $x$ is considered to be the same across instances of two different static types $T_1$ and $T_2$, if (say) $T_1$ is a subtype of $T_2$. Heap arrays capture abstractions at the level of fields not at the level of precise object references. Hence, they are *field-insensitive*. This level of abstraction provides faster analysis in dynamic compilation as they avoid complex interprocedural alias analysis for objects.

### 4.1.2 Method Level Side-effect

As discussed earlier, the goal of a side-effect analysis is to determine for each call site, a safe approximation of the side-effects that the method involved at that call site may have. This recursively includes any side effects of the methods called from that site. We capture this using the generalized flow-insensitive side-effect formulation proposed by Banning [9]. The generalized flow-insensitive side-effects of a method are represented using $\textit{GMOD}$ and $\textit{GREF}$ sets. Using the heap array representation, the $\textit{AMOD}$, $\textit{AREF}$, $\textit{GMOD}$ and $\textit{GREF}$ side-effects described in Section 2.5 can be rewritten as:
In Banning’s original formulation, MOD and REF sets are defined for specific call sites and were computed using both the parameter bindings at the call site and the GMOD of the callee. Since our analysis uses the heap array representation for modeling side effects, we do not pay special attention to parameter bindings.

The complete lattice for heap array GMOD and GREF sets is shown in Figure 4.2. It illustrates the lattice structure for heap array sets, GMOD and GREF, with lattice ordering defined by the subset relationship.

Figure 4.3 presents the fast side-effect analysis algorithm. As we analyze the body of a method $m$ in flow-insensitive manner, we accumulate the field accesses for GETFIELD/GETSTATIC and PUTFIELD/PUTSTATIC instructions in GREF and GMOD sets respectively. For CALL $p$ instruction, we determine the target of the method call. The side effect of a method $m$ also carries an inProgress flag to prevent re-definition of side-effects for recursive method calls. The side-effects for recursive methods are
function SideEffectAnalysis()
  Input  : Method m and its IR
  Output: Compute side-effect for m and its called procedures. Return
          \( GMOD(m) \) and \( GREF(m) \)

  Initialize summary information for method m;
  \( GREF(m) = GMOD(m) = \{\} \);
  inProgress(m) = true;
  for instruction I in IR do
    switch I do
      case GETFIELD/GETSTATIC a.f
        resolve the target of the field access a.f;
        \( GREF(m) = GREF(m) \lor \{a.f\} \);
      case PUTTFIELD/PUTSTATIC a.f
        resolve the target of the field access a.f;
        \( GMOD(m) = GMOD(m) \lor a.f \);
      case CALL p()
        resolve the target of the method access p;
        if the target of p is unknown or there are more than one target of
        p then
          \( GREF(m) = GMOD(m) = \bot \);
        else if inProgress(p) is set OR p is already analyzed then
          //Has already been analyzed
          \( GREF(m) = GREF(m) \lor GREF(p) \);
          \( GMOD(m) = GMOD(m) \lor GMOD(p) \);
      inProgress(m) = false;
  return GMOD(m) and GREF(m)

Figure 4.3: Simple Flow-Insensitive and Field-Insensitive Interprocedural Side-Effect Analysis to compute \( GREF \) and \( GMOD \) summaries for each method \( m \) via a top-down traversal of the call chain
iterated until a fixed point is reached (as in [9]).

If the target method does not have a precise static type or has several targets, we reset both the $GMOD$ and $GREF$ summaries of $m$ to $\bot$. Otherwise, we analyze the target method of callee $p$ and unify the respective $GMOD$ and $GREF$ sets of the callee with the caller. This unification process is described as the meet operator ($\bigvee$) in the algorithm presented in Figure 4.3. Any method whose summary is computed as $\bot$ will propagate this information up in the call chain.

In general, determining the target of a method call can be complicated in the presence of virtual methods calls and dynamic class loading. However, since HJ does not share Java's dynamic class loading semantics, we can separate the HJ classes from the Java classes and assume that it is safe to pre-load HJ classes. Specifically, we determine the target of a call to an HJ method as follows. First, we check if the method call has been resolved and has exactly one target. Second, we check if the method can be resolved using the existing set of classes loaded in the VM. Third, we trigger loading of the HJ class if necessary to resolve the target (Note that pre-loading will not need any further class loading). Finally, for virtual calls, we use whatever type information we have available for the this parameter to try and resolve the call to a single target. If the above steps do not yield a single unique target, we conservatively propagate $\bot$ as summaries for the given method. Merging side effects from multiple targets is a subject for future work. Currently, we limit our attention to HJ classes only, and conservatively propagate $\bot$ for all methods in Java classes.

### 4.1.3 Complexity

Since we use heap array representation, the worst case size of a side-effect set is bounded by the number of fields $F$ in a program. Let there be $N$ number of procedures and $E$ number of call sites. For recursive calls, we need to compute strongly connected components in the call graph, $G = \langle N, E \rangle$, and iterate until a fixed point is reached. The computation of strongly connected components using Tarjan's depth-first search
Figure 4.4: Even with interprocedural side-effect information we can not ascertain that the memory load on line 9 can be eliminated; We need an additional differently-different (DD) analysis to guarantee that \( a \) and \( b \) are guaranteed to be pointing to different objects in every execution of the program.

Algorithm [117] is of \( \mathcal{O}(N + E) \) complexity. Each step of the depth-first search algorithm involves \( F \) field operations using a standard bit-vector implementation. Thus the overall complexity of Algorithm 4.3 is \( \mathcal{O}(N + E) \times F \).

### 4.1.4 Discussion

Let us consider the example shown in Figure 4.4. Using Algorithm 4.3, \( \text{GMOD}(\text{foo}) = \{ \mathcal{H}[b] \} \). To scalar replace the memory load on line 9, \( i.e., \) heap array \( \mathcal{H}[a] \), we need to guarantee that \( a \) and \( b \) are pointing to different objects (in this case it is \( true \) as \( a \) and \( b \) are assigned two different objects in lines 5 and 6 respectively). So we need additional analysis to compute if two heap arrays are \( same \) or \( different \) that refines the side-effect analysis results. The details of these analyses are provided in Section 2.6.3.

### 4.2 Extended Side-effect Analysis for Parallel Constructs

Consider the example HJ program shown in Figure 4.5, which will serve as a running example to demonstrate side-effect analysis of parallel constructs in HJ programs. The example program has a \texttt{main} method that invokes two asyncs (one in line 6 and
```c
void main () {
    p.x = ... 
    s.w = ... 
    finish { //finish_main
        if (...) {
            async { //async_main
                p.x = ...
                isolated { q.y = ...; ... = q.y; }
                ... = p.x
            }
        }
        ... = p.x
        foo()
    }
    ... = p.x
    ... = s.w
}
void foo() {
    async bar() //async_foo
    isolated { q.y = ... }
    ... = s.w
}
void bar() {
    r.z = ...
    ... = r.z
}
```

Figure 4.5: Example HJ program for side-effect analysis in the presence of parallel constructs.

another in line 19 via the call to foo) and awaits for their termination using the finish construct that spans lines 4-14. Both the asyncs use isolated constructs to perform read-modify-write operations on the shared object field q.y. The call graph for the example program is shown in Figure 4.6. Using the method level side-effect analysis described in Section 4.1, GMOD(bar) and GREF(bar) can be computed as \( \mathcal{H}^{r}[r] \). Since other methods main and foo contain parallel constructs like finish, async, and isolated, we need additional machinery to compute side-effects of these methods.

We describe our proposed side-effect analysis for finish constructs, methods with escaping async’s feature, and isolated constructs in subsections 4.2.1 – 4.2.3
respectively, and then present the complete parallelism-aware side-effect analysis algorithm in Section 4.3.

The side-effect analysis for async constructs can be directly obtained from the GMOD and GREF sets of the target method of the async. In HJ, an async call is translated to low-level HJ runtime method call.

4.2.1 Side-Effects for Finish Scopes

Finish scopes in HJ impose the constraint that any async created within its scope must be completed before the statement after the finish scope is executed. Compiler optimizations such as code motion must pay attention to finish scope boundaries as it may be incorrect in general, to perform code motion into the body of the finish scope or out of the finish scope without knowing the effect of the finish scope. Hence, we introduce FMOD(f) and FREF(f) to represent the set of heap arrays modified and referenced within the async constructs inside a finish scope f respectively. The GMOD and GREF sets for any async invoked within a finish scope f, either directly or indirectly, is propagated to the finish scope by unifying them with the FMOD(f) and FREF(f) sets respectively. Each dynamic instance of an HJ statement has a unique Immediately Enclosing Finish (IEF) instance [107]. In our static analysis, we define IEF(s) to be the closest enclosing finish scope for statement s in the same
method. \( IEF(s) \) is undefined, \( i.e., \bot \), if \( s \) does not have an enclosing finish statement in the same method. We conservatively propagate \( \bot \) information for both \( FMOD \) and \( FREF \) sets, when any method invoked inside a finish scope has unknown target or multiple targets. Formally,

\[
FMOD(f) = \begin{cases} 
\bigcup_{\exists s \in f, s \text{ invokes } q} \{GMOD(q) \cup EMOD(q)\} & \text{if } q \text{ is an async call} \\
\bigcup_{\exists s \in f, s \text{ invokes } q} \{EMOD(q)\} & \text{otherwise}
\end{cases}
\]

\[
FREF(f) = \begin{cases} 
\bigcup_{\exists s \in f, s \text{ invokes } q} \{GREF(q) \cup EREF(q)\} & \text{if } q \text{ is an async call} \\
\bigcup_{\exists s \in f, s \text{ invokes } q} \{EREF(q)\} & \text{otherwise}
\end{cases}
\]

The async calls that are directly or indirectly (via other function calls) invoked inside a finish scope \( f \) are captured in the \( FMOD \) and \( FREF \). \( EMOD \) and \( EREF \) capture \( MOD \) and \( REF \) sets for \textit{escaping asyncs} described below.

Consider the method \texttt{main} in Example 4.5. The finish scope encompasses the side-effects of all the methods and asyncs invoked within it. Ignoring the isolated constructs on line 8 and 20 (which will be discussed later), the \( FMOD(\text{finish}_\text{main}) \) can be computed as \( \{\mathcal{H}^x[p], \mathcal{H}^x[r]\} \). \( \mathcal{H}^x[r] \) is added to the side-effect due to the method call \texttt{foo} in the finish scope. Similarly, \( FREF(\text{finish}_\text{main}) \) is computed as \( \{\mathcal{H}^x[p], \mathcal{H}^x[r], \mathcal{H}^w[s]\} \).

4.2.2 Side-Effects for Methods with Escaping Asyncs

HJ permits methods with \textit{escaping asyncs}, \( i.e., \) asyncs that have no enclosing finish scopes in the same method. We define an \textit{async-escaping} method as: 1) a method which contains an \textit{async} invocation that is not enclosed in a \textit{finish} scope, or 2) a method which invokes another \textit{async-escaping} method that is not wrapped in a finish scope. The \( GMOD \) and \( GREF \) sets for async-escaping methods are propagated to their enclosing finish scopes as their termination is guaranteed only at the end of
the enclosing finish scopes. We introduce escaping $EMOD$ and $EREF$ sets along with $GMOD$ and $GREF$ to handle async-escaping methods. Async-escaping methods continue to be async-escaping in the call graph chain until an $IEF$ is encountered. Note that the side-effects of non-escaping async’s will be collected by normal side-effect analysis of methods.

Let $\mathcal{F}(s, p)$ denote a predicate that indicates if $s$ is executed within a finish scope in $p$, i.e., $IEF(s)! = \bot$. We can formally define $EMOD$ and $EREF$ as:

$$EMOD(p) = \begin{cases} \bigcup \exists_{s \in p, s \text{ invokes } q} \{ \neg \mathcal{F}(s, p) \land (GMOD(q) \cup EMOD(q)) \} & \text{if } q \text{ is an async call} \\ \bigcup \exists_{s \in p, s \text{ invokes } q} \{ \neg \mathcal{F}(s, p) \land EMOD(q) \} & \text{otherwise} \end{cases}$$

$$EREF(p) = \begin{cases} \bigcup \exists_{s \in p, s \text{ invokes } q} \{ \neg \mathcal{F}(s, p) \land (GREF(q) \cup EREF(q)) \} & \text{if } q \text{ is an async call} \\ \bigcup \exists_{s \in p, s \text{ invokes } q} \{ \neg \mathcal{F}(s, p) \land EREF(q) \} & \text{otherwise} \end{cases}$$

The $EMOD$ and $EREF$ side effects for unfinished asyncs are propagated along the call chain until a finish scope $f$ is encountered. This ensures that the effect of these async-escaping methods are only visible at the end of $IEF$, thereby, allowing code reordering compiler transformation around these methods.

The method $\text{foo}$ in Example 4.5 invokes an async on line 19 that is not wrapped in a finish scope and is an async-escaping method. The $EMOD(\text{foo})$ and $EREF(\text{foo})$ are computed using the side effect sets $GMOD$ and $GREF$ of $\text{bar}$, i.e., $\{H^z[v]\}$.

### 4.2.3 Side-Effects for Isolated Blocks

The isolated synchronization primitive enforces mutual exclusion among async’s. Usually code motion optimizations across synchronization primitives are strongly tied to the memory model, e.g., in Java, the memory operations which were visible to a
thread before exiting a synchronized block are visible to any thread after it enters a synchronized block protected by the same monitor, since all the memory operations happen before the release, and the release happens before the acquire. To allow or disallow code motion around isolated blocks, we introduce $AMOD$ and $AREF$ sets that represent all the heap arrays modified and referenced respectively across all isolated blocks in the program. Note that, this is an overly conservative approximation as some of the isolated blocks may never execute in parallel with other isolated blocks due to a “happens-before” relationship. Further refinement of $AMOD$ and $AREF$ sets using May-Happen-in-Parallel (MHP) information [2] is a subject for future work.

Let $P$ denote all the procedures in the call graph. Let $I(s, p)$ predicate denote if $s$ is executed within an isolated block in procedure $p$. Additionally, $I(s, p)$ is set to $true$ for all the statements in the body of an isolated method. $AMOD$ and $AREF$ can be defined formally as:

\[
AMOD(p) = \{ H^c[a] | \exists s \in p, I(s, p) \land s \in \{ PUTFIELD a.x, PUTSTATIC a.x \} \}
\]

\[
AREF(p) = \{ H^c[a] | \exists s \in p, I(s, p) \land s \in \{ GETFIELD a.x, GETSTATIC a.x \} \}
\]

\[
AGMOD(p) = AIMOD(p) \bigcup_{s \in p, s \text{ invokes } q} \{ I(s, p) \land GMOD(q) \}
\]

\[
AGREF(p) = AIREF(p) \bigcup_{s \in p, s \text{ invokes } q} \{ I(s, p) \land GREF(q) \}
\]

\[
AMOD = \bigcup_{p \in P} AGMOD(p)
\]

\[
AREF = \bigcup_{p \in P} AGREF(p)
\]

Note that $HJ$ does not permit $async$ and $finish$ constructs inside the body of an isolated block or an isolated method. Hence $EMOD$ and $EREF$ side effects do not need to be incorporated into the $AGMOD$ and $AGREF$ sets.

Going back to the example program in Figure 4.5, the isolated blocks on lines 8
and 20 modify and reference heap array $\mathcal{H}^y[q]$. Hence, $AMOD = AREF = \{\mathcal{H}^y[q]\}$.

4.3 Parallelism-aware Side-Effect Analysis Algorithm

The overall side-effect analysis algorithm in the presence of finish, async, and isolated constructs is presented in Figure 4.7. This algorithm is designed to be performed on the Java bytecode performed by the HJ compiler, which translates each async construct to a runAsync call in the Java-based HJ runtime, which in turn calls the runHJTask method in an inner class that contains the body of the async.

Further, every finish scope is translated into a pair of startFinish() and stopFinish() runtime calls.

For statements/methods executed in isolated blocks, we unify the AMOD and AREF sets using the meet operator $\bigvee_i$. The $\bigvee_i$ is a conditional meet operation which is performed only if the current statement/method call is in an isolated block. Note that the HJ language does not permit any usage of async or finish constructs in the body of isolated sections [38].

The algorithm presented in Figure 4.7 walks over the IR in a flow-insensitive manner and unifies the GMOD and GREF sets for heap arrays accessed in GETFIELD (GETSTATIC) and PUTSTATIC (PUTFIELD) instructions respectively (as shown in steps 11 and 15). If these instructions are accessed within an isolated section, we unify them with AMOD and AREF respectively (as shown in steps 12 and 16). For Call $p()$ instructions, we take different actions for different function calls due to parallel constructs. For startFinish function call demarcating the start of a new finish scope, we create, i.e., $FMOD(IEF(I))$ and $FREF(IEF(I))$ sets for the finish scope (as shown in steps 7-10 in Figure 4.8). At stopFinish function call indicating activity termination, $FMOD(IEF(I))$ and $FREF(IEF(I))$ are merged into the GMOD$(m)$ and GREF$(m)$ sets for the current caller $m$ (as shown in steps 12-15 in Figure 4.8). This unification implies that the side-effect of the asyncs created within the finish scope can only be visible at the end of the finish scope.
function ParallelSideEffectAnalysis()
    Input : Method $m$ and its $IR$
    Output: (1) Compute side-effect for $m$ and its called procedures: Return $GMOD(m)$ and $GREF(m)$; (2) Compute side-effect for finish scopes: Return $FMOD$ and $FREF$; (3) Compute side-effect for isolated constructs: Return $AMOD$ and $AREF$
    Initialize information for method $m$;
    $GREF(m) = \top$ and $GMOD(m) = \top$;
    $inProgress(m) = true$;
    //A stack containing active finish scopes
    $S := \phi$;
    Set $IEF$ of the first instruction in $IR$ to $\bot$;
    for instruction $I$ in $IR$ do
        switch $I$ do
            case $GETFIELD$/GETSTATIC $a.f$
                Resolve the target of the field access $a.f$;
                $GREF(m) = GREF(m) \lor \{H^f[a]\}$;
                $AREF = AREF \lor_i \{H^f[a]\}$;
            case $PUTFIELD$/PUTSTATIC $a.f$
                Resolve the target of the field access $a.f$;
                $GMOD(m) = GMOD(m) \lor \{H^f[a]\}$;
                $AMOD = AMOD \lor_i \{H^f[a]\}$;
            case CALL $p()$
                HandleCall();
            inProgress($m$) = false;
        return
        $GMOD(m), GREF(m), AMOD(m), AREF(m), FMOD, FREF, EMOD(m), EREF(m)$

Figure 4.7 : ParallelSideEffectAnalysis($m$): Side-effect analysis in the presence of HJ parallel constructs for method $m$
function HandleCall()
Resolve the target of the method access p;

if the target of p is unknown or has several targets then
    GREF(m) = ⊥ and GMOD(m) = ⊥;
    EREF(m) = ⊥ and EMOD(m) = ⊥;
else if the target of p is startFinish then
    f := Create a new finish scope;
    IEF(I) := f;
    S.push(f);
    FMOD(IEF(I)) = ⊤ and FREF(IEF(I)) = ⊤;
else if the target method is stopFinish then
    GMOD(m) = GMOD(m) ∨ FMOD(IEF(I));
    GREF(m) = GREF(m) ∨ FREF(IEF(I));
    f := S.pop();
    Set IEF of statements following stopFinish to f;
else if the target method is runAsync then
    HandleAsync();
else if inProgress(p) is set OR GMOD(p) and GREF(p) sets are available OR
    recursively invoke ParallelSideEffectAnalysis(p) for p then
    HandleNormalMethodCall();

Figure 4.8 : Additional function to handle method calls for ParallelSideEffectAnalysis(m)
For the `runAsync` function call (as shown in Figure 4.9), we determine the target `runHJTask` method and recursively compute side-effects for this method. The `GMOD`, `GREF`, `EMOD` and `EREF` sets of the `runHJTask` method are then unified in the caller’s enclosing finish scope’s `FMOD` and `FREF` sets as shown in steps 8-9 in Figure 4.9. If the `runAsync` method call was not enclosed in a finish scope, the `GMOD`, `GREF`, `EMOD` and `EREF` sets of `runHJTask` are unified with the `EMOD` and `EREF` for the caller. This is shown in steps 5-6 in Figure 4.9. Unification with `EMOD` and `EREF` sets indicates that all the side effect for the callee async remain escaping for the caller. If the async call is enclosed in a finish scope, the async call’s side effects can only be visible at the end of the finish scope and hence added to `FMOD(IEF(I))` and `FREF(IEF(I))` as shown in steps 8-9 in Figure 4.9.

Normal method calls that are not related to parallel constructs (as shown in Figure 4.9) are handled by unifying the `GMOD`, `GREF`, `EMOD` and `EREF` sets of the callee with their corresponding side-effect sets in the caller if there is no `IEF` for the call instruction. In case of `IEF(I)` is undefined, the `EMOD` and `EREF` sets of the callee are unified with respective `GMOD` and `GREF` sets of the immediately enclosing finish scope summary. If the callee is an isolated method or is invoked in an isolated block, the `GMOD` and `GREF` sets are unified with the global `AMOD` and `AREF` sets respectively.

For the example program shown in Figure 4.5 and its corresponding call graph in Figure 4.6, the final side-effect sets are shown in Table 4.1.

The complexity of Algorithm 4.7 is similar in nature to that of Algorithm 4.3. Hence, the overall complexity of Algorithm 4.7 is $O(N + E) \times F$, where $N$ accounts for both normal method calls and parallel constructs in a program.

### 4.3.1 Discussion

As discussed in Section 4.2.3, we compute global side-effects for isolated blocks and methods. However, it is possible to refine it with the May-Happens-in-Parallel infor-
function HandleAsync()
    Determine the target runHJTask, t;
    Obtain GMOD(t) and GREF(t) by invoking ParallelSideEffectAnalysis(t); //recursive call
    if IEF(I) is undefined then
        EMOD(m) = EMOD(m) \lor GMOD(t) \lor EMOD(t);
        EREF(m) = EREF(m) \lor GREF(t) \lor EREF(t);
    else
        FMOD(IEF(I)) = FMOD(IEF(I)) \lor GMOD(t) \lor EMOD(t);
        FREF(IEF(I)) = FREF(IEF(I)) \lor GREF(t) \lor EREF(t);
    
function HandleNormalMethodCall()
    GMOD(m) = GMOD(m) \lor GMOD(p);
    GREF(m) = GREF(m) \lor GREF(p);
    AMOD = AMOD \lor_i GMOD(p);
    AREF = AREF \lor_i GREF(p);
    if IEF(I) is undefined then
        EMOD(m) = EMOD(m) \lor EMOD(p);
        EREF(m) = EREF(m) \lor EREF(p);
    else
        FMOD(IEF(I)) = FMOD(IEF(I)) \lor EMOD(p);
        FREF(IEF(I)) = FREF(IEF(I)) \lor EREF(p);

Figure 4.9: Additional functions to handle async calls and normal method calls for ParallelSideEffectAnalysis(m)
Table 4.1 : Side-effect results of parallel constructs and method calls for example program shown in Figure 4.5

Table 4.1 : Side-effect results of parallel constructs and method calls for example program shown in Figure 4.5

| GMOD(bar) | GREF(bar) = \{H^z[r]\} |
| EMOD(bar) | EREF(bar) = \top |
| GMOD(async_foo) | GREF(async_foo) = \top |
| EMOD(async_foo) | EREF(async_foo) = \{H^z[r]\} |
| GMOD(foo) | EREF(foo) = \{H^w[s]\} |
| EMOD(foo) | EREF(foo) = \{H^z[r]\} |
| GMOD(async_main) | GREF(async_main) = \{H^x[p]\} |
| EMOD(async_main) | EREF(async_main) = \top |
| FMOD(finish_main) | {H^c[p], H^c[r]} |
| FREF(finish_main) | {H^c[p], H^c[r]} |
| GMOD(main) | GREF(main) = \{H^x[p], H^z[r], H^w[s]\} |
| EMOD(main) | EREF(main) = \top |
| AMOD | AREF = \{H^v[q]\} |

Table 4.1 : Side-effect results of parallel constructs and method calls for example program shown in Figure 4.5

In this chapter, we introduced side-effect analysis for object field references in HJ programs having parallel constructs. In particular, we discussed side-effects of async, finish, and isolated constructs. The side-effects of these constructs will be used in the next chapter to perform scalar replacement transformation using a new Isolation...
1: class isolatedMHP {
2:     int x;
3:     int y;
4:     void main (isolatedMHP aa) {
5:         aa.y=10;
6:         aa.x=10;
7:         finish async {
8:             aa.y++;
9:             isolated { aa.x++; }
10:             ... = aa.y; // Can you eliminate this load?
11:         }
12:         async {
13:             aa.x++;
14:             isolated { aa.y++; }
15:             ... = aa.x; // Can you eliminate this load?
16:         }
17:     }
18: }

Figure 4.10: Improving the precision of global isolated side-effects.

Consistency memory model.

In future, it is possible to extend our side-effect analysis to advanced constructs of HJ like places and phasers. Additionally, side-effect analysis for object field references can be extended to arrays using bounded regular section analysis [62, 120].
Chapter 5

Isolation Consistency Memory Model and its Impact on Scalar Replacement

Due to recent software trends in parallel programming languages for multi-core processors, it is necessary for a compiler to be aware of the parallel constructs in the input parallel program to be able to perform any code reordering transformation. For example, the most commonly used common subexpression elimination (CSE) data flow framework can not be easily adapted to parallel programs for the following reasons:

1. Parallel constructs introduce additional kill expressions, i.e., side-effects (addressed in Chapter 4).

2. Interferences between shared data accesses of multiple threads may prohibit some subexpression elimination scenarios.

The legality of compiler transformations in the presence of interferences between shared accesses is typically defined using a memory model. In this chapter, we introduce a new weaker memory model called Isolation Consistency that permits several code reordering scenarios compared to many existing weaker memory models. Using this memory model we present an algorithm to perform scalar replacement for load elimination of HJ parallel programs.

5.1 Program Transformation and Memory Model

As stated by [78], a parallel program transformation is correct “if the set of possible observable behaviors of a transformed program is a subset of the possible observable
behaviors of the original program.” The possible observable behaviors of a parallel program and consequently the permissible program transformations are determined by the underlying memory model. There exists a wide range of memory models. A strong memory consistency model limits the observable orderings of memory operations among threads and is viewed by programmers as being easier to reason about, but difficult to implement efficiently. On the other hand, a relaxed or weaker memory model permits several reordering of memory operations and may be less easy for programmers to reason about, but is easy to implement efficiently.

Sequential Consistency (SC) defined by [72] introduced the stronger memory consistency model used in practice. Lamport’s paper states that “the result of any execution is the same as if the operations of all the processors were executed in some sequential order, and the operations of each individual processor appear in this sequence in the order specified by its program”. This definition of sequential consistency disallows reordering of memory operations of a thread even if there is no intra-thread control and data dependence between the memory operations. Consider the example program shown in Figure 5.1. A standard compiler reordering transformation that reorders statements 1 and 2 in Thread 1 can produce the result $r_1 = 2$ and $r_2 = 1$ and this result is not consistent in SC model. Note that the statements within each of the threads Thread 1 and Thread 2 do not have any dependence between them. This small example demonstrates that sequential consistency limits concurrent program transformation by disallowing reorderings that are legal under sequential execution. Additionally, sequential consistency implementations have led to a strong memory coherence assumption in the hardware, i.e., “all writes to the same location are serialized in some order and are performed in that order with respect to any processor” [58]. This enforces some form of serializability on the write operations to the same locations. These hardware memory coherence assumptions are often ignored in the software memory consistency model.

The main approach taken in recent memory consistency models is to permit com-
a.x = a.y = 0

<table>
<thead>
<tr>
<th>Thread 1</th>
<th>Thread 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: r₁ := a.x</td>
<td>3: r₂ := a.y</td>
</tr>
<tr>
<td>2: a.y := 1</td>
<td>4: a.x := 2</td>
</tr>
</tbody>
</table>

Figure 5.1: Sequential consistency is violated if the results $r₁ = 2$ and $r₂ = 1$ are obtained. A standard code reordering compiler transformation can produce this result by reordering the statements in Thread 1.

Several weak memory models have been proposed in recent years including release consistency (RC) [58], Java memory model (JMM) [84], C++ memory model (C++MM) [21], OpenMP memory model [64], and Location Consistency (LC) [53].

Neither the X10 or HJ language has a clear definition of a memory model. Since the HJ programming language uses a serial subset of the Java v1.4 language with new support for concurrency, one could imagine using Java memory model for HJ. There are several reasons why we chose not to use JMM for HJ programs:

- HJ’s concurrency support using async-finish is more general than the start-join model used in Java. The async-finish constructs allow features such as escaping-asyncs (described in Chapter 4) that have very different semantics than Java’s concurrency model;

- Current definitions of JMM are driven by informal examples [103] and lack rigorous formal mechanisms for reasoning in many cases. This has lead to many counter examples to compiler transformations allowed in JMM [121]. Table 5.1 depicts some of the code reordering transformations that are allowed (or disallowed) by the SC and JMM models;

- The JMM is mainly driven by the requirements in the software such as out-of-thin air and to our knowledge, it does not provide a memory coherence
Table 5.1: Comparison of SC and JMM for compile reordering transformations [121]. Redundant read after read elimination is the classic case of eliminating input dependences. Redundant read after write elimination removes flow dependences. Redundant write before write elimination removes redundant stores. Redundant write after read elimination removes the writes that write the same value as that of the read. Roach motel reordering allows code motion into and out of synchronized blocks.

guarantees;

Driven by the above reasons and the fact that a memory consistency model must be a contract between both the hardware and software, we describe a new Isolation Consistency memory model for HJ programs that builds on the formalism of Location Consistency (LC) [53] and extends LC model appropriately to provide weaker semantics for isolated constructs and volatile variables. One of the main reasons for choosing the LC model as the foundation is that the LC model does not require serialized ordering of memory operations to the same location (as required by SC), but instead imposes a partial ordering. Due to this, LC model allows more compiler transformations and is easier to implement. For example, going back to our code fragment in Figure 5.1, \( r_1 = 2 \) and \( r_2 = 1 \) is a possible legal outcome since the updates to \( a.x, a.y, r_1, \) and \( r_2 \) occur to different memory locations.

## 5.2 Isolation Consistency Memory Model

There is a range of memory models that have been studied in the literature including Sequential Consistency (SC) [72], Release Consistency (RC) [58], the Java Memory Model (JMM) [84], the OpenMP memory model [64], and Location Consistency
(LC) [53]. It is well known that all these models yield the same semantics for data-race-free programs, but may exhibit different semantics for parallel programs with races. A major research challenge lies in dealing with the common case when a compiler (especially a dynamic compiler) does not know for sure that the input parallel program is data-race-free. To address this case, we define a weak memory model, Isolation Consistency (IC), for which the scalar replacement for load elimination transformation described in Chapter 5 are guaranteed to be correct even in the presence of data races. They will also be correct for any data-race-free parallel program with a stronger memory model, but the optimizations may not be correct for parallel programs with data races that must obey a stronger memory model.

5.2.1 Abstraction

The definition of IC builds on the operational semantics used to define the Location Consistency (LC) in [53]. Each shared memory location $L$ is modeled as a partial order using a partially ordered multiset (pomset), $\text{state}(L) = (S, \prec)$, where $S$ is a multiset and $\prec \subseteq S \times S$ is a partial order on $S$. Pomset captures the sequencing of memory and synchronization operations. Each element $e$ of the multiset $S$ is one of the following (We use the term worker to refer to the thread/processor executing the async):

- **Write operation**: if worker $P_i$ writes value $v$ in location $L$, it performs a $\text{write}(P_i, v, L)$ operation;

- **Finish synchronization**: finish constructs in HJ impose a happens-before ordering since any async created inside the body of a finish scope completes execution before the statement after the finish scope is executed. This can be captured using a directed signal-wait synchronization primitive. If worker $P_2$ needs to wait for worker $P_1$, then $P_1$ performs a $\text{signal}(P_2)$ operation and $P_2$ performs a
corresponding \textit{wait}(P_1) operation\footnote{The \texttt{signal} and \texttt{wait} operations used in the \texttt{phaser} construct of \texttt{HJ} language are different from the ones we consider here.}.

- \textit{Isolation}: if workers $P_1, \ldots, P_k$ need exclusive access within an \texttt{HJ} place using isolated constructs, then each worker performs an \texttt{acquire}(P_i, \ast) operation followed by a \texttt{release}(P_i, \ast) operation, where $\ast$ represents all the shared memory locations in all the places defined in the isolated scope.

- \textit{Volatile Variables}: Volatile variables in \texttt{HJ} are similar to \texttt{Java}, \textit{i.e.}, when one worker writes to a volatile variable, and another worker sees that write, the first worker is communicating the second about all of the contents of memory up until it performed the write to that volatile variable. The volatile variables in the \texttt{IC} model are modeled by wrapping each volatile access using an isolated block. Hence, an \texttt{acquire}(P_i, \ast) operation followed by a \texttt{release}(P_i, \ast) is performed for every volatile access.

Let \textsl{workers}(e) be the set of workers involved in the operation $e$. For example, for $e = \text{write}(P_i, v, L)$, \textsl{workers}(e) = \{P_i\}. For an $e = \text{acquire}(P_i, \ast)$ operation, \textsl{most recent release}(e) = \{e_1, \ldots, e_k\}, where $e_i$ is the most recent release operation $e_i = \text{release}(P_i, \ast)$ performed prior to acquire $e$. For an wait operation $e$, \textsl{signal sources}(e) = \{e_1, \ldots, e_k\} where $e_i$ signals its completion of execution to $e$ during run-time.

### 5.2.2 State-Update rules for $L$

When new memory or synchronization operations are performed, the pomset $(S, \prec)$ is updated. The rules for updating a pomset based on an operation $e$ are provided using three rules namely \textit{default rule}, \textit{wait rule}, and \textit{acquire rule}. Each of the rules are described below:
Definition 5.2.1 Default Rule: The default rule for computing the new pomset $(S_{new}, \prec_{new})$ from the old pomset $(S_{old}, \prec_{old})$ after operation $e$, is as follows:

\[
S_{new} := S_{old} \cup \{e\}
\]

\[
\prec_{new} := \prec_{old} \cup \{(x, e) \mid x \in S_{old} \land \text{workers}(x) \cap \text{workers}(e) \neq \emptyset\}
\]

The default rule states that the new operation, $e$, is inserted into the multiset $S$, and the partial order $\prec$ is updated so that $x$ precedes $e$ in $S$, if $\text{workers}(e)$ and $\text{workers}(x)$ have a non-empty intersection.

Definition 5.2.2 Wait Rule: The rule for computing the new pomset $(S_{new}, \prec_{new})$ from the old pomset $(S_{old}, \prec_{old})$ after a wait operation $e$, is as follows:

\[
S_{new} := S_{old} \cup \{e\}
\]

\[
\prec_{new} := \prec_{old} \cup \{(x, e) \mid x \in S_{old} \land \text{workers}(x) \cap \text{workers}(e) \neq \emptyset\}
\]

\[
\cup \{(x, e) \mid x \in \text{signalsource}(e)\}
\]

The wait rule adds the directed synchronization ordering in the pomset along with the default rule.

Definition 5.2.3 Acquire Rule: The rule for computing the new pomset $(S_{new}, \prec_{new})$ from the old pomset $(S_{old}, \prec_{old})$ after a acquire operation $e$, is as follows:

\[
\prec_{new} := \prec_{old} \cup \{(x, e) \mid x \in S_{old} \land \text{workers}(x) \cap \text{workers}(e) \neq \emptyset\}
\]

\[
\cup \{(e', e) \mid e' \in \text{mostrecent-release}(e)\}
\]

Similar to wait rule, the acquire rule adds the mutual exclusive ordering in the pomset along with the default rule.
5.2.3 State Observability for \( L \)

Given a read operation \( e = \text{read}(P_i, L) \), the pomset \((S, \prec)\) is extended as follows:

\[
S' = S \cup \{e\} \\
\prec' = \prec \cup \{(e', e) \mid e' \in S \land P_i \in \text{workers}(e')\}
\]

Now, the set of values \( V(e) \) for read operation \( e = \text{read}(P_i, L) \) in \((S', \prec')\) can be obtained as follows,

\[
V(e) = \{ v \mid \exists w = \text{write}(P_j, v, L) \in S' \\
\text{and } w \text{ satisfies Condition 1 or 2 listed below} \}
\]

where conditions 1 and 2 are:

- **Condition 1:** \( w = \text{write}(P_j, v, L) \) and \( w \prec' e \) and \( w \in \text{MRW}(S, \prec, e) \)

  If \( w \) is a write operation that precedes \( e \) in \( \prec' \), then it can only be included in the value set \( V(e) \) if it is a most recent write (MRW) operation with respect to the read operation \( e \). \( w = \text{write}(P_j, v, L) \) is said to be a most recent write operation in extended pomset \((S', \prec')\) for location \( L \) with respect to read operation \( e \) if \( w \) is a predecessor of \( e \) (\( w \prec' e \)) and there is no other write operation on shared location \( L \), \( w' \neq w \), such that \( w' \) precedes \( e \) (\( w' \prec' e \)) and \( w \) precedes \( w' \) (\( w \prec w' \)).

- **Condition 2:** \( w = \text{write}(P_j, v, L) \) and \( w \not\prec' e \)

  If \( w \) is a write operation that does not precede \( e \) in \( \prec' \), then it is automatically included in \( V(e) \) (even though \( w \) may precede other write operations to location \( L \) in \( \prec' \)).

In summary, LC models the state of each shared location as a partially ordered multiset (pomset) of write and synchronization operations in an abstract interpreter.
In any execution that satisfies the LC model, the result returned by a read operation \( R \) must belong to the value set of the location, \textit{i.e.}, it must have been written by a write operation that is a "most recent write" with respect to \( R \) in the pomset or a write operation that is unrelated to \( R \) in the pomset.

However, the LC model also placed the restriction that the abstract interpreter executes each instruction in a thread in its original order, thereby ensuring that \textit{causality} is not violated.

In Isolation Consistency (IC), we assume that only the control and data dependences within a thread need to be preserved in the abstract interpreter. Thus the abstract interpreter is allowed to execute instructions out-of-order within a thread so long as intra-thread dependences are not violated. These intra-thread dependences are defined using a \textit{weak atomicity} model \cite{85} which ensures the correct ordering of load and store operations from multiple threads when they occur in isolated sections. For load and store operations that occur outside an isolated section, the only inter-thread ordering constraints arise from the "happens-before" relationships enforced by the finish construct.

\subsection{5.2.4 Example Scenarios}

Consider the four example parallel code fragments shown in Figure 5.2. Cases 1 and 2 demonstrate the potential for scalar replacement for load elimination across \texttt{async} constructs, Case 3 across a \texttt{finish} construct, and Case 4 across \texttt{isolated} constructs. In each case, we want to know if the load of \( a.f \) in statement 4 can be eliminated by substituting the value of a prior store operation. (The \ldots notation represents computations that do not contain accesses to any instances of field \( f \).) The IC model permits scalar replacement in all four cases, but that is not the case for the SC and JMM models. Cases 1 and 4 have no data races, but for the non-IC memory models, the onus is on the compiler to establish that there are no data races in those cases.

Case 1 appears to be an easy case because the async body is assumed to not
perform any access to field f. Both the JMM and IC models permit scalar replacement for load elimination of the a.f getfield operation in statement 4 by using the value stored in statement 2. However, an additional delay set analysis [106] is necessary for the SC model to ensure that there is no other access to field f elsewhere in the program that could contribute to a cycle and result in an execution that is potentially inconsistent with the SC model. Delay set analysis is a time-consuming whole program analysis that will be impractical for use in a dynamic optimizing compiler.

In Case 2, there is a potential data race between the conditional store of a.f in statement 3 and the load in statement 4. With the IC model, the compiler can conclude that the value stored in a.f in statement 2 will always be part of the value set for the load in statement 4, therefore making it legal to perform a load elimination accordingly. The SC and JMM models will not permit scalar replacement in this case, but the OpenMP [64] model will.

Case 3 demonstrates the scope of eliminating loads across finish boundaries. In this case, the load in statement 4 may not be eliminated with respect to statement
2. The \textit{finish} scope in statement 3 demarcates the completion of the execution of the async body in statement 3 and hence is visible to the rest of the program.

Case 4 shows the effect of scalar replacement for load elimination in the presence of \textit{isolated} constructs. The load in statement 4 cannot be eliminated in the SC and JMM models due to the \textit{isolated} construct. However, if we can analyze the side effect of the isolated construct, we should be able to eliminate the load in statement 4. In this case, the async only updates field \texttt{a.x}. Hence, eliminating the load of \texttt{a.f} in statement 4 is safe in the IC model.

5.3 Scalar Replacement for Load Elimination

Now, we will describe the scalar replacement for load elimination algorithm that analyzes parallel constructs. It follows the same basic steps of the load elimination algorithm described in Section 2.6, but embeds side-effects of parallel constructs and permits scalar replacement using isolation consistency model described in the previous section. The side-effects for method calls and parallel constructs are computed using the algorithms presented in Chapter 4.

Algorithm 5.3 presents the complete scalar replacement for load elimination algorithm in the presence of parallel constructs. Steps 4-21 determine the type of method call based on the parallel constructs and inserts appropriate pseudo-def and pseudo-use instructions for their \textit{GMOD} and \textit{GREF} sets. Each entry into the \textit{isolated} block is annotated with pseudo-defs to fields in \textit{AMOD}. This prohibits any load reuse in the \textit{isolated} block for fields that may be modified in any isolated scope. Each exit of an isolated construct is annotated with pseudo-uses of fields in \textit{AREF}. This permits loads to be eliminated in and after the isolated block exit. \textit{startFinish} and \textit{runAsync} method calls are handled by side-effect analysis and act as a no-op for load elimination algorithm. At \textit{stopFinish}, pseudo-def and pseudo-use instructions are added for \textit{FMOD} and \textit{FREF} finish side-effect sets of the current finish scope. Other normal method calls insert pseudo-def and uses for \textit{GMOD} and \textit{GREF} summary sets.
function ParallelismAwareLoadElim()

Input: Method $m$ and its $IR$

Output: Transformed $IR$ after Load Elimination

Compute side-effect summary information by invoking $ParallelSideEffectAnalysis$ presented in Figure 4.7:

for instruction $I$ in $IR$ do

    switch $I$

    case $isolatedenter$
        Insert pseudo-defs for each heap array in $AMOD$ at $I$;
    case $isolatedexit$
        Insert pseudo-uses for each heap array in $AREF$ at $I$;
    case $startFinish$
        no-op;
    case $stopFinish$
        Insert pseudo-defs for each field in $FMOD(IEF(I))$;
        Insert pseudo-refs for each field in $FREF(IEF(I))$;
    case $async$
        no-op;
    case $CALL p$
        if target of $p$ is an isolated method then
            Insert pseudo-defs for each field in $AMOD$ before $I$;
            Insert pseudo-uses for each fields in $AREF$ after $I$;
        else
            Insert pseudo-defs and pseudo-uses for each field in $GMOD(p)$ and $GREF(p)$ respectively at $I$;

    Construct extended array ssa form for each heap operand access including the pseudo-def and pseudo-use accesses introduced above;

    Perform global value numbering to compute definitely-same (DS) and allocation site information to compute definitely-different (DD) relations;

    Perform data flow analysis to propagate uses of heap arrays to defs;

    Create data flow equations for $\phi$, $d\phi$, and $u\phi$ nodes;

    Iterate over the data flow equations until a fixed point is reached;

    Perform scalar replacement for load elimination;

    For a load of a heap operand, if the value number of the associated heap operand is available, then replace the load instruction;

Figure 5.3: Scalar replacement for load elimination algorithm in the presence of parallel constructs of HJ. Legality of the elimination is provided by Isolation Consistency memory model.
if the target of the method call is not an isolated method. Otherwise, pseudo-defs for fields in $IMOD$ and pseudo-uses for fields in $IREF$ are inserted before and after the method call.

Steps 22-28 in Algorithm 5.3 first construct an extended array ssa form representation of the $IR$ over which a global value numbering is performed to compute object accesses that may be definitely-same ($DS$) or definitely-different ($DD$). In Step 24, a data flow analysis is performed that propagates uses of heap operands to their definition points. Finally, actual load elimination is performed by replacing the memory load operation by a compiler generated temporary in cases where the load is already fully available. The steps 22-28 are described in details in [52].

5.3.1 Example

Consider the example program shown in Figure 5.4. The same example was used in Chapter 4 to demonstrate side-effect analysis of parallel constructs. Table 4.1 in Section 4.3 summarizes the side-effects of both method calls and parallel constructs. Using these side-effect information and the scalar replacement algorithm with IC memory model described in Algorithm 5.3 produces the transformed code shown in Figure 5.5. Note that, $p.x$ is used as a scalar variable for replacing a memory load of $p.x$. The memory load operations on lines 8, 9, 12, 16, and 25 can now be scalar replaced. The memory load on line 15 can not be scalar replaced due to the finish ordering imposed by the $finish$ construct on lines 4-14. One interesting scenario is that the memory load on line 12 can be scalar replaced even though the program can have a data race with respect to the $async$ created on line 6.

5.4 Summary

This chapter introduces a new weak memory consistency model as Isolation Consistency memory model (IC) that favors compiler reordering transformations for racy programs while producing sequentially consistent behaviors for race-free programs.
void main () {
  p.x = ...
  s.w = ...
  finish { //finish_main
    if (...) {
      async { //async_main
        p.x = ...
        isolated { ... = q.y; ...; q.y = ...; }
        ... = p.x
      }
    }
    ... = p.x
    foo()
  }
  ... = p.x
  ... = s.w
}
void foo () {
  async bar () { //async_foo
    isolated { q.y = ...; }
    ... = s.w
  }
  void bar () {
    r.z = ...
    ... = r.z
  }
}

Figure 5.4 : Example HJ program demonstrating scalar replacement for load elimination in the presence of parallel constructs.

The ordering constraints imposed by IC model are also described. Finally, a scalar replacement for load elimination algorithm is presented whose legality is guaranteed by the IC memory model. In Section 8.1, we will present the performance improvements using our parallelism-aware scalar replacement for load elimination algorithm.

Possible directions for future work include extensions for array accesses, i.e., perform scalar replacement for arrays and define Isolation Consistency model in the presence of arrays. To handle advanced features of HJ such as remote async in distributed systems, we would like to perform scalar replacement of fields and array accesses across remote async boundaries to be able to optimize communication among the asyncs.
Figure 5.5: Transformed program after scalar replacement for load elimination is performed on the code fragment provided in Figure 5.4. Note that, the memory load of p.x on line 13 cannot be eliminated due to the finish construct.
Chapter 6

Space-Efficient Register Allocation

Typically, a register allocator consists of two tasks: allocation and assignment. Allocation ensures that no more than $k$ symbolic registers are residing in physical registers at any program point (where $k$ is the total number of physical registers available in the target machine), assignment produces the actual physical register names required to generate the executable code. Both these tasks are NP-hard at the global (i.e., procedure) level.

Today’s architectures pose new challenges to register allocation due to hardware features such as register classes, register aliases, pre-coloring, and register pairs. For example, the Intel x86 architecture provides eight integer physical registers, of which six are usable by the compiler. These six physical registers are further divided into four register classes based on calling conventions and 8-bit operand accesses. To produce high-quality machine code, a register allocator must consider these hardware features in both the allocation and assignment phases.

In this chapter, we make the following contributions:

1. We introduce a Bipartite Liveness Graph (BLG) representation as an alternative to the interference graph (IG) representation. Allocation with the BLG is formulated as an optimization problem and a greedy heuristic is presented to solve it. Our allocation phase is independent of the move-coalescing optimization that is usually performed along with allocation in an IG based Graph Coloring algorithm.

2. We present a spill-free register assignment that reduces the number of spill instructions by using register-to-register move and exchange instructions wherever
possible to maximize the use of registers.

3. We formulate spill-free register assignment with move coalescing as a combined optimization problem that maximizes the benefits of move coalescing while finding an assignment for every symbolic register. Move coalescing is performed on a Coalesce Graph (CG) that models both IR move instructions and additional register-to-register moves/exchanges needed to generate correct code. A local greedy heuristic is presented to address the assignment optimization problem.

4. We extend the register assignment approach from (3) above to handle register classes. An optimization version of the assignment problem is presented that minimizes the additional spilled symbolic registers and at the same time maximizes the benefits of move coalescing. A prioritized bucket-based greedy heuristic is presented to address this problem.

5. Finally, we present an Extended Linear Scan (ELS) register allocation algorithm that avoids building the BLG explicitly to save space. ELS retains the compile time and space efficiency of Linear Scan register allocation algorithms.

6.1 Notions Revisited

For convenience, a program point can be split into two program points based on the values read and written at that program point [105]. A register allocation problem can then be defined with respect to the split program points as follows.

**Definition 6.1.1** Each program point $p$ is split into $p^-$ and $p^+$, where $p^-$ consists of the variables that are read at $p$ and $p^+$ consists of the variables that are written at $p$.

**Definition 6.1.2** Given a set of symbolic registers, $S$, and $k$ uniform physical registers that are independent and interchangeable, determine if it is possible to assign each symbolic register $s \in S$ to a physical register at every program point where $s$
is live. If so, report the assignment as \( \text{reg}(s, x) \) that indicates the physical register assigned to \( s \) at each program point \( x \).

The number of simultaneously live symbolic registers at a program point \( p \) is denoted by \( \text{numlive}(p) \). \( \text{MAXLIVE} \) represents the maximum number of simultaneously live symbolic registers in any program point. A program point \( p \) is said to be constrained if \( \text{numlive}(p) > k \). In the presence of register classes, we call a program point \( p \) as constrained if it violates any of the register requirements of any of the register classes of the symbolic registers that are live at \( p \).

Linear Scan (\( LS \)) \cite{100, 105, 119} is a compile time and space efficient approach to register allocation that is suitable for dynamic compilation. It assumes a linear ordering of the IR instructions (typically depth-first order \cite{4}). The central data structure in \( LS \) is the notion of live interval. A live interval may contain program points where a variable \( v \) may not be live, i.e., it does not contain any useful value but it is included in the live interval of \( v \). The sub-interval during which a variable is not live is known as a hole \cite{119}.

**Definition 6.1.3** \([x, y]\) is a basic interval for variable \( v \) (denoted as \( \text{BI}(v) \)) iff \( \forall p, p \geq x, p \leq y \) and \( v \) is live at \( p \). Note that \( \text{BI}(v) \) does not include any holes. \( \text{Lo}(\text{BI}(v)) \) and \( \text{Hi}(\text{BI}(v)) \) denote the unique start and end points respectively of basic interval \( \text{BI}(v) \).

**Definition 6.1.4** A compound interval for a variable \( v \) (denoted as \( \text{CI}(v) \)) consists of a set of basic intervals for \( v \). Note that \( \text{CI}(v) \) accommodates holes.

**Definition 6.1.5** Two basic intervals, \([x_1, y_1]\) and \([x_2, y_2]\), are said to be intersecting if one of the following holds:

1. \( x_2 \geq x_1 \) and \( x_2 \leq y_1 \)

2. \( y_2 \geq x_1 \) and \( y_2 \leq y_1 \)
Let $B$ denote the set of all basic intervals and $C$ denote the set of all compound intervals in the program. Let $L$ denote the set of start points, \textit{i.e.,} $L_0$ of all the basic intervals and $H$ denote the set of end points, \textit{i.e.,} $H_i$ of all the basic intervals in the program.

### 6.2 Example

Figure 6.1 presents an example code fragment with its basic and compound intervals and $IG$. Let us assume that we have 2 physical registers, $r_1$ and $r_2$. We can easily
see that the IG has a clique of size 3, i.e., the cycle comprising of nodes $c$, $d$, and $e$. Now, consider a Graph Coloring register allocator that performs coalescing along with allocation. Both aggressive [35] and conservative [25, 28] coalescing will be able to eliminate the move edges $(a, c)$, $(b, d)$, and $(e, f)$ without increasing the colorability of the original interference graph. Since there are two physical registers, we have to spill one among the coalesced nodes $ac$, $bd$, and $ef$. Coalescing has worsened the situation because it would have sufficed to spill just one of the nodes in the coalesced node that is part of the cycle, e.g., it would have been enough to just spill $d$ instead of $bd$. The un-coalescing approach used in an optimistic coalescing technique [96] will be able to just spill one of the nodes involved in the cycle as it tries all possible combinations of assigning colors to individual nodes of a potentially spilled coalesced node. The points to note here are that we can not color the IG using 2 physical registers and that opportunities for coalescing can be missed due to not being able to color certain nodes.

A close look at the code and the intervals reveal the fact that none of the program points have more than 2 variables live simultaneously. If this is the case, two questions come to mind: 1) can we generate spill-free code with 2 physical registers that does not give up any coalescing of symbolic registers? 2) if the answer to the first question is yes, then why did Graph Coloring generate spill code and also miss the coalescing opportunity?

The answer to the first question is yes. The Bipartite Liveness Graph (BLG) shown in Figure 6.1(c) captures the fact that every basic interval end point in $V$ has degree less than or equal to 2 indicating no more than 2 compound intervals are simultaneously live. Using this information, the following register assignment is possible:

$$\text{reg}([1^+, 3^-]) = r_1, \text{reg}([2^+, 4^-]) = r_2, \text{reg}([4^+, 5^-]) = r_2, \text{reg}([3^+, 7^-]) = r_1,$$

$$\text{reg}([6^+, 9^-]) = r_2, \text{reg}([9^+, 10^-]) = r_2, \text{reg}([8^+, 13^-]) = r_1,$$

and $\text{reg}([11^+, 14^-]) = r_2$. 

This assignment requires an additional register exchange operation since the register assignment for the basic intervals of both $CI(c)$ and $CI(d)$ were exchanged when the code after the if condition was executed, *i.e.*, $CI(c) : \text{reg}([3^+ , 7^-]) = r_1\), $\text{reg}([11^+ , 14^-]) = r_2$ and $CI(d) : \text{reg}([4^+ , 5^-]) = r_2\), $\text{reg}([8^+ , 13^-]) = r_1$. We need to insert an $\text{exchg } r_1, r_2$ instruction on the control flow edge between 4 and 13. Also note that none of the coalescing opportunities on lines 3, 4 and 9 were given up during such an assignment.

Now let us try to answer the second question. Looking at the code fragment, we observe that in program point 13$, d$ interferes with two values of $c$ assigned on lines 3 and 11. Similarly, $c$ interferes with two values of $d$ assigned on lines 4 and 8. During runtime if the if-branch is taken then assignments on lines 8 and 11 will be visible to the code following the if condition, otherwise assignments on lines 3 and 4 will be visible. This notion can not be precisely captured using the definition of live ranges in an interference graph unless we convert the program to SSA form. However, an SSA based approach inserts extra copy statements during out-of-ssa translation which pose an additional challenge as discussed in Section 6.3.

Figure 6.2 presents another example code fragment with its basic and compound intervals and $IG$. Similar to the previous example, we can easily see that the $IG$ has a clique of size 3, *i.e.*, the cycle comprising of nodes $b$, $c$, and $d$. Graph Coloring register allocator will end up spilling one of the nodes $b$, $c$ or $d$ and give up the corresponding coalescing opportunity. However, it is possible to generate spill-free assignment for this program that does not need to spill any variable.

The following assignment is possible:

\[
\begin{align*}
\text{reg}([1^+ , 2^-]) &= r_1, \\
\text{reg}([2^+ , 5^-]) &= r_1, \\
\text{reg}([4^+ , 7^-]) &= r_2, \\
\text{reg}([6^+ , 7^-]) &= r_1, \\
\text{reg}([8^+ , 10^-]) &= r_1, \\
\text{reg}([11^+ , 13^-]) &= r_1, \\
\text{reg}([9^+ , 14^-]) &= r_2, \\
\text{reg}([13^+ , 15^-]) &= r_1, \\
\text{reg}([14^+ , 16^-]) &= r_2.
\end{align*}
\]

This assignment requires an additional register-to-register move as the two basic intervals of the compound interval $CI(d)$ are allocated in two different physical registers,
\( \text{i.e., } \text{reg}([6^+, 7^-])=r_1 \text{ and } \text{reg}([9^+, 14^-])=r_2. \) We need mov \( r_1, r_2 \) instruction to be added on the control flow edge between program points 7 and 13. Additionally, we observe that the all the move instructions on lines 2, 13 and 14 can be removed in the generated code as both the source and destination have the same physical register. If we analyze the code fragment, we observe that \( b \) interferes with \( c \) on the true branch of the if condition and \( b \) interferes with \( d \) on the false branch. During runtime only one path is taken and hence, one of the interferences but not both will be held. Once again, this notion was not precisely captured in the notion of live ranges in an IG representation.

The above examples illustrate cases in which none of the program points has more than 2 simultaneously live symbolic registers, yet the interference graph contains a clique of size 3 (forcing the need to spill a symbolic register). This raises a question about the general approach of formulating the register allocation problem as the graph coloring problem on the IG using live ranges. Even though the interference graph provides a global view of the program, it is less precise than intervals, and is also known to be a space and time bottleneck. Additionally, when coalescing is performed along with register allocation on an interference graph, the degree of some nodes may increase. (The colorability of the interference graph may increase if we do not use conservative coalescing.) This unnecessarily complicates the allocation phase of the register allocation process. We believe that the allocation phase should only focus on choosing a set of symbolic registers that need to be allocated to physical registers and should have nothing to do with coalescing. Coalescing should ideally be a part of the assignment phase of a register allocation algorithm.

### 6.3 Overall Approach

The register allocator presented in this chapter is depicted in Figure 6.3. The first step in the allocator is to build data structures for Basic Intervals, Compound Intervals and the Bipartite Liveness Graph (BLG) defined in Section 6.4. Then, allocation is
a) Code fragment with basic and compound intervals:

```plaintext
1  a = ...
2  b = a
3  if (...) {
4    c = ...
5    d = b
6    ...
7 }  
8  else {
9    d = ...
10   ...
11   c = ...
12 }
13 e = c
14 f = d
15 ...
16 ...
```

b) Interference Graph (dashed lines show move instructions):

```
\begin{center}
\begin{tikzpicture}
\node (a) at (0,0) {a};
\node (b) at (1,0) {b};
\node (c) at (2,0) {c};
\node (d) at (3,0) {d};
\node (e) at (4,0) {e};
\node (f) at (5,0) {f};
\draw[dashed] (a) -- (b);
\draw (b) -- (c);
\draw (c) -- (d);
\draw (d) -- (e);
\draw (e) -- (f);
\end{tikzpicture}
\end{center}
```

c) Bipartite Liveness Graph (BLG):

```
\begin{center}
\begin{tikzpicture}
\node (a) at (0,0) {a};
\node (b) at (1,0) {b};
\node (c) at (2,0) {c};
\node (d) at (3,0) {d};
\node (e) at (4,0) {e};
\node (f) at (5,0) {f};
\node (v) at (0,-2) {U};
\node (w) at (5,-2) {V};
\draw (a) -- (v);
\draw (b) -- (v);
\draw (c) -- (v);
\draw (d) -- (w);
\draw (e) -- (w);
\draw (f) -- (w);
\end{tikzpicture}
\end{center}
```

Figure 6.2: a) Example code fragment with basic and compound intervals; the dotted lines represent end-points of basic intervals. b) Interference Graph (IG); the solid lines in IG represent interference and the dashed lines represent move instructions. c) Bipartite Liveness Graph (BLG); the vertices on the left of the graph represent compound intervals, and the vertices on the right represent basic interval end points.

```
\begin{center}
\begin{tikzpicture}
\node (build) at (0,0) {Build (BI, CI, BLG)};
\node (potential) at (3,0) {Potential Spill};
\node (actual) at (6,0) {Actual Spill};
\node (select) at (9,0) {Select};
\node (moves) at (12,0) {Moves/Exchange Insertion};
\node (spill) at (15,0) {Spill code};
\node (coalesce) at (9,-3) {Coalesce};
\draw[->] (build) -- (potential);
\draw[->] (potential) -- (actual);
\draw[->] (actual) -- (select);
\draw[->] (select) -- (moves);
\draw[->] (moves) -- (spill);
\draw[->] (coalesce) -- (build);
\end{tikzpicture}
\end{center}
```

Figure 6.3: Overall Space Efficient Register Allocator using BLG.
Figure 6.4: SSA based Register Allocation. This figure is adapted from [22].

performed on the $BLG$ to determine a set of compound intervals that need to be spilled everywhere (as shown in the blocks for potential spill and actual spill). A combined phase of assignment and coalescing is then performed until all the remaining symbolic registers are assigned physical registers or spilled. Then register move and exchange instructions are added to the $IR$ to produce correct code. Finally, spill code is added to the $IR$.

As a comparison, Figure 6.4 depicts various components of an SSA-based register allocation [22, 59, 98]. Comparing our allocator with that of an SSA-based register allocator, an SSA register allocation typically demands for high compile-time overhead due to the interference graph and has additional complexity in optimizing copy statements during out-of-ssa translation. In particular, the allocator proposed in [22, 60]:

1. requires an interference graph for coloring and spilling. As seen in past work [42, 105], the interference graph is typically the space and time bottleneck in the register allocator. In contrast, our approach requires a Bipartite Liveness Graph ($BLG$) that is of lower space complexity than an interference graph in practice.

2. needs to pay attention to out-of-ssa translation after register allocation [97]. Additional copies for SSA $\phi$-node translation may be added that may degrade the quality of machine code produced. Also, it is not clear how issues such as the swap and lost copy problems [44] in out-of-ssa translation interfere with coloring and spilling. Our approach is more direct, as the register allocation is performed on a linear $IR$, not an SSA $IR$.

3. requires coalescing be performed after coloring assignment. This in turn advo-
cates the need for re-coloring in the IG during the coalescing pass, leading to
color clashes [60]. Backtracking on color assignments in the IG is an expensive
operation that may be undesirable in a dynamic compilation environment. In
contrast, our approach performs move coalescing over a Coalesce Graph that
combines the moves present in the IR and the moves that are needed by the
assignment phase into a single optimization problem. The assignment and
coaescing passes go hand-in-hand in our approach.

4. may require additional heuristics such as those described in [110] on the in-
terference graph to handle advanced features like register classes and register
aliasing. These features are easier to model in a BLG than on an interference
graph.

6.4 Allocation using Bipartite Liveness Graphs

The Bipartite Liveness Graph (BLG) is a new representation that captures program-
point specific liveness information as an alternative to the interference graph. We take
an all-or-nothing approach for spills, i.e., if spilled, every occurrence of the symbolic
register in the program will perform either a fetch or a store of the memory location
assigned to the symbolic register.

Definition 6.4.1 A Bipartite Liveness Graph (BLG) is a undirected weighted bipar-
tite graph, \( G = (U \cup V, E) \), where \( V \) denotes all the basic interval end points\(^1\) in \( \mathcal{H} \), \( U \)
denotes all the compound intervals in \( \mathcal{C} \) and an edge \( e = (u, v) \in E \) indicates that the
compound interval \( u \in U \) is live at the interval end point \( v \in V \). Each \( u \in U \) has an
associated non-negative weight \( \text{SPILL}(u) \) that denotes the spill cost of \( u \). Similarly,
each \( v \in V \) has an associated non-negative weight \( \text{FREQ}(v) \) that denotes the execution
frequency of the IR instruction associated with basic interval end point \( v \).

\(^1\)The choice of interval end points is arbitrary. We could have used interval start points instead.
Definition 6.4.2 Allocation Optimization Problem: Given a BLG, $G$, and $k$ uniform physical registers, find a spill set $S \subseteq U$ and $G' \subseteq G$ induced by $S$ such that:

1. $\forall v \in V$, $v$ is unconstrained, i.e., $\text{DEGREE}(v) \leq k$; and
2. $\sum_{s \in S} \text{SPILL}(s)$ is minimized. For each compound interval $s \in S$ and basic interval $b \in s$, set $\text{spilled}(b) := \text{true}$.

Note that, it is a waste of space to capture liveness information at every program point in the $BLG$. From an allocation perspective, it suffices to consider either the basic interval start points alone, or end points alone but not both. This is because spilling/allocation decisions only need to be taken at those points.

Given a $BLG$, the register allocation problem now reduces to an optimization problem whose solution ensures no more than $k$ physical registers are needed at every interval end point and at the same time spills as few compound intervals as possible. Algorithm 6.5 provides a greedy heuristic that solves the allocation optimization problem. Steps 3-12 choose Potential Spill (as shown in Figure 6.3) candidates using a max-min heuristic. Steps 13-17 unspill some of the potential spill candidates producing Actual Spill (as shown in Figure 6.3) candidates. Depending on the quality of potential spill candidate selection, the unspilling of spill candidates provide a way of rectifying the obvious spilling decisions (akin to unspilling in Graph Coloring).

Theorem 6.4.3 Algorithm 6.5 ensures that every program point has $k$ or fewer symbolic registers simultaneously live.

Proof: The algorithm continues to execute the while loop in Steps 3-12 until there are constrained nodes $v \in V$ in the $BLG$. This is guaranteed by Steps 3, 10, and 12. □

Theorem 6.4.4 Algorithm 6.5 requires $O(|B| \times |C|)$ space.
function GreedyAlloc()  

**Input**: Weighted Bipartite Liveness Graph $G = \langle U, V \rangle$ and $k$ uniform physical registers  
**Output**: Set $T \subseteq U$ which needs to be spilled to ensure all interval end points $v \in V$ be unconstrained, i.e., $\forall b \in T, \text{spilled}(b) = true$  

Stack $S := \phi$;  

//Potential spill selection  
Choose a constrained node $n \in V$ with largest $FREQ(n)$;  
while $n \neq null$ do  
  Choose a compound interval $s \in U$ having an edge to $n$ and has smallest $\text{SPILL}(s)$;  
  Push $n$ onto $S$;  
  Delete edge $(s, n)$;  
  Choose a constrained node $n \in V$ having an edge to $s$ and has largest $FREQ(n)$;  
  if $n == null$ then  
    Choose a constrained node $n \in V$ with largest $FREQ(n)$;  
    Delete all edges incident on $s$;  
    Remove $s$ from $G$;  

//Actual spill selection  
while $S$ is not empty do  
  $s := \text{pop}(S)$;  
  if $\forall v \in V, v$ does not become constrained by reverting $s$ and its edges in $G$ then  
    $\text{spilled}(s) := true$;  
    $T := T \cup \{s\}$;  

return $T$  

Figure 6.5 : Greedy heuristic to perform allocation using *max-min* strategy.
**Proof:** The main data structure for Algorithm 6.5 is the BLG. A BLG, \( G = (U \cup V, E) \) can be represented using an adjacency list representation that uses \( \mathcal{O}(|U| + |V| + |E|) \) space. Since \( |U| = |C|, |V| = |B|, \) and \( |E| \) in the worst case can be \( |B| \ast |C| \), the overall space requirement for a BLG is \( -\mathcal{O}(|B| \ast |C|) \). Additional \( |C| \) stack space is required for unspilling. □

Note that though \( \mathcal{O}(|B| \ast |C|) \) is a worst-case quadratic size, in practice, we do not need to consider every interval end point of the program in a BLG for Algorithm 6.5 – only constrained end points are sufficient. Hence, we expect the BLG to be a sparse graph with even lower space requirements.

**Theorem 6.4.5** Algorithm 6.5 requires \( \mathcal{O}(|C| \ast (M A X L I V E - k) \ast |C|) \) time.

**Proof:** Every interval end point of \( B \) is traversed at most \( (M A X L I V E - k) \) number of times to make it unconstrained. Each visit of an interval end point needs to visit all its outgoing edges and choose a minimum spill cost compound interval. This requires at most \( |C| \) edge visits. □

### 6.4.1 Eager Heuristic

As a comparison to the heuristic presented in Figure 6.5, we present another greedy heuristic in Figure 6.6 to perform eager allocation. This approach is compile-time efficient, as each interval end point in BLG is traversed exactly once. The basic idea is to find the next largest frequency interval end point and make the end point unconstrained before traversing another interval end point.

**Theorem 6.4.6** Algorithm 6.6 requires \( \mathcal{O}(|B| \ast |C|) \) time.

**Proof:** Every interval end point of \( B \) is traversed exactly once. Each such visit needs to visit all its outgoing edges and choose a set of minimum spill cost compound intervals to make the end point unconstrained. This requires at most \( |C| \) edge visits. □

The key advantage of performing register allocation using BLG is that the allocation process only focuses on spilling decisions to produce unconstrained interval
function EagerUniformAllocation ()

Input : Weighted Bipartite Liveness Graph \( G = \langle U, V \rangle \) and \( k \) uniform physical registers

Output: Set \( T \subseteq U \) which needs to be spilled to ensure all interval end points \( v \in V \) be unconstrained, i.e., \( \forall b \in T, \text{spilled}(b) = true \)

Stack \( S := \phi \);

//Potential spill selection
Choose a constrained node \( n \in V \) with largest \( \text{FREQ}(n) \);

while \( n \neq \text{null} \) do

Choose a compound interval \( s \in U \) having an edge to \( n \) and has smallest \( \text{SPILL}(s) \);

while \( s \neq \text{null} \) do

Push \( n \) onto \( S \);

Delete all edges incident on \( s \);

Remove \( s \) from \( G \);

if \( n \) is constrained then

Choose a compound interval \( s \in U \) having an edge to \( n \) and has smallest \( \text{SPILL}(s) \);

Choose a constrained node \( n \in V \) with largest \( \text{FREQ}(n) \);

//Actual spill selection

while \( S \) is not empty do

\( s := \text{pop}(S) \);

if \( \forall v \in V, v \) does not become constrained by reverting \( s \) and its edges in \( G \) then

\( \text{spilled}(s) := true \);

\( T := T \cup \{s\} \);

return \( T \)

Figure 6.6 : Eager heuristic to perform allocation. Each interval end point is traversed exactly once and during each visit the end point is made eagerly unconstrained.
end points/program-points and does so in a space efficient manner. In contrast, an interference-graph-based allocation has to focus on both spilling decisions and move-coalescing in an attempt to reduce the colorability of the interference graph. Move coalescing using aggressive and optimistic coalescing approaches often increase the colorability of the interference graph, thereby leading to more spill decisions. The $BLG$ based approach introduced in this section advocates that the coalescing be instead performed in the assignment phase.

After the allocation phase ensures that every program point needs $k$ or fewer physical registers, in the next section we describe how assignment for basic intervals can be performed by possibly adding extra register moves/exchanges to the $IR$ without spilling any symbolic registers. In the presence of move coalescing and register-to-register moves, we state the assignment problem as an optimization with the goal of removing the most register-to-register moves whether they come from assignments or moves that were originally present in the $IR$.

### 6.5 Assignment using Register Moves and Exchanges

In this section, we present a spill-free assignment that allows register-to-register moves and exchange instructions. We first formulate the basic assignment of intervals and then present an advanced heuristic to perform coalescing along with assignment in the presence of register-to-register moves.

#### 6.5.1 Spill-Free Assignment

**Definition 6.5.1 Spill-free Assignment:** Given a set of basic intervals $b \in \mathcal{B}$ with spilled$(b) = \text{false}$, and $k$ uniform physical registers, find a register assignment $\text{reg}(b)$ for every basic interval $b \in \mathcal{B}$ including any register-to-register copy or exchange instructions that need to be inserted in the $IR$.

The algorithm to perform register assignment for basic intervals is provided in Algorithm 6.7. The algorithm sorts the basic intervals in increasing start points.
function RegMoveAssignment()

Input: \( IR \), Set of basic intervals \( b \in B \) with \( \text{spilled}(b) = \text{false} \) and \( k \)
uniform physical registers

Output: \( \forall b \in B \), return the register assignment \( \text{reg}(b) \) and any register
moves and exchange instructions

\[ M := \varnothing; \]
\[ \text{avail} := \text{set of physical registers}; \]
\[ \text{for each basic interval } b := [x, y], \text{ in increasing start points, i.e., } L \text{ do} \]
\[ \quad \text{for each basic interval } b' := [x', y'] \text{ such that } y' < x \text{ do} \]
\[ \quad \quad \text{avail} := \text{avail} \cup \text{reg}(b'); \]
\[ \quad r := \text{find a physical register } p \in \text{avail} \text{ that was assigned to another basic} \]
interval of the same compound interval;
\[ \quad \quad \text{//if there are more than one choices, choose the physical} \]
\[ \quad \quad \text{register that reduces the cost of extra move/exchange} \]
\[ \quad \quad \text{instructions added} \]
\[ \quad \quad \text{if } r == \text{null then} \]
\[ \quad \quad \quad \text{Assert avail} \text{ is not empty}; \]
\[ \quad \quad \quad r := \text{find a physical register } p \in \text{avail}; \]
\[ \quad \quad \quad \text{reg}(b) := r; \text{avail} := \text{avail} \setminus \{r\}; \]
\[ \text{for each control flow edge, } e \text{ do} \]
\[ \quad \text{for each compound interval } c \in B \text{ that is live at both end points of } e \text{ do} \]
\[ \quad \quad \quad b_1 := \text{basic interval of } c \text{ at the source of } e; \]
\[ \quad \quad \quad b_2 := \text{basic interval of } c \text{ at the destination of } e; \]
\[ \quad \quad \quad \text{if } b_1 != \text{null and } b_2 != \text{null then} \]
\[ \quad \quad \quad \quad r_1 := \text{reg}(b_1); r_2 := \text{reg}(b_2); \]
\[ \quad \quad \quad \quad \text{if } r_1 != r_2 \text{ then} \]
\[ \quad \quad \quad \quad \quad m := \text{generate a new move instruction that moves } r_1 \text{ to } r_2, \]
\[ \quad \quad \quad \quad \quad \text{i.e., } \text{mov} \, r_1, r_2; \]
\[ \quad \quad \quad \quad \quad M := M \cup \{m\}; \]
\[ \quad \quad \quad \text{//Add Move and Exchange instructions in the } IR \]
\[ \quad \quad \quad \text{GenerateMoves}(IR, M, e); \]
\[ \text{return } T \text{ and } IR \]

Figure 6.7: Assignment using register-to-register moves and exchange instructions.
Steps 4-11 perform assignment to basic intervals using a *avail* list of physical registers. The assignment to a basic intervals first prefers getting the physical register that was previously assigned to another basic intervals of the same compound interval (as shown in Step 7). This avoids the need for additional move/exchange instructions. However, in cases where the already assigned physical register is unavailable, we assign a new available physical register (as shown in Step 10). Assigning such a new physical register may produce incorrect code without additional move/exchange instructions on certain control flow paths.

Steps 12-21 of Algorithm 6.7 creates a list of move instructions that need to be inserted on a control flow edge. These move instructions form the nodes of a directed anti-dependence graph $D$ in Algorithm 6.8. The edges in $D$ represent the anti-dependence between a pair of move instructions. Steps 5-10 add the anti-dependence edges to $D$. A strongly connected component (SCC) search is performed on $D$ to generate efficient code using *exchange* instructions for SCCs of size 2 or more (as shown in Steps 12-18). The nodes in an SCC are collapsed to a single node with exchange instructions. Finally, a topological sort order of $D$ produces the correct code for a control flow edge $e$.

The outputs produced by Algorithm 6.7 are an extension of the outputs for the Graph Coloring algorithm. The register map, *reg* is finer-grained for our approach than for Graph Coloring since it is capable of assigning different physical registers to different basic intervals of a compound interval. Another output of our approach is a set of register-to-register move or exchange instructions needed to support the assignment in the *reg* map. We assume that it is preferable to generate register-to-register moves than spill loads and stores on current and future systems, even for loads and stores that results in cache hits. This is because many processors incur a coherence overhead for loads and stores, compared to register accesses. Further, register-register moves can be optimized by move coalescing described in the next section.
function GenerateMoves()
    Input: IR, Set of move instructions $M$ and a control flow edge $e$
    Output: Modified IR with register move and exchange instructions added
    // $D$ is the anti-dependence graph
    $D := \phi$;
    for $m_1 \in M$ do
        Add a node for $m_1$ in $D$;
        //Add the anti-dependence edges
        for $m_1 \in D$ do
            for $m_2 \in D$ and $m_2 \neq m_1$ do
                src$_1 :=$ source of the move instruction in $m_1$;
                dest$_2 :=$ destination of the move instruction in $m_2$;
                if src$_1 == dest$_2$ then
                    Add a directed edge $(m_1, m_2)$ to $D$;
        $S :=$ Find strongly connected components in $D$;
        for each $s \in S$ do
            Collapse all the nodes in $s$ to a single node $n$ in $D$;
            while number of move instructions in $s > 1$ do
                $m_1 :=$ Remove first move instruction from $s$;
                $m_2 :=$ First move instruction in $s$;
                $x :=$ Generate an exchange instruction between the destinations of $m_1$ and $m_2$;
                Append $x$ to the instructions of $n$;
        for each node $n$ in $D$ in topological sort order do
            Add the move or exchange instructions of the node $n$ to the IR along the control flow edge $e$;
    return Modified IR

Figure 6.8 : Algorithm to insert of register-to-register move and exchange operations on a control flow edge.
Figure 6.9: Anti-dependence graph \((D)\) for example program shown in Figure 6.1. \(D\) has a strongly connected component (SCC) which can be reduced to a single node with \(xchg\ r_1, r_2\) instruction.

6.5.2 Example

Consider our previous example program shown in Figure 6.1. After applying Algorithm 6.7 to the sorted interval start points, we get the following assignments:

\[
\begin{align*}
reg([1^+, 3^-]) &= r_1, \\
reg([2^+, 4^-]) &= r_2, \\
reg([4^+, 5^-]) &= r_2, \\
reg([3^+, 7^-]) &= r_1, \\
reg([6^+, 9^-]) &= r_2, \\
reg([9^+, 10^-]) &= r_2, \\
reg([8^+, 13^-]) &= r_1, \\
\text{and } reg([11^+, 14^-]) &= r_2.
\end{align*}
\]

We can now observe that along the control-flow edge between statements 4 and 13, the assignment for compound interval \(CI(c)\) changes from \(r_1\) to \(r_2\). This results in the creation of a move instruction \(m_1: mov\ r_1, r_2\) (due to Step 19 of Algorithm 6.7). Similarly, along the same control flow edge, the assignment for compound interval \(CI(d)\) changes from \(r_2\) to \(r_1\). This yields another move instruction \(m_2: mov\ r_2, r_1\).

The move instructions \(m_1\) and \(m_2\) result in an anti-dependence graph \(D\) shown in Figure 6.9. \(D\) contains a strongly connected component involving \(m_1\) and \(m_2\). We can generate efficient code using exchange instructions according to Steps 11-20 of Algorithm 6.8. The \(xchg\ r_1, r_2\) instruction is added along the control-flow edge between statements 4 and 13.

The example program shown in Figure 6.2 is much simpler. After applying algorithm 6.7 to the sorted interval start points, we get the following assignments:

\[
\begin{align*}
reg([1^+, 2^-]) &= r_1, \\
reg([2^+, 5^-]) &= r_1, \\
reg([4^+, 7^-]) &= r_2, \\
reg([6^+, 7^-]) &= r_1, \\
reg([8^+, 10^-]) &= r_1, \\
reg([9^+, 14^-]) &= r_2, \\
\text{and } reg([13^+, 15^-]) &= r_1, \\
\text{and } reg([14^+, 16^-]) &= r_2.
\end{align*}
\]
This assignment needs a \textit{mov} \( r_1, r_2 \) instruction to be inserted on the control-flow edge between statements 7 and 13 as the compound interval \( \text{CI}(d) \) is switched from \( r_1 \) to \( r_2 \) along that path.

**Lemma 6.5.2** The assertion in Step 9 of Algorithm 6.7 never fails.

**Proof:** Follows from the fact that every interval end point has no more than \( k \) symbolic registers simultaneously live. This fact was ensured by the allocation phase. □

**Theorem 6.5.3** Given \( k \) uniform physical registers, spill-free assignment always finds an assignment for every \( b \in \mathcal{B} \) whose \( \text{spilled}(b) = \text{false} \).

**Proof:** Follows from the previous lemma. □

**Theorem 6.5.4** Spill-free assignment takes \( \mathcal{O}(|\mathcal{E}| \times |\mathcal{C}|) \) space where \( \mathcal{E} \) represents the set of control flow edges in a program.

**Proof:** The additional space requirement in assignment phase is due to the anti-dependence graph \( D \). For every control-flow edge \( e \in \mathcal{E} \), in the worst case we need to insert \( |\mathcal{C}| \) register-to-register move instructions. These are the number of nodes in \( D \). The number of edges in \( D \) are bounded by the square of the number of physical registers since it represents all possible anti-dependences between all possible pairs of physical registers in the worst case. Hence, the overall space complexity is \( \mathcal{O}(|\mathcal{E}| \times |\mathcal{C}|) \). □

**Theorem 6.5.5** Spill-free assignment takes \( \mathcal{O}(|\mathcal{B}| + (|\mathcal{E}| \times |\mathcal{C}|)) \) time.

**Proof:** Similar in nature to the proof for Theorem 6.5.4. □
6.5.3 Assignment with Move Coalescing and Register Moves

Move coalescing is an important optimization in register-allocation algorithms that assign the same physical register to the source and destination of an IR move instruction when possible to do so. The register assignment phase must try to coalesce as many moves as possible so as to get rid of the move instructions from the IR. As we saw in the preceding section, additional register moves may be inserted in the assignment phase instead of spilling. In this section we first present a Coalesce Graph that models both the IR move instructions and register-to-register moves. Then the register assignment phase on the coalesce graph is formulated as an optimization problem that tries to maximize the number of move instructions removed after assignment.

**Definition 6.5.6** A Coalesce Graph \((CG)\) is an undirected weighted graph, \(G = \langle V, E_m \cup E_r \rangle\), where \(V\) represents the basic intervals in \(B\) and an edge \(e \subseteq V \times V\) corresponds to the following two types of move instructions between a pair of basic intervals:

1. \(E_m\): the move instructions already present in the IR. The weight of such an edge \(W(e)\) is the frequency of the corresponding move instruction.

2. \(E_r\): the register-to-register move instructions that need to be added on control-flow edges for which the two interval end points have different register assignments for the same compound interval. The weight of such an edge \(W(e)\) is the frequency of the control-flow edge on which the move instruction is added. An exchange instruction can be viewed as consisting of two move instructions.

**Definition 6.5.7 Assignment Optimization Problem:** Given a set of basic intervals \(b \in B\) with \(\text{spilled}(b) = \text{false}\), \(CG = \langle V, E = \{E_m \cup E_r\}\rangle\), IR, and \(k\) uniform physical registers, find a register assignment \(\text{reg}(b)\) for every basic interval \(b\) such that the following objective function is minimized:
The assignment guides which additional register-to-register copy or exchange instructions need to be inserted in the IR.

Algorithm 6.10 presents a greedy heuristic to select a physical register for a basic interval \( b \) given the coalesce graph and the available set of physical registers. Map is a data structure that maps a physical register to a cost. Steps 3-7 find the physical registers (and their associated costs) that are already assigned to the neighbors of \( b \) in the coalesce graph (similar to the idea of biased coloring [27]). The additional cost of register-to-register moves that need to be inserted for correct code generation by assigning a physical register to a basic interval is penalized in Steps 8-11. The greedy heuristics chooses a physical register \( \text{reg}(b) \) with maximum cost, i.e., the benefit of assigning the physical register to basic interval \( b \).

**Theorem 6.5.8** Register assignment using Algorithm 6.10 requires \( \mathcal{O}(|\mathcal{B}| + |\mathcal{IR}| + (|\mathcal{C}| \cdot \text{max}_c)) \) space where \( \text{max}_c \) denotes the maximum number of basic intervals that a compound interval has.

**Proof:** The additional space requirement is due to the coalesce graph \( CG \) containing \( |\mathcal{B}| \) number of nodes. \( E_m \) in the worst case ends up creating \( |\mathcal{IR}| \) edges. \( E_r \) adds edges between basic intervals of the same compound interval and hence needs \( |\mathcal{C}| \cdot \text{max}_c \) number of edges. □

**Theorem 6.5.9** Register assignment using Algorithm 6.10 takes \( \mathcal{O}((|\mathcal{B}| \cdot \text{max}_c) + |\mathcal{IR}| + (|E_r| \cdot |\mathcal{C}|)) \) time.

**Proof:** In addition to Theorem 6.5.5, before deciding a physical register for each basic interval \( b \) it is required to traverse each of the neighbors in \( CG \). For all basic intervals, this adds over all \( |\mathcal{IR}| \) time complexity for \( IR \) move instructions and \( |\mathcal{B}| \cdot \text{max}_c \) time complexity for \( E_r \) edges in \( CG \). □
function GetPreferredPhysical()
    Input : A basic interval \( b \in \mathcal{B} \), coalesce graph \( G = \langle V, E = \{ E_m \cup E_r \} \rangle \) and a set \( R \) currently available uniform physical registers
    Output: Find the assignment \( \text{reg}(b) \)
    Initialize \( \text{Map} \) for every physical register to 0;
    //Maximize the IR moves that can be removed
    for each edge \( e = (b_1, b) \in E_m \) do
        if \( b_1 \) and \( b \) do not intersect then
            \( p := \text{reg}(b_1) \);
            if \( p \neq \text{null} \) and \( p \in R \) then
                \( \text{Map}(p) := \text{Map}(p) + \mathcal{W}(e) \);
    //Minimize the new register-to-register moves that needs to be inserted
    for each edge \( e = (b_1, b) \in E_r \) do
        \( p := \text{reg}(b_1) \);
        if \( p \neq \text{null} \) and \( p \in R \) then
            \( \text{Map}(p) := \text{Map}(p) - \mathcal{W}(e) \);
    \( \text{ret} := \text{Find } p \text{ with maximum cost in } \text{Map} \);
    if \( \text{ret} == \text{null} \) then
        \( \text{ret} := \text{Find any free physical register from } R \);
    \( \text{reg}(b) := \text{ret} \);
    return \( \text{reg}(b) \);

Figure 6.10 : Greedy heuristic to choose a physical register for a basic interval that maximizes copy removal.
Figure 6.11: Four register classes for integer operands in Jikes RVM for x86 architectures. The register class comprising of EBX represents the 8 bit non-volatile class. The register class with EBX, EDI and EBP represents the non-volatile register class. The register class with EAX, EBX, ECX, EDX represents the 8 bit volatile class. All the six available integer physical register form a class of their own.

Figure 6.12: Example program demonstrating assignment problems using register classes. Given $\text{regclass}(a) = \text{regclass}(d) = [r_1, r_2]$ and $\text{regclass}(c) = [r_1]$, if we assign $\text{reg}(a) = r_2$ and $\text{reg}(d) = r_1$, then we will have to spill $c$ where as if we assign $\text{reg}(a) = r_1$ and $\text{reg}(d) = r_2$, we would have obtained an assignment for $c$, i.e., $\text{reg}(c) = r_1$.

6.6 Allocation and Assignment with Register Classes

In the preceding sections, we have described register allocation and assignment for $k$ physical registers that are uniform, i.e., they are independent and interchangeable. However, due to advances in architecture, machines do not typically provide a uniform set of physical registers. For example, Figure 6.11 shows the register classes used in Jikes RVM for integer registers in Intel x86 architecture\(^2\). Note that, the register classes for x86 architecture are not disjoint, this implies that we can not consider allocation and assignment phases separately for each register class.

\(^2\)Register classes are referred to as “register preferences” in Jikes RVM.
Figure 6.13: Example demonstrating problems in coalescing due to register classes: if we coalesce $b_1$ with $b_2$ then we would have to give up any other coalescing opportunities with $b_3$, $b_4$ and $b_5$.

Register classes add new challenges to both the allocation and assignment problems. Consider the example code shown in Figure 6.12. Let us assume that the register class for $a$ and $d$ consists of $[r_1, r_2]$ and the register class of $c$ consists of $[r_1]$. During the assignment phase, if we assign $r_2$ to $a$ and then $r_1$ to $d$, this leads to $c$ being spilled since $c$ can only be assigned to $r_1$ and $r_1$ is occupied by $d$. Had we assigned $r_1$ to $a$, we would have been able to perform a spill-free assignment.

In another scenario, consider the coalesce graph shown in Figure 6.13 that depicts the additional complexity that arises while dealing with register classes and coalescing. If we coalesce $b_1$ and $b_2$ during a coalescing phase, then we would have to forgo remaining coalescing opportunities with $b_3$, $b_4$ and $b_5$. In this case, it could have been better to coalesce $b_1$ with intervals other than $b_2$.

As we have seen in Section 2.7, both the allocation and assignment problems for $k$ uniform physical registers are difficult, i.e., NP-hard to perform at all levels of compilation including global level. The new challenges as discussed in the examples above due to the register classes adds to the complexity of each problem. The reality is that we can not ignore their presence. In this section, we describe how allocation and assignment can be performed in the presence of register classes (esp. in a dynamic compilation environment).
6.6.1 Constrained Allocation using BLG

Allocation in the presence of register classes can be performed using the following two approaches:

1. Build BLG for each register class and apply Algorithm 6.5 to each BLG in a particular order (starting with the most constrained register class having fewer physical registers in a class). For example, in the x86 architecture, we need to build four BLGs and apply Algorithm 6.5 in the order 8 bit non-volatile, non-volatile, 8 bit volatile and then for the complete integer register class. If a compound interval is spilled in a BLG for a register class, that decision should be propagated to other BLGs of other classes.

2. An alternative approach is to build a single BLG. During every visit of an interval end point in Algorithm 6.5, we make it unconstrained with respect to all register classes before another interval end point is visited. This approach is space-efficient as it builds only one BLG but can eagerly generate more spills than (1).

6.6.2 Constrained Assignment

Register Assignment in the presence of register classes can be a challenging task esp. when performed along with move coalescing. Given a coalesce graph (as defined in Section 6.5), when we try to find an assignment for a basic interval b, the register classes of the neighbors of b in the coalesce graph along with the register class of b play a key role in selecting a physical register for b. An IR move instruction can be coalesced if source and destination basic intervals have a non-null intersection in their register classes.

Another key point in register assignment is that we no longer can rely on the increasing start point order for assignment of basic intervals since an early decision of physical register assignment of a register class may result in more symbolic registers
being spilled later on or giving up other opportunities for coalescing (as shown in Figure 6.12).

**Definition 6.6.1 Constrained Assignment Optimization Problem:** Given a set of basic intervals \( b \in \mathcal{B} \) with \( \text{spilled}(b) = \text{false} \), \( \text{regclass}(b) \) indicating physical registers that can be assigned to each \( b \), \( \mathcal{CG} = \langle V, E = \{E_m \cup E_r\} \rangle \), and \( \mathcal{IR} \), find a register assignment \( \text{reg}(b) \) for a subset of basic intervals \( S \subseteq \mathcal{B} \) such that the following objective function is minimized:

\[
\sum_{\forall b \in \mathcal{B} - S} \text{SPILL}(b) + \sum_{\forall e \in E, \ e = (b_1, b_2) \land \text{reg}(b_1) \neq \text{reg}(b_2)} W(e)
\]

Insert additional register-to-register copy or exchange instructions in the IR.

Algorithm 6.14 presents a bucket-based approach to register assignment that tries to strike a balance between register classes and spill cost. The \( \text{toColor} \) is a data structure that holds sorted basic intervals according to register classes in a two dimensional array. Steps 8-13 fill in elements of \( \text{toColor} \) array in the next available bucket. Steps 14-17 find an assignment for basic intervals by traversing the \( \text{toColor} \) array in row major order.

Algorithm 6.15 describes a heuristic for selecting an assignment for a basic interval. Steps 6-14 compute costs for physical registers that are already assigned to neighbors and that are not yet assigned to neighbors but will be assigned in future. The reason we consider the unassigned neighbors is to avoid eager decisions of move coalescing. Steps 15-18 penalize the cost of additional register-to-register moves inserted on control flow edges. Finally, Steps 19-21 find a physical register if one is available. Steps 22-24, spills the basic interval if there is no physical register available.

The space and time requirement of Algorithm 6.14 is similar to those of Algorithm 6.10.
function ConstrainedAssignment ()

\[\text{Input} : \text{Set of basic intervals } b \in B, \forall b \in B \text{ regclass}(b), \text{coalesce graph } G = \langle V, E = \{E_m \cup E_r\} \rangle, \text{a set of physical register classes } K, \text{a constant } \text{num\_bucket}\]

\[\text{Output: Find the assignment } \text{reg}(b) \text{ and spill decision } \text{spilled}(b)\]

//Find total number of elements per regclass
for \(b \in B\) do
    //getClassId returns the unique class id
    cid := getClassId (regclass(b));
    perClass[cid] += 1;

//Decide per bucket number of elements
for \(i := 0; i < |K|; i++\) do
    perBucket[i] := \lfloor\frac{\text{perClass}[i]}{|K|}\rfloor + 1;
    availBucket[i] := 0;

//toColor is a 2-d array whose each element is a list of basic
//intervals; Determine the bucket to which a given \(b\) should
//belong
for \(b \in B\) in decreasing order of \text{SPILL}(b)\) do
    cid := getClassId (regclass(b));
    bucket := availBucket[cid];
    Append \(b\) to toColor[bucket][cid];
    if number of elements in toColor[bucket][cid] is higher then
        perClass[cid] then
            availBucket[cid] += 1;

//Assign physical registers as dictated by the 2-d toColor array
for \(i := 0; i < |K|; i++\) do
    for \(j := 0; j < \text{num\_bucket}; j++\) do
        for \(b \in \text{toColor}[i][j]\) do
            findAssignment (b);

Figure 6.14: Greedy heuristic to perform register assignment in the presence of
register classes that prefers to spill new compound intervals in order to maximize
copy removal.
function findAssignment ()

Input : A basic interval $b \in \mathcal{B}$, $\forall b \in \mathcal{B}$ regclass($b$), coalesce graph
\[ G = (V, E = \{E_m \cup E_r\}) \], a set of available physical registers $R$

Output: Find the assignment $\text{reg}(b)$ and spill decision $\text{spilled}(b)$

1. if $R == \emptyset$ or $R \cap \text{regclass}(b) == \emptyset$ then
2. Spill the compound interval corresponding to $b$;
3. return;
4. Initialize $\text{Map}$ for each physical register to 0;
5. for each edge $e = (b_1, b) \in E_m$ do
6. \hspace{1em} if $b_1$ and $b$ do not intersect then
7. \hspace{2em} $p := \text{reg}(b_1)$;
8. \hspace{3em} if $p != \text{null}$ and $p \in R$ and $p \in \text{regclass}(b)$ then
9. \hspace{4em} $\text{Map}(p) := \text{Map}(p) + \mathcal{W}(e)$;
10. \hspace{2em} else if $p == \text{null}$ then
11. \hspace{3em} for $p' \in \text{regclass}(b_1)$ do
12. \hspace{4em} if $p' \in R$ and $p' \in \text{regclass}(b)$ then
13. \hspace{5em} $\text{Map}(p) := \text{Map}(p) + \mathcal{W}(e)$;

14. for each edge $e = (b_1, b) \in E_r$ do
15. \hspace{1em} $p := \text{reg}(b_1)$;
16. \hspace{2em} for $p' \neq p$ in $\text{Map}$ with cost $> 0$ do
17. \hspace{3em} $\text{Map}(p') := \text{Map}(p') - \mathcal{W}(e)$;
18. $\text{ret} := \text{Find p with maximum cost in Map}$;
19. if $\text{ret} == \text{null}$ then
20. \hspace{1em} $\text{ret} := \text{Find a physical register from regclass}(b) \cap R$;
21. if $\text{ret} == \text{null}$ then
22. \hspace{1em} Spill the compound interval corresponding to $b$;
23. \hspace{1em} return;
24. $\text{reg}(b) := \text{ret}$;
25. return $\text{reg}(b)$;

Figure 6.15: Heuristic to choose a physical register that maximizes copy removal.
6.7 Extended Linear Scan (ELS)

In the preceding sections, we have described space-efficient register allocation using the Bipartite Liveness Graph. In this section, we will describe an Extended Linear Scan (ELS) register allocation algorithm that uses even less space than a space-efficient register allocation algorithm. The key point addressed in the Extended Linear Scan is that it does not need an explicit representation for the Bipartite Liveness Graph, but uses $\text{numlive}$ information at interval end points.

Figure 6.16 summarizes our Extended Linear Scan algorithm. Steps 3-8 are the Potential Spill pass. We use the observation that the only interval end points $p$ for which spill decisions need to be made are those for which $\text{numlive}[p] > k$. The heuristic used in Step 4 is to process these interval end points in decreasing order of $\text{FREQ}[q]$. As in Chaitin’s Graph Coloring algorithm, Step 5 selects the compound interval with the smallest spillcost for spilling. A key difference with graph coloring is that this decision is driven by the choice of interval end point $p$, and allows for assigning different physical registers to the same symbolic register at different program points. After Step 3 has completed, a feasible register allocation is obtained with $\text{numlive}[p] \leq k$ at each interval end point $p$. The set of compound intervals selected to be spilled are identified by $\text{spilled}(s) = \text{true}$, and are also pushed onto stack $S$. Steps 9-14 is the Actual Spill pass. It examines the compound intervals pushed on the stack to see if any of them can be “unspilled”. Step 15 is the Select pass. The algorithm for register assignment with register-to-register move and exchange instructions is already discussed in Section 6.5.3.

**Theorem 6.7.1** The ELS algorithm takes $O(|\mathcal{B}| + |\mathcal{C}|)$ space and $O(|\mathcal{B}| + |\mathcal{B}|(\log(\max_c) + \log|\mathcal{B}|))$ time, where $\max_c$ is the maximum value of $\text{numlive}[p]$ at any interval end point $p$.

**Proof:** For Step 3 (Potential Spill), the selection in Step 4 of program point $q$ with $\text{numlive}[q] > k$ and largest estimated frequency, $\text{FREQ}[q]$, contributes $O(|\mathcal{B}| \log|\mathcal{B}|)$
time and Step 5 contributes $O(|B|\log(max_c))$ time, assuming that a heap data structure (or equivalent) is used in both cases. Finally, Step 9 (Actual Spill) and Step 15 (Register Assignment) contribute at most $O(|B|)$ time.

\section*{6.8 Summary}

In this chapter, we addressed the problem of compile time and space efficient register allocation. Most approaches to register allocation involve the construction of an interference graph, which is known from past work to be a major space and time bottleneck \cite{42,105}. A notable exception is the Linear Scan algorithm which is favored by many dynamic and just-in-time compilers because it avoids the overhead of constructing an interference graph. We introduced a new approach to register allocation that improves on the runtime performance delivered by Linear Scan, without exceeding its space bound. To that end, we introduced a \textit{Bipartite Liveness Graph} representation as an alternative foundation to the interference graph. Allocation with the BLG is formulated as an optimization problem and a greedy heuristic is presented to solve it. We also formulated spill-free register assignment combined with move coalescing as a combined optimization problem using the \textit{Coalesce Graph}, which models both IR move instructions and additional register-to-register moves/exchanges arising from register assignment. We then extended the above register allocation and assignment approaches to handle register classes. Experimental evaluation of our proposed allocator is provided in Section 8.2.
function ExtendedLinearScan ()

Input : IR, numlive[p] for every interval end point p, and k uniform physical registers

Output: Set $T \subseteq U$ which needs to be spilled to ensure all interval end points $v \in V$ be unconstrained, i.e., $\forall b \in T, spilled(b) = true$ and if $spilled(b) = false$, then $reg(b)$ specifies the physical register assigned to $b$.

Stack $S := \phi$;

//Potential Spill Selection

while $\exists$ an interval end point $p$ with $numlive[p] > k$ do

$q :=$ choose an interval end point with $numlive[q] > k$ and largest estimated frequency, $FREQ[q]$;

$s :=$ compound interval that is live at $q$, has $spilled(s) = false$, and has the smallest value of $SPILL(s)$;

Set $spilled(s) := true$ and push $s$ on stack $S$;

for each interval end point $x$ where $s$ is live do

$numlive[x] := numlive[x] - 1;$

//Actual Spill Selection

while stack $S$ is non-empty do

$s :=$ pop($T$);

if $numlive[q] < k$ at each interval end point $q$ where $s$ is live then

Set $spilled(s) := false$;

for each interval end point $x$ where $s$ is live do

$numlive[x] := numlive[x] + 1;$

//Register Assignment using Figure 6.7

RegMoveAssignment()

Figure 6.16 : Overview of Extended Linear Scan algorithm (ELS) with all-or-nothing approach
Chapter 7

Bitwidth-aware Register Allocation

Bitwidth-aware Register allocation [116] extends the traditional register allocators by packing subword data values, i.e., data values with narrower width than the standard data width (word size) supported by the underlying processor. The packed data values can be allocated in the same physical register, thereby moderating the register pressure of the program. Various applications, in particular from the embedded domain, make extensive use of sub-word sized values and can benefit significantly by bitwidth-aware register allocation.

In this chapter, we propose three key contributions to bitwidth-aware register allocation: 1) a limit study that compares the Tallam-Gupta bitwidth analysis algorithm [116] with a dynamic profile-driven bitwidth information, and show significant opportunities for enhancements; 2) an enhanced bitwidth analysis algorithm that performs more detailed scalar and array analysis for improved bitwidth information than in Tallam-Gupta; 3) an enhanced packing algorithm that improves packing algorithm of Tallam-Gupta by a new set of safe bitwidth estimates.

Figure 7.1: Overall Bitwidth-aware register allocation framework.
7.1 Overall Bitwidth-aware register allocation

Figure 7.1 depicts the overall flow of a bitwidth-aware register allocation algorithm. The core of a bitwidth-aware register allocation lies in the bitwidth analysis that computes actual width requirements for every variable at every program point. This bitwidth information is used to annotate the edges of the interference graph during the Build phase. Iterative Packing is then applied over the interference graph to pack narrow width live ranges as long as they can be packed onto the same physical register. Additional move instructions are added in the IR for extracting individual variables from the packed variables and finally, the global register allocation algorithm is applied for the variables that includes the new packed variables created in Packing step.

7.2 Limit Study

Our first step in studying bitwidth-aware register allocation was to perform a limit study that compares the bitwidth usage computed by the static compile-time bitwidth analysis algorithm in Tallam-Gupta (also described in Section 2.8.1) with dynamic bitwidth information obtained from an execution profile. The infrastructure used for this study was based on the GCC compiler, as depicted in Figure 7.2. The register allocation phase in GCC was modified to accept input from the box labeled “Bitwidth Analysis”, which can either generate compile-time or profile-driven bitwidth information. Using Tallam-Gupta bitwidth analysis, the width of a variable at a program point can be represented by three parts: a leading part of unused bits \( l \), a middle part of active bits, and a trailing part of unused bits \( t \). We implemented the Tallam-Gupta bitwidth algorithm in the GCC compiler to obtain this information for the compile-time case. For the profile-driven case, we instrumented the code generated by GCC so as to perform a “logical or” of the values dynamically assigned to each variable. The major motivation for performing the limit study is that the prior work by Tallam-Gupta reported static benefits of bitwidth-aware register allocation (fewer
registers used, smaller cliques in the interference graph), but did not provide any indication of what additional opportunities remain for improved bitwidth analysis.

The benchmarks used in our evaluation were all taken from the Bitwise benchmark set [17], so as to be representative of embedded applications. Our evaluation was performed on 9 out of the 15 programs in the full benchmark set. The following five programs were not used because they did not contain a return value, thereby making it possible for GCC to optimize away the entire program as dead code — *bilint, levdurb, motiontest, sha, softfloat*. In addition, the *life* program was not used, because the Bitwise benchmark set already contains a *newlife* program which is very similar to *life*. All experiments were performed using the *-O3* option and the *-param max-unroll-times=0* option\(^1\) with version 4.1 of GCC targeted to the x86 platform.

Table 7.1 lists the total number of variables (symbolic registers) available for

\(^1\)This option disables loop unrolling. Loop unrolling can create more candidates for register allocation, but the relative impact of unrolling depends on the benchmark so it was disabled.
Table 7.1: Comparison of compile-time and profile-driven bitwidth analysis: Number of and percentage of variables with bitwidth less than 32 bits.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Total # variables</th>
<th>Total # and % of variables with variable bitwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Bitwidth analysis)</td>
<td>(Profile-driven)</td>
</tr>
<tr>
<td>adpcm</td>
<td>26</td>
<td>20 (76.92%)</td>
</tr>
<tr>
<td>bubblesort</td>
<td>20</td>
<td>11 (55.00%)</td>
</tr>
<tr>
<td>convolve</td>
<td>8</td>
<td>6 (75.00%)</td>
</tr>
<tr>
<td>edge_detect</td>
<td>107</td>
<td>20 (18.69%)</td>
</tr>
<tr>
<td>histogram</td>
<td>29</td>
<td>16 (55.17%)</td>
</tr>
<tr>
<td>jacobi</td>
<td>36</td>
<td>13 (36.11%)</td>
</tr>
<tr>
<td>median</td>
<td>33</td>
<td>9 (27.27%)</td>
</tr>
<tr>
<td>mpegcorr</td>
<td>30</td>
<td>13 (43.33%)</td>
</tr>
<tr>
<td>newlife</td>
<td>62</td>
<td>19 (30.64%)</td>
</tr>
</tbody>
</table>

register allocation in each benchmark, followed by the number of variables that were identified to have varying bitwidth by static analysis, and next by the number of variables that were identified to have variable bitwidth by profile information. The results in the table indicate that there is opportunity for significant improvement in compile-time bitwidth analysis, compared to the static analysis obtained from the Tallam-Gupta algorithm.

We now introduce another metric called the active compression factor (ACF) to measure the effectiveness of the bit width analysis. Let $AB_{ij}$ denote the number of active bits in register operand $j$ at statement $i$ (obtained either from static analysis or from profile information), and $TB_j$ denote the number of total bits in register operand $j$ (in other words, the statically defined size of $j$). Let $FREQ_i$ denote the dynamic frequency of statement $i$. We define the active compression factor as follows:

$$ACF = \frac{\sum_{i \in \text{INSN}} \sum_{j \in \text{REGOPERAND}} FREQ_i \times TB_j}{\sum_{i \in \text{INSN}} \sum_{j \in \text{REGOPERAND}} FREQ_i \times AB_{ij}}$$

Note that $ACF$ must be $\geq 1$ since $TB_j \geq AB_{ij}$. 


Table 7.2: Active Compression Factor (ACF) comparison across static and profile-driven bitwidth analysis without loop unrolling.

Table 7.2 shows ACF values for the compile-time and profile-driven cases. The same execution profile information is used for the $FREQ_i$ values in both cases – the difference lies in the computation of the $AB_{ij}$ values. A larger ACF value indicates a greater opportunity for bitwidth-aware register allocation. The results in Table 7.2 show ACF values in the range of 1.0 to 1.37 for the compile-time case, and in the range of 1.45 to 3.90 for the profile-driven case. Once again, this shows opportunity for improved bitwidth analysis, compared to the results obtained from the Tallam-Gupta algorithm.

7.3 Enhanced Bitwidth Analysis

We outline two key enhancements that we made to the bitwidth analysis in the Tallam-Gupta algorithm, both of which were motivated by the opportunities identified by the limit study in the previous section.

1. Enhanced Scalar Analysis. The Tallam-Gupta bitwidth analysis includes a forward zero bit section analysis and a backward dead bit section analysis using a data flow framework. The forte of such an approach is that it first forward propagates width information from definition to use and then back propagates them
from use to definition. Although their approach analyzed variables involved in logical operations efficiently, they did not compute accurate width information for variables involved in operations such as arithmetic computations. Consider the programming scenario where a variable is incremented inside a loop by some constant value. If we knew the loop bounds, we can compute the upper bound on the value that can be assigned to the variable. This can be used to decide the useful bits of the variable. This kind of programming scenario is seen very often in Bitwise benchmark set [17].

Bitwidth analysis for variables modified inside loops require a closed-form solution. We extended the bitwidth analysis of Tallam-Gupta with a recurrence analysis that can identify general induction variables and other patterns with closed-form solutions. This is more general than the scalar range analysis presented by [115].

Each closed form expression for a variable $x$ updated inside a loop-nest is represented using a linear chrec that is expressed using $\{base, op, stride\}$ and is evaluated using Newton’s interpolation formula as $\{base, +, stride\}(x) := base + step \times x$. The $base$ represents the value computed outside the loop nest and the $stride$ represents the value added or subtracted in every iteration of the loop. Note that the $stride$ can also be a variable whose bitwidth information is already available or whose closed-form expression can be/is being computed. The idea of using chrec representation is that it can be evaluated quickly using Newton’s interpolation formula. Note that the recurrence analysis can analyze improper loop nests.

Given chrec representation for each statement of a loop nest $l$, the high level algorithm to compute the range of possible values for variables defined in $l$ using SSA-based intermediate representation is provided in [15]. Typically an SSA based intermediate representation is unavailable during register allocation level. We present a recurrence analysis using the Program Dependence Graph
(PDG). The steps are provided in Algorithm 7.3. A cycle in the PDG represents the need for a closed-form solution. Since each statement in the PDG has an associated chrec, we combine multiple chrecs of the statements in the PDG. The combined chrecs can then be evaluated using Newton’s interpolation formula given the base value and the loop bounds.

2. Enhanced Array Analysis. A key limitation of the Tallam-Gupta algorithm is that it performs no analysis of array variables. We added an array range analysis that tracks the values being assigned to arrays, and integrates the array analysis with the enhanced scalar analysis. This enhancement performs a flow-insensitive analysis of all accesses to an array variable.

```verbatim
function RecurrenceAnalysis ()
    Input : IR, loop nest l
    Output: Width of the variables defined inside loop nest l
    Perform renaming in the IR;
    Compute the Program Dependence Graph (PDG);
    for each SCC in PDG do
        Combine the chrecs of the statements in the SCC;
        Collapse the SCC into a single node in the PDG;
        Perform a topological sort of nodes in the PDG and propagate the chrecs;
        Using the upper and lower bounds of l and the initial values of variables outside l, evaluate the combined chrecs;
    
    Figure 7.3 : Recurrence analysis for computing bitwidth information of variables accessed inside loops.
```

We use two code examples to illustrate the benefits of these two enhancements, and how they are used in conjunction with each other. Figure 7.4 contains a code fragment from the Bitwise adpcm benchmark. While it may not be standard practice in general, it is common practice in embedded applications for loop iterations to be bounded by compile-time constants defined in the program. When analyzing the
#define  NSAMPLES  2407
...
int  sbuf[NSAMPLES];
...
for (i=0; i<NSAMPLES; i++) {
    sbuf[i]=i & 0xFFFF;
}
...
for (i = 0; i < NSAMPLES; i++) {
    val = sbuf[i];
    ...
}
...

Figure 7.4: Code fragment from BITWISE adpcm benchmark.

...
for (i=0; i<SIZE/2; i++) {
    sortlist_even[i] = (SIZE-(i << 1)) | (1 << (WIDTH-1));
    sortlist_odd[i] = (SIZE-((i << 1) | 1)) | (1 << (WIDTH-1));
}
for (top=SIZE-1; top>0; top--) {
    for (i=0; i<top; i++) {
        io = i >> 1;
        ie = io + (i & 1);
        s1=sortlist_even[ie];
        s2=sortlist_odd[io];
        if (s1 > s2 ^ (i & 1)) {
            sortlist_even[ie] = s2;
            sortlist_odd[io] = s1;
        }
    }
}
...

Figure 7.5: Code fragment from BITWISE bubblesort benchmark.
expression, \( i \ & \ 0xFFFF \), our enhanced scalar analysis determines that variable \( i \) must be in the range, \( 0 \ldots 2406 \). Further, the constant \( 0xFFFF \) value has a bitwidth of 16 bits. Hence, each element assigned to the \( \text{sbuf} \) array has a lower bound of 0, and an upper bound of \( \min(2406, 65535) = 2406 \), or a maximum bitwidth of 12 bits (The \( \min \) function is applied when a bitwise-and operator is being analyzed.). Scalar variable \( \text{val} \) is then bounded by a maximum bitwidth of 12 bits.

Figure 7.5 contains a code fragment from the Bitwise bubblesort benchmark. There are two static definitions each for arrays \( \text{sortlist\_even} \) and \( \text{sortlist\_odd} \). However, the values of \( \text{s2} \) and \( \text{s1} \) that appear in the right-hand-side of the second pair of definitions originate from the same arrays. Therefore, our array analysis determines that the bitwidth of the array elements must be bounded by their initial definition, \( i.e. \), 17 bits.

### 7.4 Enhanced Packing

In this section, we outline improvements in the variable packing heuristic used in the bitwidth-aware register allocation. Figure 7.8 contains a summary of the Tallam-Gupta bitwidth-aware register allocation algorithm. The key step that implements the packing heuristic is Step 7. Packing heuristic plays a significant role on the effectiveness of bitwidth-aware register allocation. Note that the packing step in bitwidth-aware register allocation is different from coalescing in classical register allocation. In classical register allocation, two non-interfering variables can be coalesced so as to use the same physical register. In contrast, packing permits two interfering variables to be combined provided the sum of their bitwidths does not exceed the register word size.

We now discuss three key characteristics of the Tallam-Gupta algorithm, and outline how they were extended/replaced in our algorithm:

1. Packing is performed \( \text{conservatively} \) in the Tallam-Gupta algorithm, \( i.e. \), packing is restricted to cases when the node created by packing two nodes has fewer
1: A := ...
2: B := ...
3: ... := A & 0x1fff // 17 bits needed
4: C := ...
5: ... := B & 0xff // 8 bits needed
6: ... := C & 0xff // 8 bits needed
7: D := true
8: ... := A & 0x7fff // 15 bits needed
9: ... := D // 1 bit needed

Figure 7.6: Example program for demonstrating imprecision in Tallam-Gupta packing

Figure 7.7: Interference Graph for Example program shown in Figure 7.6 that shows overly conservative estimation of EMIW width information using Tallam-Gupta approach.

than \( k \) neighbors with degree of \( k \) or more (where \( k \) is the number of registers available for allocation). Even though this guarantees that the colorability of the interference graph is not increased, our experimental results (described in Section 8.3) show that this restriction is too conservative in many cases, so we use aggressive approach proposed by [35].

2. As described in Figure 2.16, if nodes \( A \) and \( B \) are packed, and both have an edge to another node, \( C \), Tallam-Gupta conservatively estimates the new label for
the edge from the new packed node, $AB$ to $C$, using $EMIW$. We observe that the $EMIW$ estimates used in the Tallam-Gupta algorithm can result in edge labels that are overly conservative, thereby precluding some possible packing opportunities. Let us consider the example program shown in Figure 7.6 and its corresponding interference graph with edge labeling shown in Figure 7.7. Using Tallam-Gupta approach described as $E_{int}$ in Figure 2.16, the $EMIW$ of the cycle $ABC$ can be computed as $(25, 8)$ since $E_A = 17 + 8 + 8 = 33$, $E_B = 8 + 15 + 8 = 29$, and $E_C = 8 + 17 + 8 = 33$ imply $E_{int} = 33$. This will prevent packing $A$, $B$, and $C$ to a single physical register since the combined width exceeds the 32-bit size of a physical register that is assumed in this example. Note that $D$ can be packed with $A$ using Tallam-Gupta heuristic shown in Figure 2.15. However, if we look at the code in Figure 7.6, the $MIW(A, B, C) = 29$. It should have been possible to pack $A$, $B$, $C$, and $D$ to a single physical register. This indicates the imprecision of their approach. We will describe a new set of $EMIW$ estimates in the next section to improve the precision of packing.

3. The priority function for live ranges used in the Tallam-Gupta algorithm for selecting nodes in Step 3 of Figure 7.8 is defined as follows:

$$
Priority(lr) = \frac{\text{Estimated Load/Store Savings}}{\text{Live Range Area}}
$$

$$
= \frac{\text{Estimated Load/Store Savings}}{\sum_p \text{width}(lr, p)}
$$

(7.1) (7.2)

However, our experience has shown that this priority function often favors short-lived live ranges which have a small area, even though they may not offer a large savings in load/store instructions. Our enhancement was to remove the denominator term in the priority function, so that all live ranges are prioritized (largest-first) according to the estimated absolute load/store savings.
function BitwidthAwareRegisterAllocation ()
    Input : IR
    Output: Transformed IR with register allocation
    Construct the interference graph IG;
    Label IG edges with width information due to interferences;
    W:=Construct a prioritized node list for the nodes in IG;
    while W ≠ φ do
        Get a node, say n, from prioritized node list;
        for each node a in n’s neighbor do
            //Packing heuristic
            Attempt packing a with n;
            if packing is successful then
                Update IG with a new packed node and edges;
                W := W - {n};
        Replace each packed variable set with a new name in IR;
        Introduce new intra-variable moves in IR;
        Perform graph coloring register allocation and assignment;
    return modified IR

Figure 7.8 : Bitwidth aware register allocation in a graph coloring scenario.
7.4.1 Improved $EMIW$ estimates

As mentioned in (2) above, the update of edge labels after variable packing can be overly conservative in the Tallam-Gupta algorithm. Tallam-Gupta uses edge labels in the interference graph to compute $EMIW$ estimates to approximate the $MIW$ information. Our observation (motivated by the example shown in Figure 7.7) is that the nodes in the interference graph can also be annotated with their maximum width information that can be used in conjunction with edge labeling to obtain more precise $EMIW$ estimates. For the example program shown in Figure 7.7, the $MIW$ of variables $A$ and $C$ is $15 + 8 = 23$. To this, if we add 8, which is the maximum width of $B$ across all program points, we will get $EMIW(A, B, C) = 21 + 8 = 29 = MIW(A, B, C)$, which is precise than Tallam-Gupta’s $EMIW(A, B, C) = 33$.

Let $NODEMAX$ denotes the maximum width of a variable across all program points. Using $NODEMAX$ for every variable in the interference graph and Tallam-Gupta’s edge labeling, we present a set of new $E_1$, $E_2$, $E_3$, $E_4$, $E_5$ and $E_6$ estimates for $EMIW$ as shown in Table 7.3. These $EMIW$ estimates are used in conjunction with the equations provided in Figure 2.16. The safety of the new precise estimates trivially follow from the intermediate value theorem of Tallam-Gupta and is described in [11].

7.5 Summary

In this chapter, we studied the problem of enhancing bitwidth-aware register allocation. Our limit study showed significant opportunities for improvement, compared to the algorithm pioneered by Tallam and Gupta. The enhanced bitwidth analysis that performs more detailed scalar analysis and array analysis results in improved bitwidth information than in Tallam and Gupta. The enhanced packing algorithm that performs less conservative (more aggressive) coalescing than in [116]. Also, the proposed improved $EMIW$ estimates result in improving the precision of packing algorithm. Section 8.3 reports experimental evaluation of our proposed enhancements.
\( E_1 = A_b + B_a + \text{NODEMAX}(C) \)
\( E_2 = A_c + C_a + \text{NODEMAX}(B) \)
\( E_3 = C_b + B_c + \text{NODEMAX}(A) \)
\( E_4 = B_a + C_a + \max(A_b, A_c), \text{ if } E_{\text{min}} = E_A \text{ and } E_4 \geq E_{\text{min}} \)
\( E_5 = A_b + C_b + \max(B_a, B_c), \text{ if } E_{\text{min}} = E_B \text{ and } E_5 \geq E_{\text{min}} \)
\( E_6 = A_c + B_c + \max(C_a, C_b), \text{ if } E_{\text{min}} = E_C \text{ and } E_6 \geq E_{\text{min}} \)

\[
\text{EMIW}(A, B, C) = \min(E_{\text{int}}, E_1, E_2, E_3, E_4, E_5, E_6)
\]

\[
(AB_c, C_{ab}) = (A_b + B_a, \text{NODEMAX}(C)) \text{ if } \text{EMIW}(A, B, C) = E_1
\]
\[
(AB_c, C_{ab}) = (A_c + \text{NODEMAX}(B), C_a) \text{ if } \text{EMIW}(A, B, C) = E_2
\]
\[
(AB_c, C_{ab}) = (B_c + \text{NODEMAX}(A), C_b) \text{ if } \text{EMIW}(A, B, C) = E_3
\]
\[
(AB_c, C_{ab}) = (B_a + \max(A_b, A_c), C_a) \text{ if } \text{EMIW}(A, B, C) = E_4
\]
\[
(AB_c, C_{ab}) = (A_b + \max(B_a, B_c), C_b) \text{ if } \text{EMIW}(A, B, C) = E_5
\]
\[
(AB_c, C_{ab}) = (A_c + B_c, \max(C_a, C_b)) \text{ if } \text{EMIW}(A, B, C) = E_6
\]

Table 7.3 : New EMIW estimates for variable packing using NODEMAX.

In future, we would like to study the overhead of bitwidth-aware register allocation in terms of the number of extra instructions added for packing and unpacking, the effect on run-time performance and energy reduction. The idea of variable packing can be used for modern architectures which provide vector physical registers to pack scalar values. For example, the Intel x86 SSE2 extension provides sixteen 128-bit physical registers which can be used to pack several 32-bit integer values to address the bandwidth bottlenecks in multi-core processors.
Chapter 8

Performance Results

In this chapter, we report on our experimental evaluation for the Side-effect Analysis (described in Chapter 4), Scalar replacement for Load Elimination (described in Chapter 5), Space-efficient Register Allocation (described in Chapter 6) and Bitwidth-aware Register Allocation (described in Chapter 7). We use two compiler infrastructures to demonstrate the effectiveness of our techniques, Jikes RVM [66] and GCC [56]. Scalar replacement techniques presented in this thesis were evaluated in a Jikes RVM dynamic compilation environment for HJ programs. The register allocation algorithms were evaluated both in Jikes RVM and GCC. Finally, the enhancements to Bitwidth-aware register allocation were evaluated in GCC alone since the standard set of benchmarks exposing bitwidth characteristics were written mostly in C programming language.

8.1 Side-Effect Analysis and Load Elimination

We present an experimental evaluation of the scalar replacement for load elimination algorithm introduced in Chapter 4 and Chapter 5 for a set of programs written in the subset of HJ consisting of the async, finish and isolated parallel constructs.

8.1.1 Experimental setup

The performance results were obtained using Jikes RVM 3.0.0 [66] on a 16-core system that has four 2.40GHz quad-core Intel Xeon processors running Red-Hat Linux (RHEL 5). The system has 30GB of memory.

For our experimental evaluation, we use the production configuration of Jikes RVM
with the following options: `-X:aos:initial Compiler=opt -X:irc:O0`. By default, Jikes RVM does not enable SSA based HIR optimizations like scalar replacement for load elimination transformation at optimization level O0. We modified Jikes RVM to enable the SSA and load elimination phases at O0. However, since the focus of our transformation is on optimizing application classes, the boot image was built with scalar replacement for load elimination turned off and the same boot image was used for all execution runs reported. The set of optimizations performed at O0 are: copy propagation, constant propagation, common subexpression elimination, inline allocation of scalar, inlining of statically resolved methods and linear scan register allocation. The `ParallelSideEffectAnalysis` procedure presented in Figure 4.7 was implemented as an HIR optimization pass in the `OptimizationPlanner`, and the new scalar replacement for load elimination algorithm from Figure 5.3 was implemented as an extension to the existing load elimination algorithm in Jikes RVM based on the FKS algorithm [52].

All results were obtained using the `-Xmx2000M` JVM option to limit the heap size to 2GB, thereby ensuring that the memory requirement for our experiments was well below the available memory on the 16-core Intel Xeon SMP. The `PLOS_FRAC` variable in `Plan.java` was set to 0.4f for all runs, to ensure that the Large Object Size (LOS) was large enough to accommodate all benchmarks. The main program was extended with a five-iteration loop within the same Java process for all JVM runs, and the best of the five times was reported in each case. This approach was chosen to reduce the impact of dynamic compilation time and other virtual machine services in the performance comparisons.

For our experiments, we used the five largest HJ programs that we could find — three Section 3 Java Grande Forum (JGF) benchmarks (Moldyn, RayTracer, Montecarlo) and two NAS Parallel (NPB) benchmarks (CG and MG). All JGF benchmarks were run with the largest data size available. Sizes “A” and “W” were used for CG and MG respectively, to ensure completion in a reasonable amount of
time. For all executions we set the NUMBER_OF_LOCAL_PLACES runtime option for HJ to 1 to obtain a single-place configuration, and also set INIT_THREADS_PER_PLACE to the number of worker threads \( k \) used in the evaluation. All executions used the work-sharing HJ v1.5 runtime scheduling system described in [13].

The five HJ benchmarks listed above use finish and async constructs, but not isolated. To evaluate our optimization in the presence of isolated constructs, we created a hybrid HJ+Java version of SPECjbb2000 benchmark that uses the async, finish and isolated constructs from HJ, but also retains the CyclicBarrier.await() construct from Java (which was modeled as an unknown method call in our analysis).

### 8.1.2 Experimental results

All the benchmarks we used offer many scalar replacement for load elimination opportunities across method calls and parallel constructs due to several usage of field accesses. Table 8.1 depicts the static count of various parallel constructs in the benchmarks. MolDyn, CG, and MG benchmarks create a large number of small tasks and await for their completion within an outer loop. Both the smaller tasks and the outer loop access several object fields which can be eliminated by our scalar replacement for load elimination approach. For Montecarlo and RayTracer benchmarks there is no outer loop and uses relatively fewer parallelization constructs. SPECjbb2000 offers opportunities for code motion around isolated constructs as these constructs are spread all around the source code.

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th># of async</th>
<th># of finish</th>
<th># of isolated</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG-A</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>MG-W</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Moldyn-B</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SPECjbb2000</td>
<td>1</td>
<td>1</td>
<td>169</td>
</tr>
</tbody>
</table>

Table 8.1: Static count of parallel constructs in various benchmarks.
We perform two additional compiler transformations that create more opportunities for scalar replacement and improved register allocation:

1. **Loop-invariant getfield code motion pre-pass**: In general, a loop-invariant `getfield` operation cannot be moved out of a loop since it may throw a `NullPointerException`. To address this case, we perform the standard transformation of replacing a while loop by a zero-trip test and a repeat-until loop so as to enable loop-invariant code motion of `getfield` operations while still preserving exception semantics. This transformation is performed as a pre-pass to scalar replacement for load elimination. We use the side-effect analysis described in Chapter 4 for method calls inside the loop to determine if a `getfield` operation is loop-invariant.

2. **Live-range splitting post-pass**: a potential negative impact of scalar replacement is that increasing the size of live-ranges can lead to increased register pressure. This in turn may cause a performance degradation if the register allocator does not perform live-range splitting. Since the Linear Scan register allocator in Jikes RVM currently does not split live-ranges, we introduce a live-range splitting pass after load elimination that only splits live-ranges of the scalar temporaries introduced by our optimizations. The live-ranges of these scalars are split around all call instructions and loop entry-exit regions. This creates smaller scalar live-ranges for which spilling and register assignment decisions can be made separately. However, in some cases, this benefit can be undone by the register allocator if it decides to coalesce the live ranges back before allocation.

Experimental results are reported for the following cases:

1. **1-thread NOLOADELIM** – Baseline measurement with no load elimination and a single worker thread;
Table 8.2: Compilation times in milliseconds of various Jikes RVM passes for NPB benchmarks (CG and MG), JGF benchmarks (Moldyn, Raytracer, and Montecarlo) and SPECjbb2000 benchmark using NO LOADELIM, FKS LOADELIM, and FKS+TRANS LOADELIM cases.

2. *k*-thread FKS LOADELIM – use of the FKS load elimination algorithm [52] with no side effect analysis and *k* worker threads.

3. *k*-thread FKS+TRANS LOADELIM – use of the FKS load elimination algorithm [52] with the two transformation passes described in the paragraph above but with no side effect analysis, and *k* worker threads.

4. *k*-thread PAR LOADELIM – use of the extended parallelism-aware scalar replacement for load elimination algorithm from Figure 5.3 with side effect analysis and *k* worker threads.

5. *k*-thread PAR+TRANS LOADELIM – use of the extended parallelism-aware scalar replacement for load elimination algorithm from Figure 5.3 combined with the two transformation passes described in the previous paragraph and *k* worker threads.

In this study, the results for 2), 3), 4), and 5) were restricted to the elimination of `getfield` operations only. Extension of these results for array-load operations is a subject for future work.
Table 8.3: Compilation times in milliseconds of various Jikes RVM passes for NPB benchmarks (CG and MG), JGF benchmarks (Moldyn, Raytracer, and Montecarlo) and SPECjbb2000 benchmark using PAR LOADELIM and PAR+TRANS LOADELIM cases.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>PAR+TRANS LOADELIM</th>
<th>PAR LOADELIM</th>
<th>Total Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>side-effect time in ms</td>
<td>ssa+loadelim time in ms</td>
<td>trans time in ms</td>
</tr>
<tr>
<td>CG-A</td>
<td>102</td>
<td>398</td>
<td>84</td>
</tr>
<tr>
<td>MG-W</td>
<td>131</td>
<td>442</td>
<td>110</td>
</tr>
<tr>
<td>Moldyn-B</td>
<td>76</td>
<td>255</td>
<td>47</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>77</td>
<td>246</td>
<td>44</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>90</td>
<td>253</td>
<td>44</td>
</tr>
<tr>
<td>SPECjbb2000</td>
<td>580</td>
<td>1153</td>
<td>329</td>
</tr>
</tbody>
</table>

Table 8.2 and 8.3 report the compile-time results for various Jikes RVM passes. The total compilation time for PAR+TRANS LOADELIM is on average 1.38× slower than FKS+TRANS LOADELIM and ranges from 1.22× (for SPECjbb2000) to 1.47× (for Montecarlo). This modest increase in compile-time establishes that the side-effect analysis based load elimination algorithm introduced in this thesis is practical for use in dynamic compilation.

Table 8.4 and 8.5 report the dynamic number of GETFIELD operations for different scalar replacement for load elimination algorithms. The second column in the table specifies the total number of GETFIELD operations in the original program. The third and fourth columns report the remaining number of GETFIELD operations in the program after using FKS LOADELIM and FKS+TRANS LOADELIM algorithms respectively. Similarly, the fifth and sixth column report the remaining number of GETFIELD operations using PAR LOADELIM and PAR+TRANS LOADELIM algorithms respectively. (Since we’re only counting dynamic GETFIELD operations in Table 8.4 and 8.5, the live range splitting post-pass in TRANS has no impact on these results.)

We observe that PAR+TRANS LOADELIM reduces the dynamic GETFIELD
<table>
<thead>
<tr>
<th>Benchmark</th>
<th># getfield (original)</th>
<th># getfield after FKS load elim.</th>
<th># getfield after FKS+TRANS load elim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG-A</td>
<td>3.89E09</td>
<td>3.10E09</td>
<td>3.03E09</td>
</tr>
<tr>
<td>MG-W</td>
<td>1.41E04</td>
<td>1.15E04</td>
<td>1.13E04</td>
</tr>
<tr>
<td>MolDyn-B</td>
<td>1.19E10</td>
<td>7.91E09</td>
<td>5.82E09</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>3.08E10</td>
<td>2.02E10</td>
<td>2.02E10</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>1.75E09</td>
<td>1.54E09</td>
<td>1.48E09</td>
</tr>
<tr>
<td>SPECjbb2000</td>
<td>1.19E09</td>
<td>1.025E09</td>
<td>8.95E08</td>
</tr>
</tbody>
</table>

Table 8.4: Dynamic counts of GETFIELD operations for NPB benchmarks (CG and MG), JGF benchmarks (MolDyn, Raytracer, and Montecarlo) and SPECjbb2000 benchmark using FKS LOADELIM and FKS+TRANS LOADELIM cases.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th># getfield after PAR load elim.</th>
<th># getfield after PAR+TRANS load elim.</th>
<th>impr. rel. to FKS+TRANS (%age)</th>
<th>impr. rel. to FKS (%age)</th>
<th>impr. rel. to original (%age)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG-A</td>
<td>2.34E09</td>
<td>3.92E05</td>
<td>99.99 %</td>
<td>99.99 %</td>
<td>99.99 %</td>
</tr>
<tr>
<td>MG-W</td>
<td>7.96E03</td>
<td>6.71E03</td>
<td>40.58 %</td>
<td>41.72 %</td>
<td>52.55 %</td>
</tr>
<tr>
<td>MolDyn-B</td>
<td>4.91E09</td>
<td>3.11E09</td>
<td>46.49 %</td>
<td>60.62 %</td>
<td>73.89 %</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>1.67E10</td>
<td>1.38E10</td>
<td>31.82 %</td>
<td>31.93 %</td>
<td>55.25 %</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>1.15E09</td>
<td>9.19E08</td>
<td>37.95 %</td>
<td>40.47 %</td>
<td>47.38 %</td>
</tr>
<tr>
<td>SPECjbb2000</td>
<td>6.65E08</td>
<td>5.78E+08</td>
<td>35.44 %</td>
<td>43.19 %</td>
<td>51.56 %</td>
</tr>
</tbody>
</table>

Table 8.5: Dynamic counts of GETFIELD operations for NPB benchmarks (CG and MG), JGF benchmarks (MolDyn, Raytracer, and Montecarlo) and SPECjbb2000 benchmark using PAR and PAR+TRANS LOADELIM cases. The improvements of PAR+TRANS LOADELIM with respect to original, FKS, and FKS+TRANS LOADELIM cases are presented in the last three columns.
counts for all benchmarks in the range of 31.93% for Raytracer and 99.99% in CG compared to FKS LOADELIM (shown in column 8). With respect to total GETFIELD operations (column 2), PAR+TRANS LOADELIM reduces the dynamic counts in the range of 47.38% for Montecarlo and 99.99% in CG (shown in column 9). For the CG benchmark, the dominant method in terms of execution time is step0. In the absence of our side effect analysis, load elimination for this function was limited due to the presence of a function call inside the inner loop.

Figure 8.1 presents the relative performance improvements of the three parallel Section 3 Java Grande benchmarks and the two Nas Parallel benchmarks\(^1\) with respect to the 1-thread NO LOADELIM case. For the 1-thread case, we observe an average of 1.29× performance improvement of PAR+TRANS LOADELIM in comparison to the FKS LOADELIM case, with a best-case 1.76× improvement (for Moldyn). While comparing with FKS+TRANS LOADELIM, PAR+TRANS LOADELIM yields an average improvement of 1.20× with best-case 1.32× improvement (for Moldyn).

For the 16-thread case, the parallelism-aware scalar replacement for load elimination algorithm in Figure 5.3 including the two optimizations (PAR+TRANS LOADELIM Thread=16) resulted in a 1.15× improvement over the FKS intraprocedural approach without optimizations, on average. For the MolDyn benchmark, we achieved a maximum of 1.39× improvement. When we compare against FKS with optimizations, on average PAR+TRANS LOADELIM Thread=16 resulted in a 1.11× improvement with best-case 1.20× improvement for Moldyn. Three of the five benchmarks (CG, MolDyn, and Montecarlo) show measurable speedup with the use of PAR LOADELIM, whereas for the remaining two (MG and Raytracer) there was no measurable speedup. Using live-range splitting as part of PAR+TRANS LOADELIM, we can see that both MG and Raytracer do not degrade performance. We believe that a

\(^1\)For SPECjbb2000, we haven’t as yet obtained a measurable difference in runtime due to the reduction in dynamic getfields shown in Table 8.4 and 8.5, because of the inability of the HJ v1.5 work-sharing runtime to work efficiently with await() calls from Java. In the future, we plan to extend our scalar replacement for load elimination algorithm to support phasers [107] which can be used as a replacement for the await() calls.
Figure 8.1: Performance improvement for NPB benchmarks (CG and MG) and JGF Benchmarks (Moldyn, Raytracer, and Montecarlo) using the scalar replacement for load elimination algorithm presented in Figure 5.3. The improvement is shown relative to the 1-thread NO LOADELIM case.
Figure 8.2 : Scaling of JGF Section 3 MolDyn Size B benchmark using the scalar replacement for load elimination algorithm introduced in Figure 5.3. The speedup is shown relative to the 1-thread *NO LOADELIM* case.

Figure 8.3 : Scaling of NPB CG Size A benchmark using the scalar replacement for load elimination algorithm introduced in Figure 5.3. The speedup is shown relative to the 1-thread *NO LOADELIM* case.
live-range splitting based register allocator could further improve the performance results reported in this thesis. Figure 8.2 and 8.3 show the speedup details for Mol-Dyn and CG benchmarks as the number of workers \( k \) increases for \( PAR+TRANS LOADELIM \).

### 8.2 Space-Efficient Register Allocation

We present an experimental evaluation of the space-efficient register allocation algorithm introduced in Chapter 6 for the SPECint2000 integer benchmark suite in GCC static compiler and the Java Grande Forum serial benchmarks in Jikes RVM dynamic compiler.

#### 8.2.1 GCC Evaluation

We report on experimental results obtained from a prototype implementation of Graph Coloring (as described in [89]) and Extended Linear Scan register allocator (as described in Section 6.7 of Chapter 6) in GCC compiler.

#### 8.2.1.1 Experimental setup

We used version 4.1 of the GCC compiler using the -03 option. Compile-time and execution time were measured on a POWER5 processor running at 1.9GHz with 31.7GB of real memory running AIX 5.3.

Experimental results are presented for eight out of twelve programs from v2 of the SPECint2000 benchmark suite. Results were not obtained for 252.eon because it is a C++ benchmark, and for the three other benchmarks — 176.gcc, 253.perlbmk, and 255.vortex — because of known issues [111] that require benchmark modification or installation of v3 of the CPU2000 benchmarks.
Table 8.6: Compile-time overheads for functions with the largest interference graphs in SPECint2000 benchmarks. \(|S| = \#\) symbolic registers, \(|IG| = \#\) nodes and edges in Interference Graph, \(|B| = \#\) intervals in interval set, Space Compression Factor (SCF) = \(|B|/|IG|\), GC = graph coloring compile-time, ELS = Extended Linear Scan with register-to-register move and exchange instructions.

| Function                  | \(|S|\) | \(|IG|\) | \(|B|\) | SCF   | GC    | ELS   |
|----------------------------|--------|--------|--------|-------|-------|-------|
| 164.gzip.build_tree        | 161    | 2301   | 261    | 11.3% | 141.4ms| 9.4ms |
| 175.vpr.try_route          | 254    | 2380   | 445    | 18.7% | 208.7ms| 9.5ms |
| 181.mcf.sort_basket        | 138    | 949    | 226    | 22.7% | 6.8ms  | 0.1ms |
| 186.crafty.InputMove       | 122    | 1004   | 219    | 21.8% | 150.2ms| 7.8s  |
| 197.parser.list_links      | 352    | 9090   | 414    | 4.5%  | 114.4ms| 7.4ms |
| 254.gap.SyFgets            | 547    | 7661   | 922    | 12.0% | 118.8ms| 8.0ms |
| 256.bzip2.sendMTFValues    | 256    | 2426   | 430    | 17.7% | 133.0ms| 7.4ms |
| 300.twolf.closepins        | 227    | 5105   | 503    | 9.8%  | 212.8ms| 9.1ms |

Figure 8.4: SPEC rates for Graph Coloring and ELS register Allocator described in Section 6.7.
8.2.1.2 Experimental results

Table 8.6 summarizes compile-time overheads of the Graph Coloring and ELS register allocation algorithm. The measurements were obtained for functions with the largest interference graphs in the eight SPECint2000 benchmarks, using the `-O3 -finline-limit=3000 -ftime-report` options in gcc. It is interesting to note that the Interference Graph size, $|IG|$, typically grows as $O(|S|^{1.5})$, whereas the number of intervals, $|B|$ is always $\leq 2|S|$. This is one of the important reasons behind the compile-time efficiency of the Linear Scan and ELS register allocation algorithms.

While it is theoretically possible for the number of intervals for a symbolic register to be as high as half the total number of instructions in the program (e.g., if every alternate instruction is a “hole” – which could lead to a non-linear complexity for the ELS register allocator), we see that in practice the average number of intervals per symbolic register is bounded by a small constant ($\approx 2$). We see that the Space Compression Factor (SCF) = $|B|/|IG|$ varies from 4.5% to 22.7%, indicating the extent to which we expect the interval set, $B$ to be smaller than the interference graph, IG. Finally, the last two columns contain the compile-time spent in global register allocation for these two algorithms. For improved measurement accuracy, the register allocation phase was repeated 100 times, and the timing (in ms) reported in Table 8.6 is the average over the 100 runs. While compile-time measurements depend significantly on the engineering of the algorithm implementations, the early indications are there is a marked reduction in compile-time when moving from GC to ELS register allocation for all benchmarks. The compile-time speedups for ELS register allocator relative to GC varied from 15× to 68×, with an overall speedup of 18.5× when adding all the compile-times.

Figure 8.4 shows the SPEC rates obtained for the Graph Coloring and ELS register allocation algorithms, using the `-O3` option in GCC. Recall that a larger SPEC rate indicates better performance. In summary, the runtime performance improved by up to 5.8% for the ELS register allocator relative to GC (for 197.parser), with an
average improvement of 2.3%. There was only one case in which a small performance
degradation was observed for the ELS register allocator, relative to GC — a slowdown
of 1.4% for 181.mcf. These results clearly show that the compile-time benefits for
Extended Linear Scan can be obtained without sacrificing runtime performance —
in fact, ELS register allocator delivers a net improvement in runtime performance
relative to GC. Further, the results indicate that the extra register-to-register moves
did not contribute a significant performance degradation.

8.2.2 Jikes RVM evaluation

We present an experimental evaluation of the Bipartite Liveness Graph (BLG) based
constrained register allocation and assignment algorithms presented in Section 6.6 of
Chapter 6.

8.2.2.1 Experimental setup

The experimental setup uses Jikes RVM 3.0.0 [66] dynamic compiler on an Intel
Xeon 2.4GHz system with 30GB of memory and running Red-Hat Linux (RHEL 5).
We used the serial version of the Java Grande Form (JGF) benchmark suite [65] to
evaluate the performance of our register allocator.

The serial programs in the JGF benchmark suite comprises of seven Section 2
benchmarks (Crypt, Heapsort, Sparsematmult, Sor, Series, LUFact, and FFT)
and five larger Section 3 benchmarks (Raytracer, Moldyn, Montecarlo, Euler, and
Search). Of these, Jikes RVM was unable to execute the FFT benchmark due to VM
errors, so we present results for the remaining eleven. Further, the execution times
were obtained for the Section 2 benchmarks at optimization level 02 and for Section 3
benchmarks at optimization level 00. (Jikes RVM was unable to execute Section 3
benchmarks at a higher optimization level than 00 due to compilation errors.) The
boot image for Jikes RVM used a production configuration with a modification to
PLOS_FRAC that was set to 0.4f to ensure that Jikes RVM had a Large Object Space
(LOS) that was large enough for these benchmarks. The execution times reported were the best of three runs within a single JVM instance for each benchmark.

Since the Jikes RVM release did not support generation of the Intel exchange instruction, we modified its assembler to add this support. Jikes RVM uses SSE registers for storing double/floating point values. However, to the best of our knowledge, there does not exist a direct exchange instruction to swap values in SSE registers, so we used three xor instructions to exchange a pair of float/double values.

### 8.2.2.2 Experimental results

Table 8.7 reports the benchmarks that used register moves and those that used register exchange operation. We can see that all the benchmarks use register-to-register move instructions. All Section 3 benchmarks and the LUFact benchmark used register-to-register exchange instructions. This suggests that larger methods offer more opportunities for generation of exchange instructions than smaller methods.

Figure 8.5 reports the relative speedup of our register allocator with that of the existing linear scan register allocator in Jikes RVM. Our register allocator resulted in

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Reg-to-Reg</th>
<th>Reg-to-Reg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crypt-C</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Heapsort-C</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Sparsematmult-C</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Sor-C</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Series-B</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>LUFact-C</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Moldyn-B</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>Euler-B</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Search-B</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Table 8.7: Benchmarks for which register-to-register move and register exchange instructions were generated.
a performance improvement in the range of 1.00× to 3.52×. The largest improvement of 3.52× was obtained for Moldyn, with improvements of 1.42× and 1.15× for Sor and Euler respectively. In no case did BLG deliver worse performance than LS. These results demonstrate that the Bipartite Liveness Graph based register allocation algorithm can deliver convincing runtime performance improvements relative to Linear Scan.

Table 8.8 compares the compile time overhead of ELS and LS register allocation algorithms. Since ELS separates allocation and assignment into two separate passes (as opposed to LS that performs both in a single pass) and also includes the option of adding register-to-register moves, the compile-time of ELS was observed to be between 2 to 3 times slower than LS. As the execution times for these benchmarks are in the order of tens of seconds, we believe this increase in compile-time for the
<table>
<thead>
<tr>
<th>Benchmark</th>
<th>LS Compile-time in ms</th>
<th>ELS Compile-time in ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crypt-C</td>
<td>24</td>
<td>68</td>
</tr>
<tr>
<td>Heapsort-C</td>
<td>19</td>
<td>41</td>
</tr>
<tr>
<td>Sparsematmult-C</td>
<td>19</td>
<td>45</td>
</tr>
<tr>
<td>Sor-C</td>
<td>19</td>
<td>44</td>
</tr>
<tr>
<td>Series-B</td>
<td>19</td>
<td>49</td>
</tr>
<tr>
<td>LUFact-C</td>
<td>22</td>
<td>51</td>
</tr>
<tr>
<td>Raytracer-B</td>
<td>35</td>
<td>101</td>
</tr>
<tr>
<td>Moldyn-B</td>
<td>47</td>
<td>114</td>
</tr>
<tr>
<td>Montecarlo-B</td>
<td>23</td>
<td>70</td>
</tr>
<tr>
<td>Euler-B</td>
<td>92</td>
<td>267</td>
</tr>
<tr>
<td>Search-B</td>
<td>23</td>
<td>56</td>
</tr>
</tbody>
</table>

Table 8.8: Compile-time comparison of ELS with LS in Jikes RVM

margin of performance improvement achieved is acceptable, in general.

8.3 Bitwidth-Aware Register Allocation

We report on experimental results obtained from our prototype implementation of bitwidth-aware register allocation based on GCC.

8.3.1 Experimental setup

Figure 8.6 depicts how the bitwidth-aware register allocator is inserted into the phases of the GCC compiler. A standard graph coloring register allocator [35] was used instead of GCC’s local and global register allocator. Note that we now have three options for Bitwidth Analysis — the Tallam-Gupta [116] algorithm, enhanced bitwidth analysis, and profile-driven information. The enhanced analysis results were obtained by our implementation of the enhanced scalar and array analysis outlined in Section 7.3. Also, there are two options for Variable Packing — the Tallam-Gupta algorithm or the enhanced packing algorithm outlined in Section 7.4.

The experimental results reported in this section will be used to compare five
different cases:

1. Bitwidth-Unaware — a standard graph coloring algorithm is used with no support for bitwidth-aware register allocation.

2. + Bitwidth-Aware — enhancement of the previous case by using the Tallam-Gupta bitwidth-aware register allocation.

3. + Enhanced Packing — addition of the enhanced packing techniques introduced in Section 7.4.

4. + Enhanced Bitwidth — addition of the enhanced scalar and array bitwidth analysis techniques introduced in Section 7.3.

5. + Profiled Bitwidth — like the previous case, but with profiled bitwidth information from the limit study used instead of statically analyzed bitwidth information.

The benchmark programs being used in this section are the same as those that were used for the limit study described in Section 7.2.

As can be seen in Figure 8.6, the same register allocator based on graph coloring is used in all cases. Therefore, the only way for the bitwidth-aware heuristics to demonstrate an improvement compared to bitwidth-unaware allocation, is for the heuristics to perform some packing of nodes.

### 8.3.2 Experimental results

Table 8.9 reports the number of node-pairs packed when processing all nine benchmark programs for number of available registers 8. Note that the packing pre-pass for Tallam-Gupta depends on the number of available registers (conservative coalescing) whereas our modified approach does not (aggressive coalescing). The results show that our combined heuristic (Case 4 above) performs significantly more packing than the Tallam-Gupta algorithm.
### Table 8.9: Comparison of number of packed node-pairs with different levels of bit-sensitive register allocation for the number of available physical registers = 8.

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th>Bitwidth-Aware (Tallam-Gupta)</th>
<th>+ Enhanced Packing</th>
<th>+ Enhanced Bitwidth</th>
<th>+ Profiled Bitwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>adpcm</td>
<td>0</td>
<td>7</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>bubblesort</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>convolve</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>edge_detect</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>64</td>
</tr>
<tr>
<td>histogram</td>
<td>1</td>
<td>1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>jacobi</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>median</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>mpegcorr</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>newlife</td>
<td>0</td>
<td>2</td>
<td>40</td>
<td>41</td>
</tr>
</tbody>
</table>

### Figure 8.6: GCC modification for register allocation
Next, Table 8.10 compares the number of dynamic load/store instructions arising from register spills for the five different cases. Each row represents the case for a certain number of available registers, and each entry represents the sum of the dynamic load/store spill instructions for the nine benchmarks.

As seen in Table 8.10, the Tallam-Gupta algorithm had zero impact on reducing the number of dynamic load/store spill instructions, for the cases studied, and essentially yielded the same dynamic spill load/store instruction count as the bitwidth-unaware. However, the techniques introduced in Chapter 7 (cases 3 and 4 above) reduced the dynamic spill load/store instruction count to 3% to 27% of the bitwidth-unaware case. This is a significant reduction.

### 8.4 Summary

This chapter provides an experimental evaluation of the memory access optimization techniques described in this dissertation. The scalar replacement for load elimination transformation described Chapter 5 show decreases in dynamic counts for GETFIELD operations of up to 99.99%, and performance improvements of up to 1.76× on 1
core, and 1.39× on 16 cores, when compared to the load elimination algorithm available in Jikes RVM. A prototype implementation of our BLG register allocation phase combined with the constrained assignment in Jikes RVM demonstrates runtime performance improvements of up to 3.52× relative to the Linear Scan on an x86 processor. An evaluation of our Extended Linear Scan register allocator in GCC show that the compile-time speedups for ELS relative to GC were significant, and varied from 15× to 68×. In addition, the resulting execution time improved by up to 5.8%, with an average improvement of 2.3% on a POWER5 processor. Finally, the enhancements to bitwidth-aware register allocation described in Chapter 7 can reduce the number of dynamic spill load/store instructions to between 3% and 27%.

The experimental evaluations combined with the foundations presented in this dissertation, we strongly believe that the proposed high-level and low-level optimizations are useful in addressing some of the new challenges emerging in efficient optimization of parallel programs for multi-core architectures.
Chapter 9

Conclusions and Future Work

In this dissertation, we have presented a combination of high-level and low-level compiler analyses and optimizations to address the Memory Wall problem in multicore architectures. The high level analyses include May-Happen-in-Parallel (MHP) analysis and Side-Effect Analysis for any language that adopts the core concepts of places, async, finish, and isolated from the HJ programming model. The low level optimizations include Scalar replacement for Load Elimination and Register Allocation.

We introduced a new algorithm for May-Happen-in-Parallel (MHP) analysis for HJ programming model. The main contributions of this work compared to past MHP analysis algorithms are as follows:

1. We introduced a more precise definition of the MHP relation than in past work by adding condition vectors that identify execution instances for which the MHP relation holds, instead of just returning a single true/false value for all pairs of executing instances.

2. Compared to past work, the availability of basic concurrency control constructs such as async and finish enabled the use of more efficient and precise analysis algorithms based on simple path traversals in the Program Structure Tree, and did not rely on interprocedural pointer alias analysis of thread objects as in MHP analysis for the Java language.

3. We introduced place equivalence (PE) analysis to identify execution instances that happen at the same place. The PE analysis helps us in leveraging the fact...
that two statement instances which occur in atomic sections that execute at the same X10 place must have $MHP = \text{false}$.

We introduced an interprocedural scalar replacement for load elimination algorithm for dynamic optimization of parallel programs. The main contributions of our work include: a) side-effect analysis of method calls, b) support for scalar replacement for load elimination in the presence of three core parallel constructs – async, finish, and isolated, c) an IC memory model that establishes the legality of our load elimination transformation for parallel constructs, and d) performance results to study the impact of scalar replacement on a set of standard HJ parallel programs. Our performance results show decreases in dynamic counts for getfield operations of up to 99.99%, and performance improvements of up to $1.76 \times$ on 1 core, and $1.39 \times$ on 16 cores, when comparing the algorithm in this paper with the load elimination algorithm available in Jikes RVM. The algorithm has been implemented in Jikes RVM for optimizing a subset of HJ parallel programs.

We addressed the problem of space-efficient register allocation. Most approaches to register allocation involve the construction of an interference graph, which is known from past work to be a major space and time bottleneck [42, 105]. A notable exception is the Linear Scan algorithm which is favored by many dynamic and just-in-time compilers because it avoids the overhead of constructing an interference graph. In this thesis, we introduced a new approach to register allocation that improves on the runtime performance delivered by Linear Scan, without exceeding its space bound. To that end, we introduced a Bipartite Liveness Graph representation as an alternative foundation to the interference graph. Allocation with the BLG is formulated as an optimization problem and a greedy heuristic is presented to solve it. We also formulated spill-free register assignment combined with move coalescing as a combined optimization problem using the Coalesce Graph, which models both IR move instructions and additional register-to-register moves/exchanges arising from register assignment. We then extended the above register allocation and assignment
approaches to handle register classes. Our experimental results for 11 serial Java Grande benchmarks compared our \textit{BLG} based register allocation with that of the existing Linear Scan (\textit{LS}) register allocator in Jikes RVM. The results show that a \textit{BLG} based register allocation can achieve runtime benefits of up to $3.52 \times$ compared to \textit{LS}.

We studied the problem of enhancing bitwidth-aware register allocation. Our \textit{limit study} showed significant opportunities for improvement, compared to the algorithm pioneered by Tallam-Gupta. We used our prototype implementation of bitwidth-aware register allocation in \textit{gcc} to compare the dynamic number of load/store instructions) resulting from a) \textit{bitwidth-unaware} allocation, b) \textit{bitwidth-aware} allocation, c) \textit{enhanced bitwidth-aware} allocation with improved bitwidth analysis and improved packing, and d) \textit{ideal profile-driven bitwidth-aware} allocation. Our results show that our enhancements can reduce the dynamic number of spill load/store instructions to 3\% to 27\% of the number obtained from the Tallam-Gupta algorithm.

\section*{9.1 Future Work}

The May-Happen-in-Parallel analysis presented in this dissertation can be enriched using distance vectors and can be applied in an interprocedural context. The same can also be improved to handle other synchronization constructs of \textit{HJ} including \textit{phasers} and \textit{delayed async}. Possible directions for future work for scalar replacement include improving the precision of our analysis using \textit{MHP} analysis. Also, our techniques can be implemented for array accesses that go beyond simple field accesses. Directions for future work in space-efficient register allocation include further study of the trade-off between register-move instructions and spill load/store instructions, and support for partial spill using live-range splitting. The bitwidth aware register allocation needs to study the overhead of bit-aware register allocation (number of extra instructions added), effect on run-time performance and energy reduction.
Bibliography


